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ABSTRACT

This manual was designed for use in conducting staff development sessions for elementary teachers of mathematics in Mississippi in grades K-6. The four topical areas treated in the document are: (1) measurement and geometry; (2) fractions; (3) procedural errors in arithmetic; and (4) problem solving. The number of instructional hours necessary for completing the activities for each topic varies from 2 to 12 hours. The entire instructional program may require from 40-60 hours, depending upon the skill and knowledge base of the participating teachers. The manual was written as an instruction guide for a staff development leader. Each topic begins with a brief overview followed by a series of instructional activities. For each activity, information for the leaders is listed in summary form, followed by the instructional materials to be used, and a description of advanced preparation. Various types of teaching aids, worksheets, record sheets, activity cards, and transparency masters are incorporated into the guide. Since most of the instructional activities require the hands-on manipulation of materials, a maximum of 30 teachers per workshop is recommended. (TW)

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MATHEMATICS / SCIENCE

GRADES K - 6

MACTE

MISSISSIPPI ASSOCIATION
OF COLLEGES FOR
TEACHER EDUCATION, INC.

1987

REVISION

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PREFACE

This manual was originally designed and has been revised for use in conducting staff development sessions for elementary school teachers of mathematics in grades K-6. In the development of the manual, mathematics was viewed not simply as a body of knowledge, but as an ongoing process in which everyone engages as he attemps to organize and systematize objects and events encountered in his environment. Strategies for teaching mathematics which reflect this view and are soundly based upon research related to the cognitive development of children are emphasized. The intent was to model in these staff development activities, wherever possible, exemplary teaching practices for elementary school mathematics. For these reasons, a heavy emphasis was placed on hands-on mathematics activities, social interaction in small groups, and teacher-guided discussion to summarize, clarify, and accurately verbalize the abstraction and/or correct errors in thinking.

Four topics are treated in the manual, Measurement and Geometry, Fractions, Procedural Errors in Arithmetic, and Problem Solving. The number of instructional hours necessary for completing the activities for each topic varies from two-twelve hours. The entire instructional program may require from 40-60 hours, depending upon the skill and knowledge base of the participating teachers.

The manual was written as an instructional guide for a staff development leader. Each topic begins with a brief overview followed by a series of instructional activities. For each activity, the workshop leader's behaviors are listed in summary form followed by the instructional materials to be used and a description of advance preparation. Various types of teaching aids, worksheets, record sheets, activity cards, and masters for transparencies, are incorporated in the manual for each topic. Since most of the instructional activities require the hands-on manipulation of materials, a maximum of thirty teachers per workshop session is recommended.

The manual addresses areas of need in mathematics teacher education identified by mathematics educators across Mississippi, grade school through higher education levels. Mathematics well taught in the elementary grades builds the thinking skills, conceptual mental structures, and positive attitudes necessary for the study of more abstract mathematical principles needed in a world of advanced technology. If somehow this staff development program promotes the teaching of mathematics in the elementary grades more effectively, we, as educators, can rejoice in a job well done.

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FOR TEACHER EDUCATION (MACTE)
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MEASUREMENT AND GEOMETRY

Dr. Jean M. Shaw

MEASUREMENT AND GEOMETRY

OVERVIEW AND RATIONALE

Can you glance across a room and tell how many feet wide and long the room is? Can you look at or lift a piece of meat in the grocery store and tell how heavy it is without reading the label? Do you associate temperatures and feelings? For example, if it's 50°F outside, do you know whether you'd want to wear gloves or not? If it were 40°C outdoors, do you know what it would feel like? Whether or not you'd be successful at answering the above questions, you are thinking about some important measurement ideas.

Measurement is widely used in everyday life. When we buy almost anything--from food to clothing, from land to hobby equipment--we buy it measured or sized according to some sort of measurement. It is impossible to imagine modern life without measurement. Because of its everyday life applications, knowledge of measurement is important to elementary students.

Even in primitive society crude measurements--usually comparisons--were used. An object was compared with another object. For instance, a rabbit was smaller than a deer, but a mouse was smaller than a rabbit. When the need for more accurate measurements developed, people established units of measure for common use. Lengths related to dimensions of the human body were often used. A cubit was from the elbow to the tip of the middle finger; a palm was the width of the hand; a foot was equal to a typical man's foot; and a fathom was from the tip of one middle finger to another with outstretched arms.

Unstandardized units of measurement were used for a long time; some such as a pinch of salt or a handful of sand are still in use. Such measurements are not very reliable, however, because they vary with different people. Gradually, standardized measurements developed and were defined by specific standards and adopted by governmental law.

The metric system or Systeme International de Units (SI), is in usage world-wide. The United States is "going metric" gradually too. Scientists and medical personnel have long used the metric system for measuring and weighing. It has the advantages of related units, units based on physical entities, and most importantly, is easy to use because it is based on the decimal system and uses tens, hundreds, and thousands to compare units.

Because teachers are probably less familiar with the metric system than the traditional English system, your work in this session will concentrate on the metric system. The ideas of building relationships around concrete objects, learning to measure by actually measuring, and developing a "metric sense" can be applied to the learning of the English as well as the metric system.

Geometry too, is important in everyday life. We are surrounded by geometric shapes--some solid or three-dimensional, some flat or plane figures and some points. Shapes vary in their sizes and proportion. For effective communication, children need to be able to describe, compare represent, relate, and name shapes they encounter in their environment. Effective teaching of basic geometric ideas also sets the stage for subsequent, more formal work in geometry.

Building understanding of measurement and geometry requires that students have experiences with real materials. Students learn to measure by actually measuring, not by discussion of measurement alone. They learn about the relationships of geometric shapes by handling and talking about models, drawing and constructing figures, and finding examples of geometric figures in the environment. Hands-on activities are vital supplements to study of textbook material and worksheets. As students deepen their understandings of measurement and geometry concepts and sharpen their skills, positive attitudes of interest, confidence, and appreciation of shapes and sizes encountered in everyday life are developed.

Measurement and geometry are long-accepted topics in the elementary curriculum. They are not topics that should be taught only when time permits. Indeed, the topics can often be used profitably to provide a break from computation. Measurement and geometry are intrinsically interesting to children. Study of the topics provides opportunities for group work and problem solving. Awareness of the topics carries over into everyday life and increases children's awareness of shapes and sizes in the environment. Mastery of measurement and geometry vocabulary allows children to be precise in their use of language both in school and out. Lets' get started on our study of these important topics.

BEHAVIORAL OBJECTIVES FOR WORKSHOP PARTICIPANTS.

Workshop participants will:

1. know background information on measurement and geometry and principles for teaching the topics.
2. participate in learning activities for measurement and geometry.
3. discuss the content and skills developed by the activities as well as the activities themselves.
4. understand principles for teaching the topics in this section.

WORKSHOP LEADER BEHAVIORS.

Workshop leaders will:

1. provide an overview of measurement content for grades K-6.
2. discuss principles for planning and carrying out measurement activities.
3. present information on the nature of measurement.
4. discuss teaching both metric and customary measurement.
5. present an overview of measurement learning activities.
6. supervise participants as they work with the activities.

7. lead a discussion of the activities after they have been completed. (This will complete your work on measurement.)
8. discuss geometry content for grades K-6.
9. present principles for teaching geometry.
10. present an overview of geometry learning activities.
11. supervise participants as they work with activities.
12. lead a discussion of the geometry activities. (This will complete your work on geometry.)
13. discuss ideas for implementation of training materials in local educational agencies. (This concludes the training session.)

PREPARING FOR THE WORKSHOP.

Prepare and gather the following materials for the workshop. Carefully read the section on workshop content and activities.

Transparencies MG-1 through MG-10 (for measurement)
MG-12 and M-13 (for geometry)

Handouts MG-11 (for measurement)
MG-31 (for geometry)
MG-40 (for home applications)

Patterns MG-12 through MG-30 (for measurement)
MG-32 through MG-39 (for geometry)

Materials for Measurement (These are listed by activity. You will want to set them out in different areas around the room.) Materials are listed for each 35 participants.

Child-Made Rulers and Tapes - 40 copies of pattern MG-14, scissors, crayons, transparent tape.
ID Cards - 40 note cards, rulers, transparent tape, scissors.
Length Graph - 12 sheets construction paper, scissors, 12 sheets plain paper, glue, rulers.
Temperature Scavenger Hunt - 6 thermometers, 7 scavenger hunt lists (MG-18).
Thermometer Model - 8 poster boards, 40 patterns (MG-20), scissors, 12 m (about 12 yards) 1 cm wide elastic (½ inch), 2 red permanent markers.
Metric Me - 20 copies of patterns MG-16 and MG-17, 5 tape measures, 3 meter sticks.
Playdough - 750 ml (3 cups) flour, 1 box salt, 1 box cooking alum, 1 small bottle cooking oil, food coloring, coffeepot for heating water, plastic bag to store dough, 2 measuring cups, 1 set measuring spoons, large bowl, wooden stirring spoon, recipe (MG-25).
Popcorn Volumes - 200 ml popcorn, 60 ml oil (if popper requires it), corn popper, salt (optional), measuring cups, hot pads, large bowl, cleanup materials, napkins.

Draw and Measure Volumes - slips of paper with volumes marked (MG-26)

Calibrated Cup - 20 transparent plastic cups, scissors, masking tape, source of water (or water in one gallon jug and an empty jug for pouring waste), six 15 or 30 ml spoons or small calibrated cups.

Use Your Calibrated Cup -

Making a Balance Scale - (Make just 3 scales for demonstration) 3 half-gallon milk cartons, 2 l sand, 1 coathanger, wire cutters or sturdy scissors, 3 rulers, 6 clothespins, 6 styrofoam cups, 2 paper clips, yarn.

Using the Balance Scale #1 and #2 - Small objects (participants can also furnish some), gram weight chart (pattern MG-27).

Mass of My . . . - 3 or 4 scales (if possible, use different types—a balance scale, step-on scale, spring scale, and/or fisherman's scale would be appropriate), chart (MG-30).

Summary of Materials for Measurement. Materials were listed by activity because you will set them up in separate stations. To help you gather equipment, the materials are also listed below in categories.

School Supplies

15-20 pairs of scissors (5 activities use scissors)
3 rolls transparent tape
40 note cards
8 paper clips
12 sheets construction paper
3 red permanent markers
2 boxes crayons
2 rolls masking tape
5 sheets plain paper
8 poster boards

Measuring Equipment

15-20 rulers (2 activities use rulers)
3 rulers with prepunched holes
6 thermometers
6 measuring cups
12 measuring spoons (you will need at least six 15 or 30 ml spoons or small calibrated cups)
1 set gram weights or 1 set of plastic cubic centimeters that weight 1 gm each
3 or 4 scales of different types

Groceries

Popcorn
Napkins
1 small bottle cooking oil
salt 1 pound (.45kg)

Groceries (continued)

- Koolaid mix
- Rice, 2 pounds (908 grams)
- Flour, 2 pounds (908 grams)
- Small container powdered alum
- 40 transparent plastic cups

Kitchen

- Corn popper
- Large bowl
- Cleanup supplies (cloth, towel, detergent)
- 1 pitcher
- 2 boxes or bowls for dry rice
- 2 empty gallon jugs if a water supply is not available in classroom
- Coathanger
- 6 clothespins
- Wire cutters or sturdy scissors
- 3 empty half-gallon milk containers

Miscellaneous

- Small objects to measure in grams
- 12m (about 12 yards) of 1cm (½ inch) wide elastic
- 2 l sand (or salt)
- Small box to put slips of paper in

Patterns

- 35 copies MG-14 (or enough for all participants)
- 6 copies MG-18 (for group use)
- 35 copies MG-20
- 1 copy MG-27
- 1 copy MG-30
- 35 copies MG-16
- 35 copies MG-17

Materials for Geometry (These are listed by activity so you can set them up in different areas of the classroom.)

Alike and Different - Wood or plastic geometric solids; common objects to represent rectangular prisms, cones, spheres, cylinders, drawings of geometric shapes on small cards (MG-32).

Geoboards - 3 or 4 geoboards, colored rubber bands, task cards (MG-33).

Closed Curves with Yarn - Flannel Board or 2 or 3 carpet squares, pieces of yarn cut 10-40 cm, scissors, geometric name cards (MG-34).

Models of Geometric Solids - Scissors, glue, paper, patterns (MG-34 a-c).

Geometric Solids of Clay - Clay (or playdough), plastic knives, wood or plastic models to copy, paper, scissors.

Folding Angles - Paper, rulers, pencils.

Choose It, Draw It - Names of lines, segments, angles (MG-36), paper, pencils, rulers.

Tangrams - Pieces and puzzles (MG-38).

Yarn Perimeters - Yarn, scissors, meter sticks, rulers.

Area Pictures - Grid paper (MG-39), crayons, paper.

Area of My Hand - Grid paper (MG-39), crayons, paper.

Summary of Materials for Measurement - To help you gather all the geometry supplies, use the following categorized list.

School Supplies

6-9 pairs scissors (3 activities use scissors)
Flannel Board or 2-3 carpet squares
2 sets geometric solids (cube, sphere, pyramid, cone, rectangular prism, etc.)
Clay or playdough
30 sheets plain paper
6-9 rulers (3 activities use rulers)
Pencils
2-3 bottles glue
2-3 meter sticks
2-4 boxes crayons
3-4 geoboards
Colored rubber bands

Miscellaneous

Common objects to represent rectangular prisms, spheres, cylinders, pyramids
20 pieces yarn cut 10-40 cm
5-6 plastic knives

Patterns

1 copy MG-32
1 copy MG-34
20 copies MG-34 a-c
1 copy MG-36
20 copies MG-38
40 copies MG-39

WORKSHOP CONTENT AND ACTIVITIES

Introduce the measurement and geometry session using remarks from the overview and rationale. Point out that measurement and geometry are two of ten areas selected by the National Council of Teachers of Mathematics as basic skill areas for K-12 mathematics programs. Measurement and geometry are interesting, pertinent, vital areas of elementary mathematics. They are areas that must be taught and taught well. They are not areas to "get around to once computation is mastered." Teachers must do a good job of teaching measurement and geometry. They may profit from some of the suggestions in this workshop.

Use the following comments as a guide for a brief lecture on some measurement ideas. Show the appropriate transparencies as your speak.

1. Measurement Content for Grades K-6

In the elementary grades students deal with length, volume temperatures and mass or weight (Transparency MG-1). Before they are ready to work formally with these ideas, they need a background of premeasurement concepts including work with comparative words such as long, short, wide, narrow, heavy, light, hot, cold, large, small, more, less, and equal (Transparency MG-2).

To measure length, students learn to use rulers, meter and yard sticks, and tape measures (Transparency MG-3). In kindergarten, children can do much comparison and informal work with measuring tools. At the first grade level, children are introduced to using rulers independently and measuring objects to the nearest inch or centimeter; at subsequent grade levels they measure in smaller gradations such as half and quarter inches and millimeters. By the fifth grade, students identify relationship between meters and centimeters and between yards, feet, and inches. As extensions of the study of length, perimeter and area are often included. For this workshop, we will deal with perimeter and area in the geometry section.

Students learn to read containers for measuring volume and identify the relationship between units of volume such as cups, pints, and quarts, and millimeters and liters (Transparency MG-4). Work with measuring volume and reading values marked on containers can begin in kindergarten in an informal way. Mastery is usually expected by the fourth grade level.

Informal work can also begin early on reading thermometers and associating temperatures with certain activities (Transparency MG-5). For example, most people need to wear a jacket when the temperature registers below 15°C; water freezes at 0°C. Children should master reading of thermometers to the nearest degree by the fifth grade level.

Work with mass or weight parallels that of measures of volume (Transparency MG-6). Children can use kilograms and pounds or grams and ounces early in their school careers. Formal work with reading scales begins at about the fourth grade level. Children also learn the relationships of grams to kilograms and ounces to pounds in the elementary school.

Along with using and reading measurement tools, students are expected to learn about appropriate units for different purposes (Transparency MG-7). To measure long distances, kilometers or miles are appropriate, whereas to measure the length of small objects such as books, centimeters or inches may be the most appropriate units. Students should also be made aware of the constant use of measurement in the world about them so they can see that the skills they are learning are worthwhile and applicable to real life.

2. Principles for Teaching Measurement. (Transparency MG-8). When planning measurement activities for elementary students, educators can be guided by the following principles:

- Help children learn by doing. Children need hands-on experiences if they are to be successful with measurement ideas and skills. Most elementary children are in the concrete operational stage of development. They must approach learning in a real, concrete way.

- Use the "Guess and Check" or "Estimate and Verify" method. Learners should constantly estimate the quantities they are to measure. They then actually measure and check themselves. Making an estimate of each quantity forces a "thinking step" before children use a measuring tool. As children estimate, they make a small "investment" in knowing what the correct measurement is; they become motivated to find out what the measurement is.

- Compare, use nonstandard measures, then standard measures. Within each type of measurement children should first make gross comparisons. For length, for example, they could see that one child is taller than another, or that two tables are about the same height. Children should next use units of measurement that they devise. (How many hands long is the desk? How many steps from Joey's desk to the teacher's chair?). Finally, pupils should use standard units of measurement such as centimeters, pounds, and degrees.

- Assist children in forming mental images of measurements. Mental pictures of measurements help learners to understand the quantities they are dealing with. Children must build up associations of quantities and their measurement names. If children know that the typical temperature in their classroom during warm weather is 25°C, they can compare other temperatures to room temperature with understanding. If fourth graders can remember that an average kindergartener is about a meter tall whereas a typical professional basketball player is two meters tall, they can judge other heights against these mental images.

- Involve students in making some measurement tools. Children learn much about units of measurement and their relationships as they help to prepare their own measuring tools. Even first graders can make simple rulers and measuring tapes. Fourth graders can make thermometer-like devices and balance scales. As children make measurement tools, the number of available devices in the classroom increases with very little cost! A sense of pride in making usable tools also results. In schools where measurement equipment is minimal, making measurement tools is a necessity!

- Think of most measurement lessons in terms of prelab, laboratory, and postlab experiences. Before hands-on work, children should know what they are to do and can receive necessary background and motivation for measurement experiences. In the laboratory portion of a measurement lesson, students actually use measuring tools and usually record data. During this phase, the teacher needs to circulate among the students to keep them on task. The teacher can also note problem areas for subsequent discussion. Lab time isn't play time! It is a time for concept development application of thinking skills, and problem solving. Lab time is essential. After hands-on experiences, students can share results, check themselves, and make generalizations in a postlab session. Students could also complete data sheets to hand in for checking by the teacher.

- Plan some home activities and communications with parents concerning measurement activities. Home-school communication enhances any topic under study, but is especially simple with measurement. Students can use measuring equipment they have made at home. They can gather data at home to share with their classmates. Students can find uses of measurement in their homes. As students work with the metric system, most parents will be glad to know that children are mastering some of the basics of this internationally-used system. We will provide a handout of home study suggestions later in this session.

3. The Nature of Measurement. Measurement has several aspects which must be understood by teachers if they are to correctly convey information to students. (Transparency MG-9)

   - Measurement may be discrete or continuous. Discrete measurement involves the use of separate or discrete variables and usually attaches integer values to quantities. The cost of stamps, number of people in a family, or the number of apples in a bag are examples of discrete measurements. Continuous measurement may be thought of as values on an uninterrupted scale. Examples of continuous measurements are heights, blood pressure, temperature, and time.

   - Continuous measurements are always approximate. No matter what unit of measurement is used, it is always possible to select a second unit of measurement smaller than the original one. Hence, it is not possible to be "exact" because one can always choose a smaller, "more exact" unit. Educators should avoid statements such as "It's exactly a meter long." instead saying, "It's very close to a meter."

- Precision of measurement depends on both tools and those who use them. A ruler calibrated in millimeters is more precise than one calibrated in centimeters for example. A scale that measures grams is more precise than one calibrated only in kilograms and half-kilograms. Homemede measuring tools are generally not as precise as ones produced more expensively by machines. Precision in measurement is attained gradually by people. As children refine their measurement skills, they are typically internally motivated to increase their precision. In measuring small objects like pencils and crayons, errors of about a centimeter may be acceptable for first graders; fifth graders, however, may be expected to measure small objects to the nearest millimeter. Different degrees of precision are appropriate for different purposes. In buying clothes we may be interested in knowing a waist measurement to the nearest inch or centimeter. Even though the measurement could be measured more precisely, a greater degree of precision is not necessary.

- Mass or weight? The distinction between mass and weight is a technical one. Although it may not interest children, conscientious teachers may want to be aware of it. Mass is the amount of matter in a material. Weight is a force; it expresses the pull of gravity on objects. Objects of equal masses will balance on a scale. Astronauts weigh less on the moon than the earth because of lesser gravitational pull on the moon, but their masses are the same in both places.

4. Metric or Customary Measurement? (Transparency MG-10) Todays' children must learn to deal with both feet and centimeters, degrees Celsius and Farenheit, grams and ounces. Both metric and customary units abound in the real world. Although the metric system is used world wide for trade and science, many customary units are in use in the United States. The metric system, based on tens, complements children's growing understanding of place value. Work with customary units provides some practice with fractions. The National Council of Teachers of Mathematics endorses the use of the metric system, but much advertising uses customary units. Children, therefore, need an understanding of both systems.

To build concepts and mental images of units of measurements, it may be best to complete a small section of work on the metric system; then come back a week or so later, and complete a small section on customary units. Confusion often results when pupils have to deal with side-by-side pages featuring both metric and customary units.

Conversions between systems are to be avoided, especially for elementary students. Youngsters in grades K-6 do not have the technical skills to convert from one system to the other, nor should they have to. Rather, they should work with one system, using its measuring tools, then later work with the other system and its appropriate tools.

5. Measurement Learning Activities. (Point out handouts MG-11 in written materials). Draw the participant's attention to the following points:

   - Each activity card contains instructions for participants on the front. On the back of each card are objectives, approximate grade level, a list of materials, and teaching notes where pertinent.

   - The activity cards may be reproduced and given to individual children or groups of children to eliminate lots of time being spent on verbal instructions. When they use task cards, children feel independent and responsible.

   - Task cards may also be placed in learning centers or stations to provide instructions for learners. This is the procedure we will use today.

   - In the room are two or three activities for each of the following areas: length, temperature, volume, and mass. Work as many activities as you can, but be sure you work with at least one or two activities in each area. The activities that we will work with today are starred on the handout. Other activities are for future use in elementary classrooms.

   - For work with children you would probably provide just two or three different activities on the same topic, such as volume. You might have children rotate from one activity to another within a given time frame.

   - Getting organized, as we're doing now, is the prelab phase. As we work of course we'll be doing the laboratory phase. We'll sum up in about an hour as our postlaboratory phase.

   Divide participants into pairs. Assign about one-fourth of the pairs to start with the length activities, one-fourth with the temperature activities and so on. Tell participants to work activities, then move on at their own pace. As participants work, circulate among them. See who needs help or encouragement. Make note of good observations or questions for the postlab discussion.

6. Discussing the Measurement Activities. Lead a discussion of the following points:

   - Specific questions about particular activities.
   - Activities that promote particular skills or concepts--start by asking for concepts or skills developed by a particular activity.

- Setting up the activities for maximum student involvement in elementary classrooms.
- Ask participants to share suggestions for classroom management. Here you may introduce some topics such as ways of assigning small group work, moving groups from one activity to another, maintaining an appropriate noise level, and sharing results.
- Ask participants to discuss activities that illustrate principles for teaching measurement particularly well. (Use transparency MG-8 to recap list of principles, transparency MG-12 recaps geometry principles.)
- Participants' reactions to the activities.
- Extensions of the activities for home use.
- Other points participants want to raise.

7. Geometry Content for Grades K-6. Before they work with traditional geometric content, kindergarteners and first graders must deal extensively with positional and relational words such as over, under, inside, outside, right, left, above, and below (Transparency MG-12a). They then work with plane figures such as circles, triangles, squares, and rectangles. In grades 2 and 3 children are expected to master the names of some solid figures--spheres, cubes, cylinders, cones, pyramids, and rectangular prisms. In grades 4,5 and 6, children explore the relationships of lines and segments and learn the terms parallel, perpendicular, and intersecting. They deal with symmetry and axes of symmetry. Upper grades youngsters also develop and use the formulas for area and perimeter of rectangles. They are introduced to terms for circles: radius, diameter and center, and also work with different sizes of angles.

8. Principles for Teaching Geometry in the Elementary School. Several principles should guide our teaching of geometry in the elementary school. Many of these principles are complementary to those that guided us in teaching measurement (Transparency MG-12b).

   - Base study on exploration. Involve children in handling, selecting, drawing, and describing the geometric figures you are studying. Build children's understanding before they are asked to memorize terms. Children need to know more about geometric figures than their names!

   - Be sure that children use 4 kinds of models for geometric figures. Use real world models. As you introduce parallel lines, ask children to point out representations of parallel lines in the classroom and to look for more representations at home. Use of examples children encounter daily deepens their understanding and lets them see geometry as a useful, pertinent, interesting topic. Use classroom manipulatives such as blocks, flannel boards, geoboards, and cardboard figures. Illustrate geometric content yourself. Be sure that children also handle the classroom manipulatives! Have children make models of the geometric figures that illustrate the relationships that you are studying. Children can trace, draw, and construct their own models. The step is essential if children are to internalize geometry content. Make children familiar with conventional symbols and names for geometric figures also. Use symbols in your work at the chalkboard and in learning centers. Have children label their own models using conventional symbols.

- Focus activities on skills where children gather and organize data. You might focus on these skills: (Transparency MG-13)
  - Listing attributes of geometric figures--Are figures flat? Do they have straight sides? Are they closed? and so on.
  - Comparing geometric figures--Are they alike? How? How are they different?
  - Identifying results as geometric figures undergo change--What happens when figures are folded? Cut apart and rearranged? Flipped? Rotated?
  - Identifying representations of geometric figures--Children name models and pictures of geometric shapes.
  - Describing relationships among geometric figures--Classifying figures according to commonalities, seeing patterns in groups of figures, exploring relationships of area and perimeter are examples of this skill.

- Require mastery of vocabulary only after extensive hands-on experience. (Back to Transparency MG-12b). Teachers should model correct geometry vocabulary, but realize that children need many experiences before they can be consistently precise in their use of words.

- Relate geometry and measurement topics wherever possible. Coordination of the two topics is not difficult. As objects are measured, explore their properties and geometric names.

9. Geometry Learning Activities. (Point out handout MG-31). Set up a workshop situation similar to the one conducted on measurement. Again, let participants work in pairs and rotate through the activities that are starred in Handout MG-31. They should complete at least one or two activities in each of these areas:

   Plane and solid figures
   Relationships
   Lines, Points, Planes, Angles
   Symmetry, Congruence, Similarity
   Transformations
   Perimeter and Area

   (As you know, perimeter and area are often considered measurement topics as well as geometry topics.)

10. Discussing the Geometry Activities. Lead a discussion similar to the one on measurement activities. Distribute handout MG-40 for more ideas for home applications. Let participants brainstorm some more ideas for students to do at home.

11. Implementation of Training for Teaching Measurement and Geometry in Local Educational Agencies. Discuss the suggestions for implementing the workshop materials in local educational agencies with participants. The following suggestions include timing, staffing, features of the written materials, and follow-up activities. Have participants note the reference section at the end of this section. Ask participants to make other suggestions for effective follow-up activities for inservice education.

Workshop leaders may wish to implement the workshop as inservice training in a full-day session similar to the one conducted in your training. If workshop leaders need to break the material into smaller segments, they could conduct two half-day workshops, presenting the measurement material one half-day and the geometry material in another half-day. The material could be broken into even smaller segments of one and one-half to two hours as follows:

Introduction to measurement and activities on length
Activities on temperature, volume and mass
Introduction to geometry and activities on polygons
Remainder of geometry activities

The workshop material could be presented to K-6 teachers as an entire group, or teachers could be broken into K-2 or K-3 and 3-6 and 4-6 subgroups.

In presenting the workshops in local educational agencies, it may be appropriate or convenient to have the lecture/discussion introductory sessions done by the person who attended this training or the staff development coordinator or the district math specialist, supervisor, or coordinator. The training leader needs to be in charge, but may designate other colleagues to assume responsibilities. For example, individual teachers could prepare materials for learning centers or specific activities. The parts of sessions that include work with teaching/learning materials could be handled by grade level teachers. If teachers prepare and gather materials for use in inservice training, they should be allowed to keep these materials for future classroom use.

Before training is instituted, leaders might do a needs assessment and focus subsequent work on areas of greatest need. For instance, on measurement materials, teachers might feel confident about teaching length and temperature, but need more work on volume and mass. Training leaders could then concentrate their efforts on providing more activities for building understanding of ways to teach mass and volume.

If workshop leaders assign follow-up or support activities in addition to the training sessions, participants might briefly report on their classroom applications as part of each training session. Workshop leaders might also provide materials and instructions for a make-it, take-it support session in which participants construct and organize games, measurement equipment, and other teaching aids for use in their classrooms.

Workshop leaders or staff development persons have probably noticed that the material in the measurement and geometry section has a number of features intended to help in organizing a workshop. The materials include an overview or introduction, descriptions of leader and participant behaviors, a discussion guide or "script" for the leader, and masters for transparencies that may be used to guide the leader's discussion. The materials include a list of supplies and equipment needed to conduct the workshop sessions. The written materials also include descriptions of numerous learning activities for children. These are reproduced in a card-like format and are intended to serve as directions for workshop participants as they are involved in learning activities. The cards may also be used by children (at second grade level or above) to follow directions and complete the activities. The backs of the cards feature guidance and instructions for teachers: approximate grade level (many activities can be used at a variety of levels with slight adaptations), objectives, materials lists, and teaching notes. To make the workshop and classroom implementation of activities easy, patterns, charts, games, and other illustrations are included.

Support or follow-up activities will vary according to school district needs and local policies. Some general suggestions for support activities based on the measurement and geometry material follow. Teachers could be required to:

1. Implement a suggested number of measurement and geometry activities in their math classes. They may report on their activities in oral presentations at subsequent staff development meetings, showing appropriate equipment or children's work. They may also turn in a brief (half-page) statement about the activity(ies) they implemented, their reactions to the success of the activity(ies) and the children's reactions.

   Principals or supervisors could check plan books to see that appropriate activities are incorporated. Teachers could invite administrators to observe classroom implementations. Teachers could display the results of their work. They might exhibit children's charts, drawings, designs, and models in their classrooms or in the halls. The administrator could check the children's work as evidence of implementation of activities. Children could write about the measurement and geometry activities they were involved in as a language arts assignment. For an example of a staff development form, see MG-41.

2. Implement a suggested number of homework activities in which children's families can become involved (Handout MG-40 has some suggestions). Teachers may report their results as suggested above. They may also wish to follow-up the activities with a brief parent questionnaire containing items such as:

What homework activities did your child carry out?

What information did your child share with you concerning the measurement or geometry activities done at home?

What was your child's attitude toward the home assignment?

What was your attitude toward the measurement or geometry homework?

3. Study the districts' AIM plan to ascertain whether principles for teaching measurement and geometry are being carried out.

   Teachers could find measurement and geometry objectives. They can evaluate the processes by which objectives are taught, comparing the local processes to the principles for teaching and activities suggested in these materials. They might want to change learning activities or incorporate additional objectives where feasible.

4. Integrate measurement and geometry activities into other curricular areas such as art, language arts, social studies, or science. Implement a suggested number of activities in the classroom. Report on the activities as suggested in number 1. For example, in science, children might keep records of measurements of plants they are growing or examine crystals to find various geometric shapes. In language arts, children could record observations of geometric shapes in the environment or write creative "Shape Stories." In social studies, children can make drawings and maps. They can record distances and heights as they work with various land forms. Children might make models and designs in art, thus using both geometry and measurement skills.

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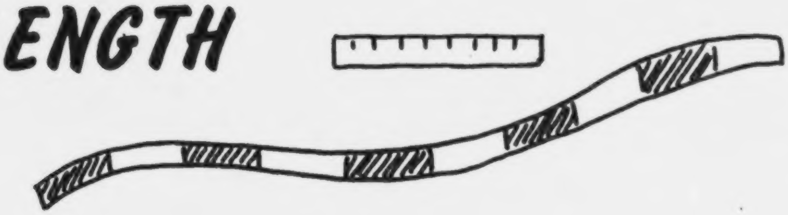
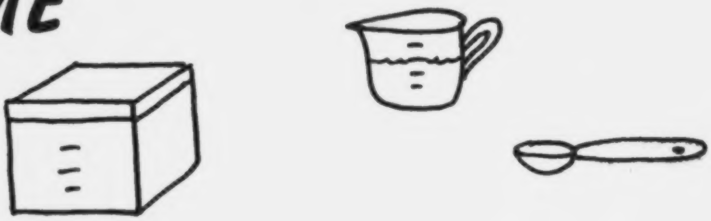
MG-1

SOME MEASUREMENT CONTENT FOR K - 6

☆ LENGTH



☆ VOLUME

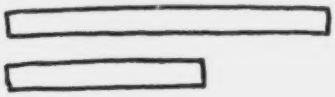


☆ TEMPERATURE

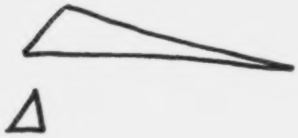
☆ MASS OR WEIGHT

SOME PREMEASUREMENT CONCEPTS

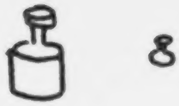
☆ LONG - SHORT



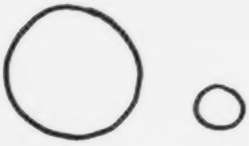
☆ WIDE - NARROW



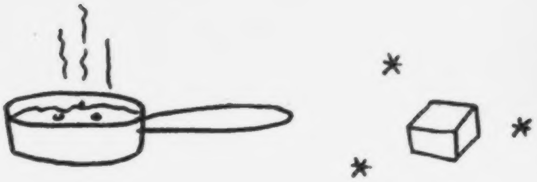
☆ HEAVY - LIGHT



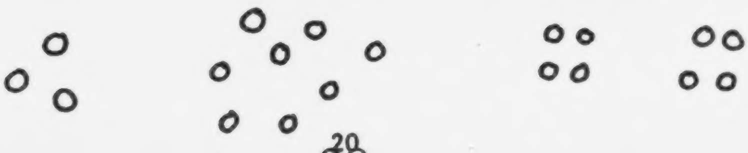
☆ LARGE - SMALL



☆ HOT - COLD



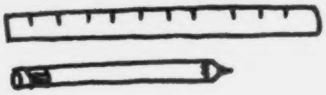
☆ LESS - MORE - EQUAL



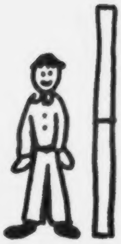
LENGTH

USE RULERS, TAPES, STICKS, ETC. TO MEASURE LENGTH.

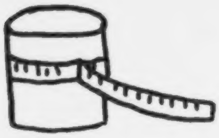
- CENTIMETERS



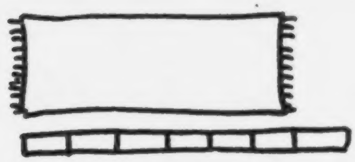
- METERS



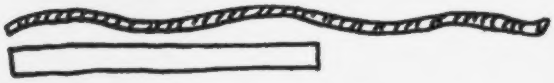
- MILLIMETERS



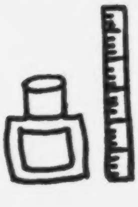
- INCHES
- FEET

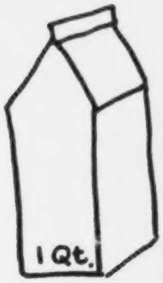


- YARDS

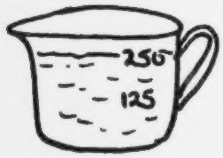


FRACTIONS OF AN INCH



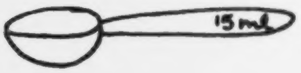


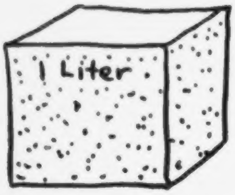
VOLUME

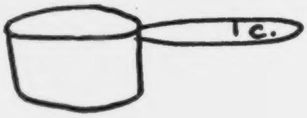


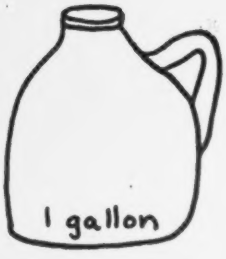
USE CUPS, SPOONS, AND OTHER CALIBRATED CONTAINERS TO MEASURE CAPACITY.

- MILLILITERS
- LITERS
- CUPS
- PINTS
- QUARTS
- GALLONS



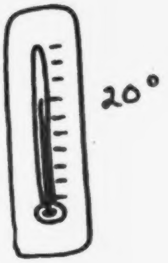




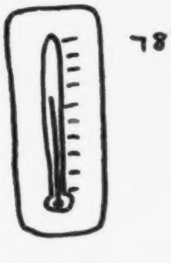


TEMPERATURE

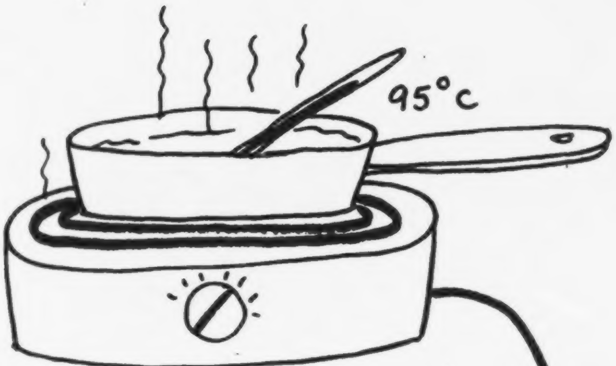
USE THERMOMETERS



DEGREES CELSIUS

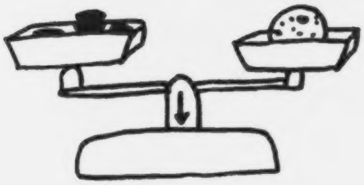


DEGREES FARENHEIT

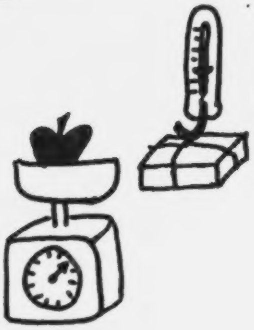


MASS OR WEIGHT

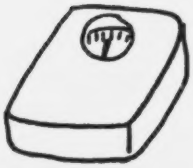
USE SCALES--



BALANCE



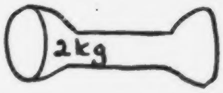
SPRING



STEP ON

- GRAMS

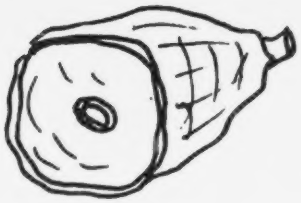
- KILOGRAMS



- OUNCES

Cheese
Snack
1oz.

- POUNDS



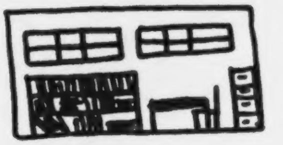
APPROPRIATE UNITS FOR DIFFERENT PURPOSES

METERS OR KILOMETERS?

Tupelo
Biloxi
Sea...

distance between towns

INCHES OR FEET?



length of classroom

KILOGRAMS OR GRAMS?

person's mass or weight?

CUPS OR TEASPOONS?

flour for cookies

PRINCIPLES FOR TEACHING MEASUREMENT

#1 HELP CHILDREN LEARN BY DOING

#2 "GUESS AND CHECK" OR "ESTIMATE AND VERIFY"

#3 COMPARE, USE NONSTANDARD AND STANDARD MEASURES

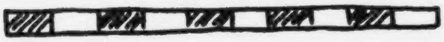
#4 FORM MENTAL IMAGES

#5 MAKE MEASURING TOOLS

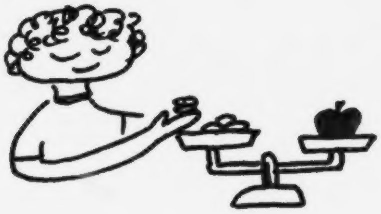
#6 PLAN LABORATORY EXPERIENCES

#7 USE HOME ACTIVITIES AND COMMUNICATIONS

PRINCIPLES FOR TEACHING MEASUREMENT

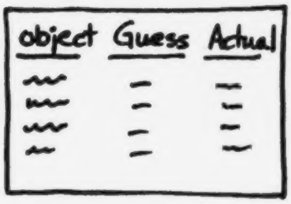


#1 HELP CHILDREN LEARN BY DOING



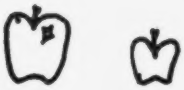
#2 USE "GUESS AND CHECK" OR

"ESTIMATE AND VERIFY"

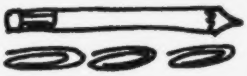


#3 USE STAGES:

COMPARE

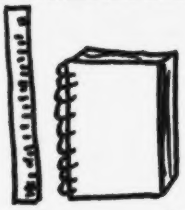


USE NONSTANDARD MEASURES



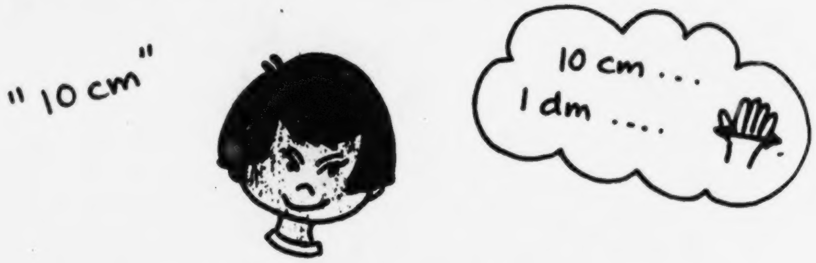
"About 3 clips long"

USE STANDARD MEASURES

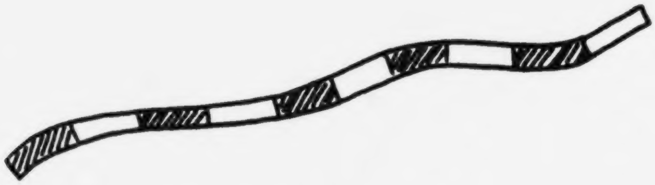


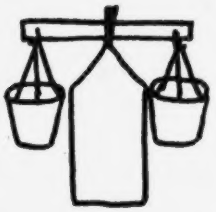
"About 18 cm high"

#4 FORM MENTAL IMAGES



#5 MAKE MEASURING TOOLS





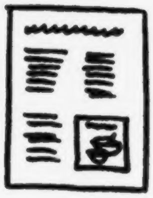
#6 PLAN LABORATORY EXPERIENCES:

PRE-LAB

LAB

POST-LAB

#7 USE HOME ACTIVITIES AND COMMUNICATIONS



MG-9

THE NATURE OF MEASUREMENT

- MAY BE DISCRETE OR CONTINUOUS
- CONTINUOUS MEASUREMENT IS APPROXIMATE
- PRECISION DEPENDS ON TOOLS AND PEOPLE
- MASS - AMOUNT OF MATTER IN AN OBJECT
- WEIGHT - A FORCE DETERMINED BY GRAVITATIONAL PULL

METRIC OR CUSTOMARY ???

meters grams degrees Celsius liters

feet quarts pounds degrees Farenheit

☆ used worldwide in science and trade	☆ used in U. S.
☆ NCTM endorses metric	☆ tradition "endorses customary units
☆ based on powers of ten, uses decimals	☆ based on various units, uses fractions

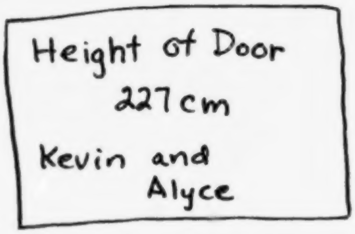
BOTH ARE USED IN EVERYDAY LIFE!

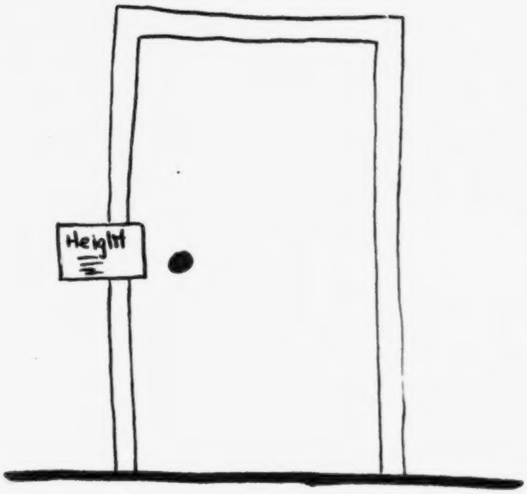
DON'T CONVERT BETWEEN SYSTEMS!

MG-11a

METRIC AWARENESS SIGNS

MAKE AND POST LITTLE SIGNS AROUND THE CLASSROOM OR BUILDING. YOUR SIGNS WILL TELL PEOPLE ABOUT DISTANCES. THEN MEASURE THE DISTANCES. MAKE YOUR SIGNS NEAT SO OTHERS CAN READ THEM.

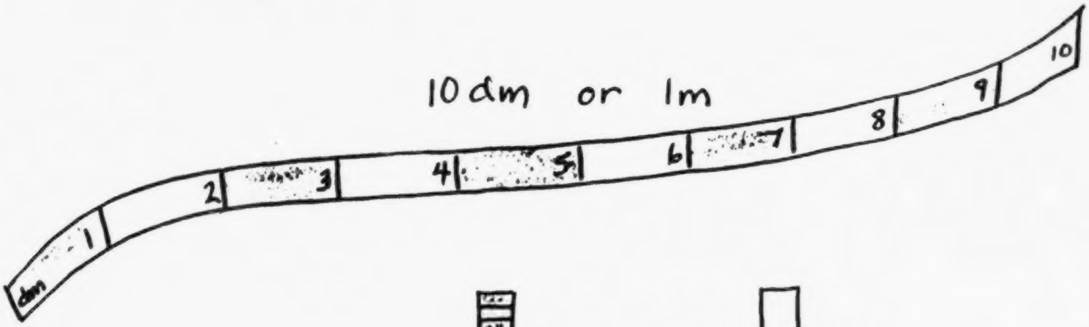

Height of Door
227 cm
Kevin and
Alyce

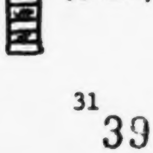


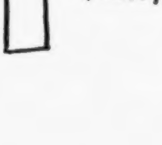
MG-11a

CHILD-MADE RULERS AND TAPES*

MAKE UP THE RULERS AND TAPES FROM THE HANDOUT SHEET. COLOR AND LABEL THEM NEATLY. FIT THE SECTIONS OF THE 1 M TAPE TOGETHER CAREFULLY.


10 dm or 1m
dm
1
2
3
4
5
6
7
8
9
10





METRIC AWARENESS SIGNS

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL ESTIMATE, MEASURE AND LABEL DISTANCES IN THE CLASSROOM

MATERIALS: RULERS OR METER STICKS, METRIC AWARENESS SIGNS (PERHAPS 2 PER GROUP), (PATTERN MG-15), TAPE.

TEACHING NOTES: BE SURE CHILDREN ARE SPECIFIC ABOUT THE DISTANCES THEY MEASURE. INSTEAD OF SAYING "DESK," THEY SHOULD SAY "LENGTH OF THE DESK." CHECK CHILDREN AS THEY WORK. IF SIGNS CAN BE PLACED OUTSIDE THE CLASSROOM, THEY WILL GENERATE A LOT OF INTEREST. YOU CAN ALSO MEASURE IN INCHES, FEET AND YARDS.

CHILD-MADE RULERS AND TAPES

APPROXIMATE GRADE LEVEL: 2-6, FIRST GRADERS CAN PARTICIPATE TOWARD THE END OF THE YEAR WITH MUCH SUPERVISION.

OBJECTIVES: CHILDREN WILL MAKE AND NEATLY LABEL MEASURING TAPES OF 10 cm, 1 dm, AND 1 m SIZES.

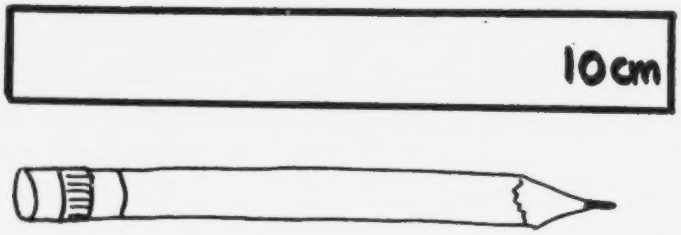
MATERIALS: A PATTERN FOR EACH CHILD (PATTERN MG-14), SCISSORS, CRAYONS, PENCILS, TRANSPARENT TAPE.

TEACHING NOTES: CHILDREN WILL NEED HELP REINFORCING THEIR TAPES. ENCOURAGE CAREFUL LABELING OF SECTIONS. HAVE CHILDREN TELL WHAT THEY HAVE MADE (10 cm = 1 dm, 10 dm = 1 m). USE THE TAPES FOR MEASURING EXPERIENCES AT HOME AND SCHOOL.

MG-11a

ABOUT 10 cm?

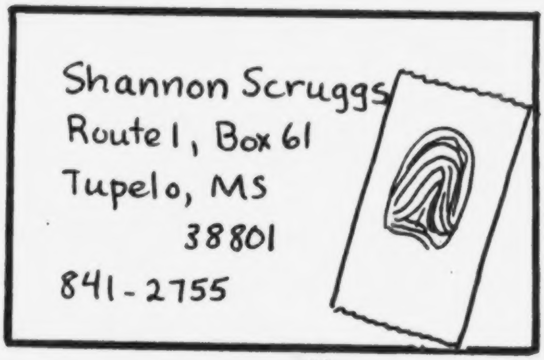
USE YOUR 10 cm STRIP. FIND TWO OBJECTS ABOUT AS LONG AS THE STRIP. FIND 2 OBJECTS SHORTER THAN THE STRIP. FIND 2 OBJECTS LONGER THAN THE STRIP. YOU WILL SHOW YOUR OBJECTS TO YOUR CLASSMATES.



MG-11a

ID CARDS*

CAREFULLY MEASURE A LITTLE RECTANGLE 5 cm BY 8 cm. CUT IT OUT. WRITE YOUR NAME, ADDRESS, AND TELEPHONE NUMBER ON IT. ADD A FINGERPRINT TO YOUR ID CARD.



Shannon Scruggs
Route 1, Box 61
Tupelo, MS
38801
841-2755

ABOUT 10 cm?

APPROXIMATE GRADE LEVEL: K-2

OBJECTIVES: CHILDREN WILL CORRECTLY CHOOSE OBJECTS LONGER THAN, SHORTER THAN, AND EQUAL TO 10 cm.

MATERIALS: 10 cm STRIPS (CHILDREN CHOULD CUT THESE FROM THE PATTERN (MG-14), CLASSROOM OBJECTS.

TEACHING NOTES: LET CHILDREN TELL ABOUT THE OBJECTS THEY PICKED, THEN ARRANGE THEM IN THREE GROUPS ACCORDING TO LENGTH. YOU COULD PIN SOME OBJECTS ON A BULLETIN BOARD. YOU MIGHT SHARE RESULTS CONCURRENTLY IN TWO GROUPS SUPERVISED BY THE TEACHER AND ASSISTANT TEACHERS. YOU MIGHT ALSO USE A DISTANCE SUCH AS 4 INCHES FOR THIS ACTIVITY.

ID CARDS

APPROXIMATE GRADE LEVEL: 1-3

OBJECTIVES: CHILDREN WILL MEASURE AND DRAW A RECTANGLE TO NEAREST cm; CHILDREN WILL WRITE NAME, ADDRESS, PHONE NUMBER LEGIBLY; CHILDREN WILL MAKE A NEAT FINGERPRINT.

MATERIALS: NOTECARDS OR PLAIN WHITE PAPER, RULERS, TRANSPARENT TAPE, SCISSORS.

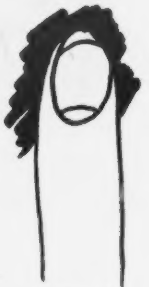
TEACHING NOTES: ID CARDS COULD ALSO BE MADE 2 BY 3 INCHES. PROVIDE ADDRESSES AND PHONE NUMBERS FOR CHILDREN WHO DO NOT HAVE THEM MEMORIZED. THIS ACTIVITY IS EASILY SUPERVISED IF THE TEACHER WORKS WITH HALF THE GROUP AND THE ASSISTANT WITH THE OTHER HALF.

MG-11b

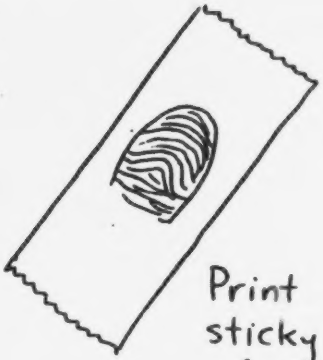
EASY FINGERPRINTS*

SCRIBBLE A SMALL SPOT WITH PENCIL LEAD. RUB YOUR FINGER INTO THE SCRIBBLE. PRINT YOUR FINGER ONTO THE STICKY SIDE OF TRANSPARENT TAPE. TAPE THE PRINT DOWN TO YOUR ID CARD.

Scribble



Rub

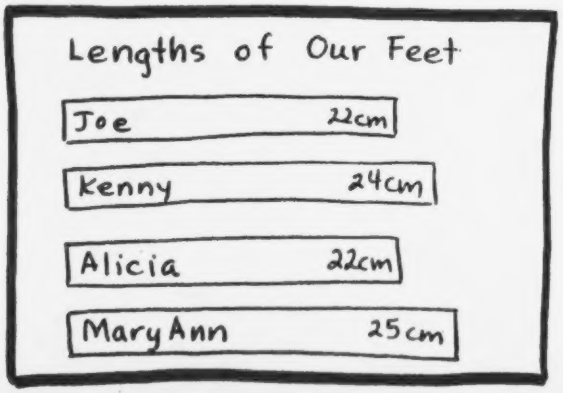


Print onto sticky side of tape

MG-11b

LENGTH GRAPH

WORK WITH A GROUP OF FOUR OR FIVE. CAREFULLY MEASURE THE LENGTH OF YOUR FEET WITH SHOES. CUT PAPER STRIPS TO THE SAME LENGTHS AS YOUR FEET. GLUE THE STRIPS TO A PAPER TO MAKE A BAR GRAPH. PUT A TITLE ON YOUR GRAPH. LABEL EACH BAR WITH YOUR NAME AND LENGTH.



EASY FINGERPRINTS

APPROXIMATE GRADE LEVEL: K-4

OBJECTIVES: CHILDREN WILL PREPARE NEAT FINGERPRINTS FOLLOWING DIRECTIONS.

MATERIALS: PENCILS, PAPER, TRANSPARENT TAPE, ID CARDS.

TEACHING NOTES: CHILDREN COULD ALSO EXAMINE FINGERPRINTS WITH A HAND LENS AND SEE SIMILARITIES AND DIFFERENCES IN THEIR PRINTS.

LENGTH GRAPH

APPROXIMATE GRADE LEVEL: K-6

OBJECTIVES: CHILDREN WILL WORK IN GROUPS AND PREPARE A BAR GRAPH INCLUDING ACTUAL SIZED BARS, LABELS, AND A TITLE.

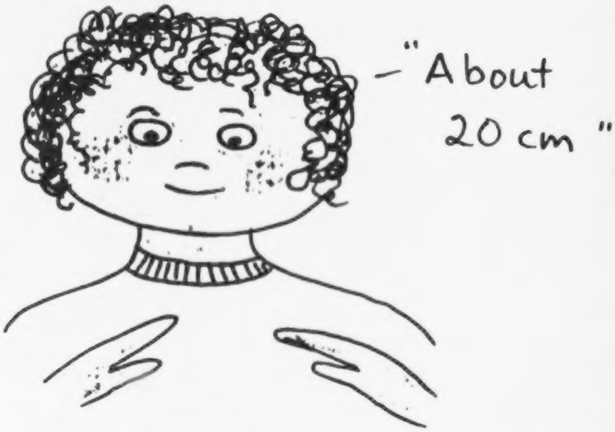
MATERIALS: CONSTRUCTION PAPER (PRECUT IN STRIPS 5 CM WIDE FOR GRADES K-2), PLAIN PAPER, GLUE, RULERS.

TEACHING NOTES: YOUNGER CHILDREN WILL NEED MORE GUIDANCE. K-1 STUDENTS MAY SIMPLY CUT BARS TO FEET SIZES, BUT NOT MEASURE THE BARS IF DESIRED. STUDENTS MAY ARRANGE THE BARS IN ORDER--SHORTEST TO LONGEST--ON THE GRAPH.

MG-11b

SHOW ME

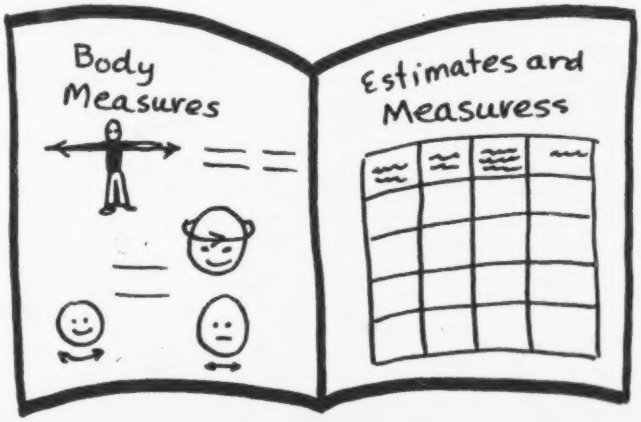
WORK WITH A SMALL GROUP. TAKE TURNS BEING THE LEADER. THE LEADER DRAWS OUT A SLIP OF PAPER AND TELLS THE OTHERS THE LENGTH. EACH TRIES TO SHOW THE LENGTH WITH THEIR FINGERS. EVERYONE CHECKS THE LENGTHS WITH A RULER.



MG-11b

"METRIC ME" BOOKLETS

GATHER AND RECORD PERSONAL MEASURES IN "METRIC ME" BOOKLETS. YOU CAN WORK IN PAIRS OR THREES TO COMPLETE THE BOOKLETS.


Body Measures
Estimates and Measuress

SHOW ME

APPROXIMATE GRADE LEVEL: 2-6

OBJECTIVES: CHILDREN WILL SHOW APPROXIMATE DISTANCES AND CHECK THEMSELVES.

MATERIALS: A RULER FOR EACH CHILD, SLIPS OF PAPER WITH DISTANCES WRITTEN ON THEM.

TEACHING NOTES: THIS GAME MAY BE PLAYED WITH THE WHOLE CLASS FOR PERIODS OF JUST 5-10 MINUTES. IT HELPS CHILDREN DEVELOP MENTAL IMAGES OF DISTANCES. FOR FIFTH AND SIXTH GRADERS, USE SOME EQUIVALENT NAMES FOR DISTANCES--INSTEAD OF 10 cm CALL OUT 100 mm. ONCE IN A WHILE, NAME AN "IMPOSSIBLE" DISTANCE--1 km--AND LET THE CHILDREN TELL YOU WHY IT IS IMPOSSIBLE TO SHOW WITH THEIR HANDS.

"METRIC ME" BOOKLETS

APPROXIMATE GRADE LEVEL: K-6

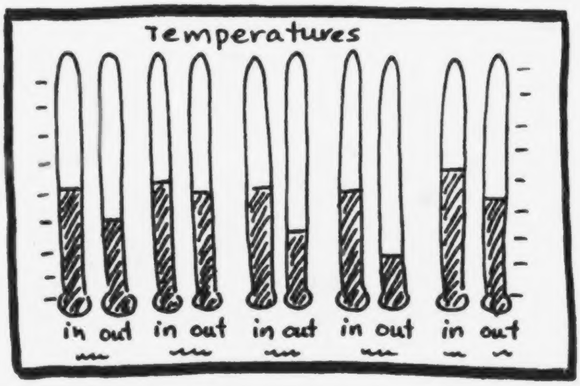
OBJECTIVES: CHILDREN WILL ESTIMATE AND MEASURE BODY MEASURES.

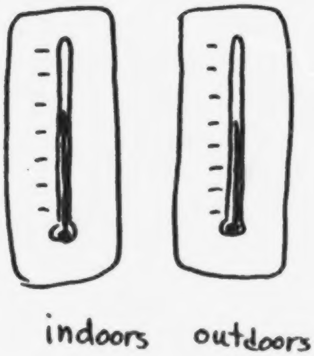
MATERIALS: BOOKLETS REPRODUCED FOR CHILDREN, RULERS, MEASURING TAPES, STEP-ON SCALES (PATTERNS MG-15 AND MG-16).

TEACHING NOTES: FOR YOUNGER CHILDREN, THE BOOKLETS MAY BE COMPLETED IN 2 OR EVEN 4 DAYS. CIRCULATE AND CHECK CHILDREN AS THEY WORK. YOUNGER CHILDREN WILL NEED MUCH HELP AND SUPERVISION.

INDOOR AND OUTDOOR TEMPERATURES

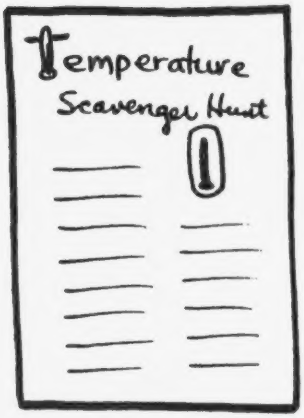
GRAPH TEMPERATURES OVER A PERIOD OF TIME. YOU COULD ALSO ADD WIND SPEED AND WIND CHILL TO YOUR WEATHER DATA.




indoors
outdoors

TEMPERATURE SCAVENGER HUNT*

WORK WITH A TEAM. USE A THERMOMETER TO FIND AS MANY OF THE TEMPERATURES ON THE SHEET AS YOU CAN. RECORD YOUR DATA ON THE SHEET.


Temperature
Scavenger Hunt

What's the Temperature?

Water fountain

boiling water

outside air

ice water

water from the "hot" faucet

INDOOR AND OUTDOOR TEMPERATURES

APPROPRIATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL APPROXIMATE TEMPERATURES, READ A THERMOMETER, AND SHOW RESULTS ON A GRAPH.

MATERIALS: THERMOMETER, GRAPH FORM (PATTERN MG-19), MARKERS, OPTIONAL WINDCHILL AND WINDSPEED CHARTS (MG-20).

TEACHING NOTES: CHILDREN CAN TAKE TURNS MEASURING TEMPERATURES OVER A PERIOD OT TIME--PERHAPS TWO WEEKS. "WEATHERPERSONS" CAN REPORT RESULTS TO THE CLASS. TRY THIS ACTIVITY AT A TIME WHEN THE WEATHER IS LIKELY TO BE ESPECIALLY CHANGEABLE.

TEMPERATURE SCAVENGER HUNT

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL CORRECTLY MEASURE TEMPERATURES IN SEVERAL LOCATIONS AND RECORD DATA.

MATERIALS: ONE THERMOMETER FOR EACH GROUP, SCAVENGER HUNT LIST (PATTERN MG-18).

TEACHING NOTES: IF ENOUGH THERMOMETERS ARE NOT AVAILABLE, STUDENTS CAN TAKE TURNS GETTING DIFFERENT PARTS OF THE DATA. DISCUSS VARIOUS ANSWERS TO TRY TO BUILD STUDENTS' MENTAL IMAGES OF THE TEMPERATURES.

MG-11c

COMPARING TEMPERATURES

CHOOSE A FARAWAY CITY AND KEEP TRACK OF TEMPERATURES, COMPARING THEM TO THOSE IN YOUR HOMETOWN. LOCATE THE CITIES ON A MAP OR GLOBE.

Jackson, MS
high 32°c
low 20°c

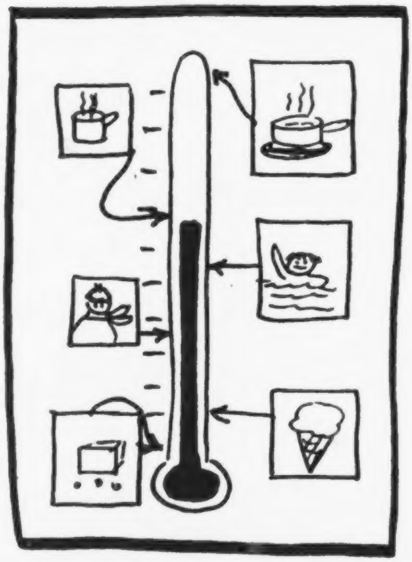
Cairo, Egypt
high 37°c
low 25°c

Brisbane, Australia
high 22°c
low 15°c

MG-11c

TEMPERATURE COLLAGE

CUT PICTURES TO REPRESENT ACTIVITIES YOU CAN DO AT DIFFERENT TEMPERATURES. GLUE THE PICTURES ONTO A BIG THERMOMETER PICTURE. ADD ARROWS FROM THE PICTURES TO SHOW APPROXIMATELY WHAT TEMPERATURE EACH PICTURE REPRESENTS.



COMPARING TEMPERATURES

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL KEEP TRACK OF TEMPERATURES IN VARIOUS CITIES FOR A PERIOD OF TIME.

MATERIALS: CHARTS (PATTERN MG-21), MARKERS, NEWSPAPERS REPORTING TEMPERATURES FOR CITIES AROUND THE WORLD.

TEACHING NOTES: CHILDREN COULD WORK IN PAIRS. THEY SHOULD KEEP DATA FOR AT LEAST A WEEK. HAVE CHILDREN REPORT WHAT THEY FOUND TO THEIR CLASSMATES AND COMPARE THE TEMPERATURES WITH YOURS. LOCATE CITIES ON THE MAP. YOU MIGHT USE CITIES IN THE SOUTHERN HEMISPHERE AND EXPLAIN DIFFERENCES IN THEIR WEATHER.

TEMPERATURE COLLAGE

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL ASSOCIATE VARIOUS TEMPERATURES WITH ACTIVITIES.

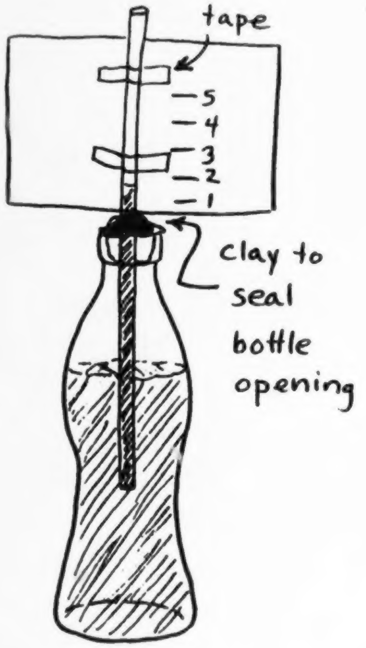
MATERIALS: LARGE PAPER OR BULLETIN BOARD SPACES; OLD MAGAZINES, CATALOGS, AND NEWSPAPERS; RULERS; GLUE.

TEACHING NOTES: CONDUCT THIS ACTIVITY ONLY AFTER SEVERAL EXPERIENCES WITH TEMPERATURE. LET CHILDREN NAME OTHER ACTIVITIES FOR VARIOUS TEMPERATURES BESIDES THE ONES THEY FIND PICTURES OF.

MG-11d

HOMEMADE "THERMOMETER"

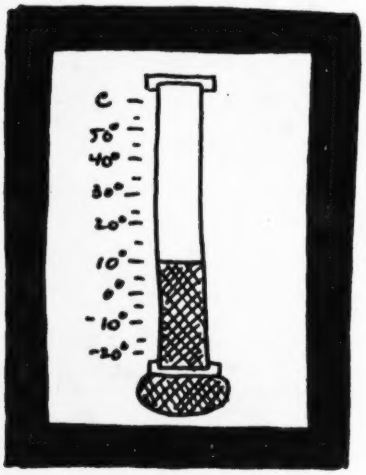
MAKE A DEVICE LIKE A THERMOMETER. USE A SODA BOTTLE, COLORED WATER, MODELING CLAY, A STRAW AND A CARD. FIX YOUR SUPPLIES TO LOOK LIKE THE PICTURE. BE SURE THE CLAY IS TIGHTLY SEALED. PLACE YOUR "THERMOMETER" IN VARIOUS PLACES IN THE CLASSROOM. RECORD DATA ON YOUR CHART.



MG-11d

THERMOMETER MODEL*

MAKE A THERMOMETER MODEL. GLUE A PATTERN TO CARDBOARD. CUT OUT THE SLITS. COLOR ½ OF A 48 cm SECTION OF ELASTIC RED. PULL ELASTIC THROUGH THE SLITS. STAPLE OR SEW THE ELASTIC IN PLACE. SET THE MODEL TO VARIOUS TEMPERATURES. LET YOUR CLASSMATES READ THE TEMPERATURES.



HOMEMADE "THERMOMETER

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: STUDENTS WILL CONSTRUCT A THERMOMETER-LIKE DEVICE AND USE IT TO COMPARE TEMPERATURES.

MATERIALS: SODA BOTTLES, FOOD COLORING, TRANSLUSCENT PLASTIC STRAWS, MODELING CLAY, NOTE CARDS, TAPE, HEATING DEVICES OR BUCKET OF ICE, CHARTS (PATTERN MG-23).

TEACHING NOTES: STUDENTS CAN WORK IN THREES ON THIS PROJECT. AFTER THE THERMOMETERS ARE MADE, HAVE STUDENTS PLACE THEM IN VARIOUS PLACES--ON THE RADIATOR, IN A PAN OF VERY HOT WATER, ON THEIR DESKS, OUTDOORS, OR IN ICE WATER. SHARE FINDINGS.

THERMOMETER MODEL

APPROXIMATE GRADE LEVEL: 4-6

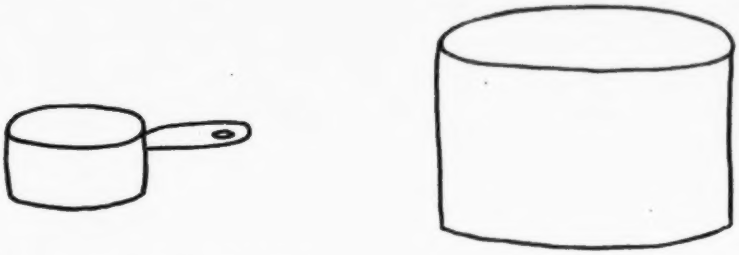
OBJECTIVES: CHILDREN WILL MAKE A THERMOMETER MODEL, SET IT TO VARIOUS TEMPERATURES, AND READ VARIOUS TEMPERATURES.

MATERIALS: CARDBOARD, PATTERNS (PATTERN MG-22), SCISSORS, ABOUT 48 CM OF 1 CM WIDE ELASTIC PER THERMOMETER MODEL, STAPLER OR NEEDLE AND THREAD, RED PERMANENT MARKER.

TEACHING NOTES: STUDENTS CAN WORK IN GROUPS OF FOUR OR FIVE ON THE MODEL. YOU MIGHT MAKE AN EVEN LARGER MODEL FOR USE BY THE ENTIRE CLASS. YOU COULD ALSO MAKE FARENHEIT THERMOMETER MODELS.

FILL 'ER UP!

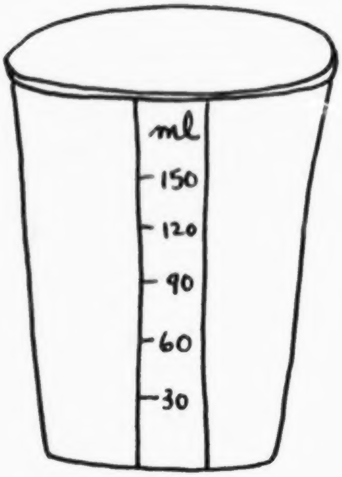
PREDICT HOW MANY OF ONE OF THE SMALLER CONTAINERS WILL FILL ONE OF THE LARGER CONTAINERS. FILL THE CONTAINERS AND TEST YOUR PREDICTIONS. WRITE DOWN YOUR RESULTS.



Prediction : 10 smaller containers will fill the larger one.

CALIBRATE A CUP*

PUT A STRIP OF MASKING TAPE ON THE CUP AS SHOWN. POUR WATER INTO THE CUP 30 ML AT A TIME. MARK THE WATER LEVEL ON THE TAPE EACH TIME. NUMBER THE MARKS. MARK "ML" AT THE TOP AS YOUR UNIT OF MEASUREMENT.



FILL 'ER UP!

APPROXIMATE GRADE LEVEL: K-4

OBJECTIVES: CHILDREN WILL PREDICT AND MEASURE VOLUME OF SEVERAL CONTAINERS.

MATERIALS: SEVERAL "SMALL" CUPS AND SEVERAL LARGE CONTAINERS, CHARTS (PATTERN MG-24).

TEACHING NOTES: HELP YOUNGER CHILDREN WITH COUNTING AND RECORDING OF DATA. USE LARGER CONTAINERS OR ODD SHAPES FOR OLDER CHILDREN.

CALIBRATE A CUP

APPROXIMATE GRADE LEVEL: 2-6

OBJECTIVES: CHILDREN WILL PREPARE A MEASURING CUP AND CALIBRATE IT IN 30 ML INTERVALS.

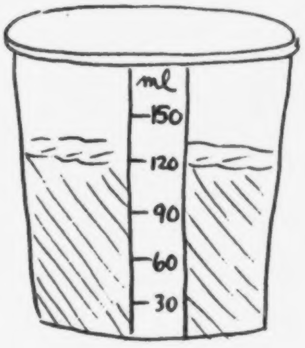
MATERIALS: CLEAR PLASTIC CUPS, MASKING TAPE, SCISSORS, SOURCE OF WATER, 15 OR 30 ML STANDARD MEASURING SPOONS OR SMALL CUPS.

TEACHING NOTES: THIS ACTIVITY DEMONSTRATES HOW TO CALIBRATE ONE MEASURING DEVICE FROM A STANDARD DEVICE. STRESS CAREFUL WORK. HAVE STUDENTS LOOK AT THE WATER AT EYE LEVEL BEFORE THEY MAKE MARKS. USE THE CUPS FOR SEVERAL OTHER ACTIVITIES; ONE IS INCLUDED IN THESE MATERIALS. YOU COULD ALSO CALIBRATE CUPS IN 2 FL. OZ. INTERVALS.

MG-11e

USE YOUR CALIBRATED CUP*

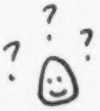
HOW MANY ML DO YOU DRINK IN A SWALLOW? FIND OUT. FILL YOUR CUP WITH KOOLAID OR MILK. NOTE THE LEVEL OF THE LIQUID. NOW TAKE A SWALLOW AND NOTE THE LEVEL. FIGURE HOW MANY ML YOU DRANK. CONTINUE THE PROCESS UNTIL YOUR CUP IS EMPTY. FIGURE YOUR AVERAGE NUMBER OF ML PER SWALLOW!

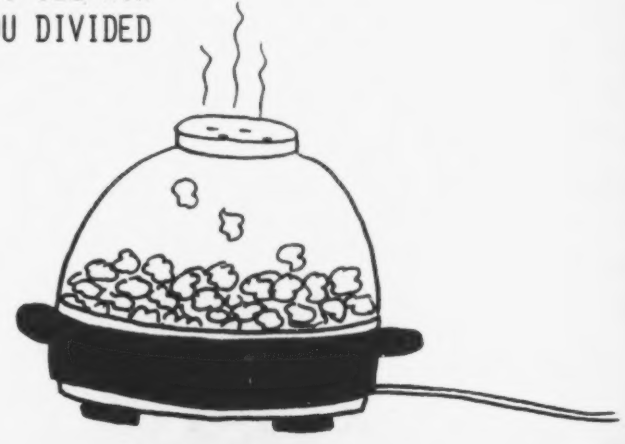


MG-11e

POPCORN VOLUMES

MAKE POPCORN FOR CLASS USING 200 ML OF POPCORN AND 60 ML OF OIL (IF NEEDED). PREDICT WHAT THE VOLUME OF POPPED CORN WILL BE. MEASURE THE POPPED CORN. DIVIDE THE VOLUME BY THE NUMBER OF PEOPLE IN YOUR CLASS TO SEE HOW MUCH EACH PERSON WOULD GET IF YOU DIVIDED IT EVENLY.





WHAT WILL THE POPPED VOLUME BE?

USE YOUR CALIBRATED CUP

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL READ THE CUP'S SCALE AND DETERMINE HOW MUCH LIQUID THEY DRINK.

MATERIALS: CALIBRATED CUPS, MILK OR KOOLAID.

TEACHING NOTES: ENCOURAGE STUDENTS TO KEEP RECORDS OF HOW MANY EACH SWALLOW. COMPARE RESULTS.

POPCORN VOLUMES

APPROXIMATE GRADE LEVEL: 3-6 (OR YOUNGER CHILDREN WITH TEACHER ASSISTANCE)

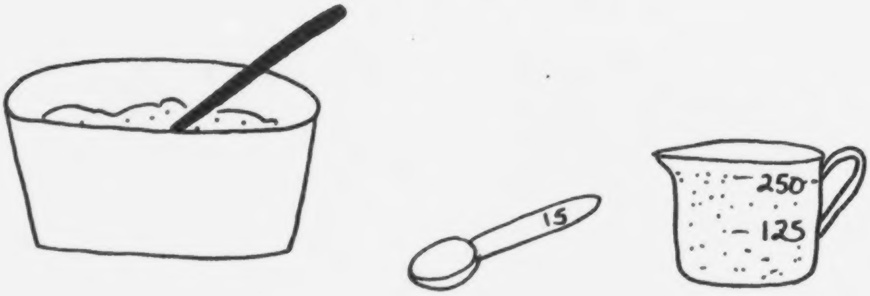
OBJECTIVES: CHILDREN WILL MEASURE UNPOPPED AND POPPED CORN: THEY WILL PERFORM A 2 DIGIT DIVISION PROBLEM.

MATERIALS: POPCORN POPPER, LARGE BOWL, MEASURING CUPS, POPCORN, COOKING OIL, SALT (OPTIONAL), HOT PADS, CLEANUP MATERIALS, NAPKINS.

TEACHING NOTES: CAUTION THE CHILDREN ABOUT WORKING WITH THE HOT POPPER. THE ENTIRE CLASS MIGHT PREDICT THE VOLUME OF POPPED CORN. YOU COULD FIND THE AVERAGE AND RANGE OF THEIR GUESSES.

PLAYDOUGH*

WORK WITH A GROUP TO MAKE PLAYDOUGH ACCORDING TO THE RECIPE. MEASURE CAREFULLY. SHARE THE WORK. WHEN YOUR PLAYDOUGH IS FINISHED, USE IT IN SOME OF THE PLAYDOUGH TASKS.

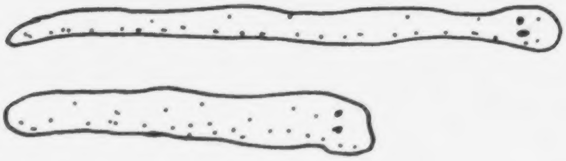


MG-11e

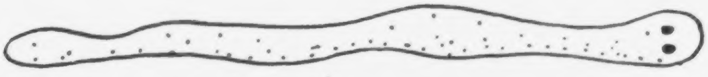
PLAYDOUGH TASK #1

MEASURE 15 ML OF PLAYDOUGH.
ROLL IT INTO SNAKE SHAPES
THIS LONG:

15 CM
30 CM
8 CM



COMPARE THE FATNESS OF THE THREE SNAKE SHAPES. CAN YOU MAKE A SNAKE 60 CM LONG?



HOW LONG IS THE SNAKE?

PLAYDOUGH

APPROXIMATE GRADE LEVEL: K-6

OBJECTIVES: CHILDREN WILL MEASURE VOLUME, WORK IN A GROUP, AND MAKE PLAYDOUGH TO USE IN SUBSEQUENT LEARNING ACTIVITIES.

MATERIALS: PLAYDOUGH RECIPE (PATTERN MG-25), INGREDIENTS, SOURCE OF HOT WATER, MIXING BOWL AND SPOON, MEASURING CUPS AND SPOONS.

TEACHING NOTES: OLDER STUDENTS CAN DO THE ACTIVITY INDEPENDENTLY: YOUNGER ONES WILL NEED GUIDANCE. FOR THE ENTIRE CLASS TO WORK WITH PLAYDOUGH AT THE SAME TIME, 3 OR 4 BATCHES MUST BE MADE.

PLAYDOUGH TASK #1

APPROXIMATE GRADE LEVEL: 2-6

OBJECTIVES: CHILDREN WILL FORM PLAYDOUGH TO GIVEN DISTANCES.

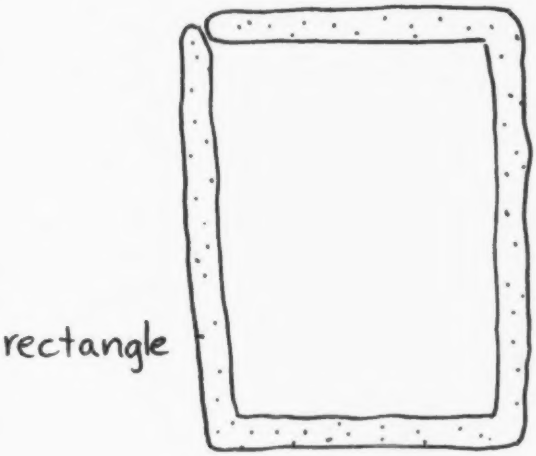
MATERIALS: RULERS, PLAYDOUGH.

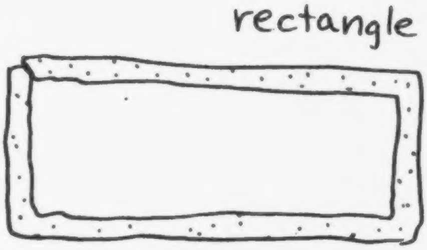
TEACHING NOTES: KINDERGARTENERS AND FIRST GRADERS MIGHT DO THE TASK WITH TEACHER DIRECTION. THEY MIGHT SIMPLY MAKE SHORT, LONG, AND MEDIUM SNAKES.

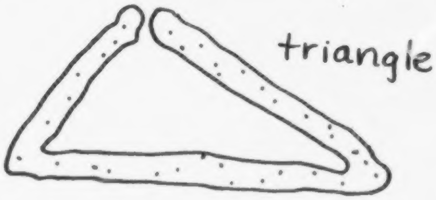
MG-11f

PLAYDOUGH TASK #2*

FORM YOUR PLAYDOUGH INTO A RECTANGLE. COMPARE SIZES OF RECTANGLES WITH SOME CLASSMATES. NOW MAKE A SQUARE. ALSO TRY A CIRCLE AND A TRIANGLE.







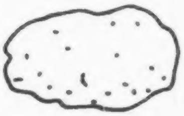
MG-11f

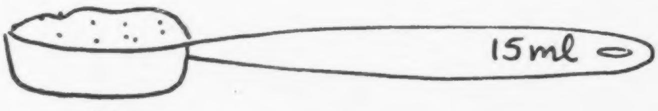
PLAYDOUGH TASK #3 *

TRY TO PINCH OFF 15 ML OF PLAYDOUGH. PRESS THE DOUGH INTO A SPOON TO SEE HOW CLOSE YOU CAME. NOW TRY THESE VOLUMES:

30 ML
5 ML

TRY TO GET SOME OTHER VOLUMES THAT YOU NAME IN ADVANCE.





PLAYDOUGH TASK #2

APPROXIMATE GRADE LEVEL: K-6

OBJECTIVES: CHILDREN WILL USE PLAYDOUGH TO FORM VARIOUS POLYGONS.

MATERIALS: PLAYDOUGH

TEACHING NOTES: CHILDREN MAY REPRESENT POLYGONS WITH SOLID "SLABS" OF DOUGH OR MAY ROLL THE DOUGH INTO A SNAKE AND THEN FORM THE OUTLINE OF EACH FIGURE.

PLAYDOUGH TASK #3

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL ESTIMATE AND CHECK VARIOUS SMALL VOLUMES.

MATERIALS: PLAYDOUGH

TEACHING NOTES: THIS ACTIVITY BUILDS CHILDRENS' MENTAL IMAGES OF SMALL VOLUMES. IT COULD ALSO BE DONE WITH TEASPOONS AND TABLESPOONS.

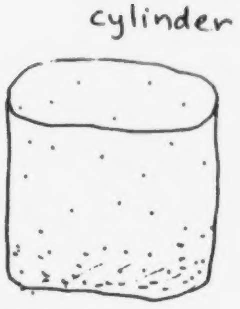
MG-11f

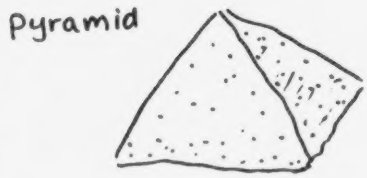
PLAYDOUGH TASK #4*

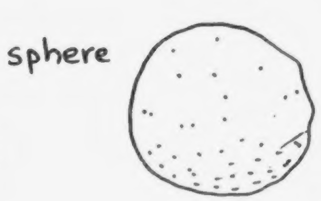
LET'S MAKE SOME THREE-DIMENSIONAL FIGURES. USE PLAYDOUGH. TRY THESE:

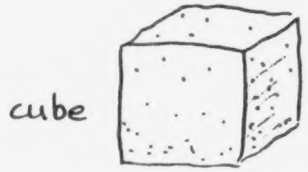
- .SPHERE
- .CUBE
- .CYLINDER
- .PYRAMID

WHAT OTHER SOLIDS CAN YOU MAKE AND NAME?





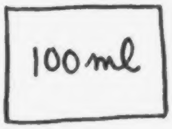


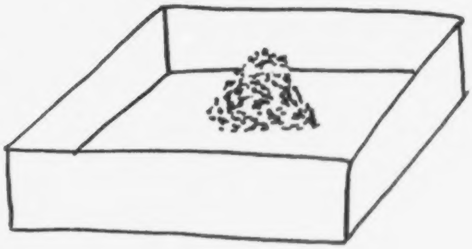


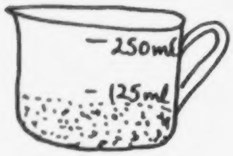
MG-11f

DRAW AND MEASURE VOLUMES*

TAKE TURNS DRAWING SLIPS OF PAPER. TRY TO SCOOP OUT THE AMOUNT OF RICE THAT THE PAPER TOLD YOU. NOW USE A MEASURING CUP AND CHECK YOURSELF.






250 ml
125 ml

PLAYDOUGH TASK #4

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL USE PLAYDOUGH TO FORM GEOMETRIC SOLIDS:

MATERIALS: PLAYDOUGH

TEACHING NOTES: CHILDREN MAY WANT TO LABEL AND DISPLAY THEIR SOLIDS.

DRAW AND MEASURE VOLUMES

APPROXIMATE GRADE LEVEL: 3-6

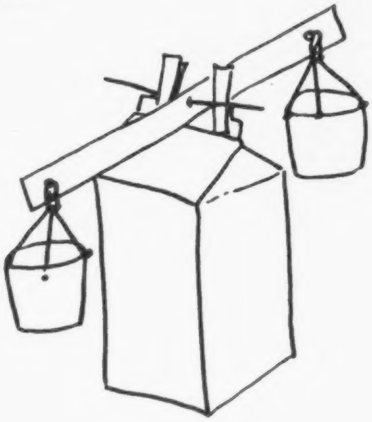
OBJECTIVES: CHILDREN WILL ESTIMATE AND MEASURE VARIOUS VOLUMES.

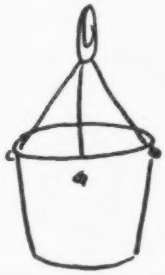
MATERIALS: SLIPS OF PAPER WITH VARIOUS VOLUMES, PATTERN MG-26 (1 L, 100 ML, 200 ML, 500 ML, .5 L, 15 ML, 30 ML, ETC.) 2 CONTAINERS FOR THE PAPER, RICE IN A LARGE CONTAINER, ANOTHER CONTAINER TO SCOOP ESTIMATES INTO MEASURING CUPS AND SPOONS.

TEACHING NOTES: ENCOURAGE THE CHILDREN TO KEEP THE RICE IN THE CONTAINERS. YOU COULD ALSO USE SAND OR BIRDSEED FOR THE ACTIVITY.

MAKING A BALANCE SCALE

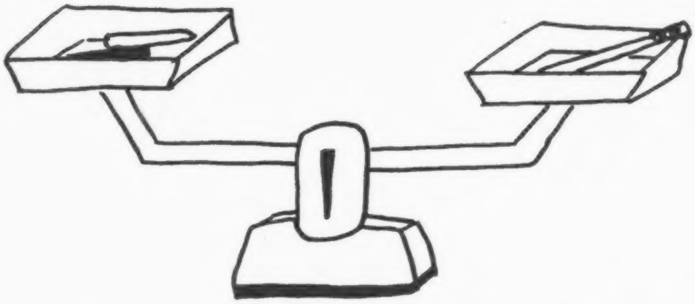
USE A RULER WITH HOLES PUNCHED, PART OF A COATHANGER, 2 CLOTHESPINS, AND A HALF-GALLON MILK CARTON TO MAKE A BALANCE SCALE. PUT SOME SAND IN THE MILK CARTON TO WEIGHT IT DOWN. MAKE SOME CUPS TO HANG ON THE SCALE WITH STYROFOAM CUPS AND YARN. ATTACH THE CUPS WITH PAPER CLIPS.





WHAT THINGS BALANCE?

USE VARIOUS SMALL OBJECTS TO BALANCE OTHER OBJECTS ON THE BALANCE SCALE. RECORD YOUR RESULTS.



2 crayons balance 1 pencil
2 crayons balance 8 styrofoam bits

MAKING A BALANCE SCALE

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL MAKE A SIMPLE BALANCE SCALE.

MATERIALS: FOR EACH SCALE: HALF-GALLON MILK CARTON, SAND, 15 cm LENGTH OF COATHANGER WIRE, 2 CLOTHESPINS, RULER WITH PREPUNCHED HOLES, 2 STYROFOAM CUPS, 2 PAPER CLIPS, YARN.

TEACHING NOTES: CHILDREN SHOULD BE ABLE TO DO ALL THE WORK IF YOU PRECUT THE WIRE. MAKE 4-6 SCALES FOR YOUR CLASSROOM.

WHAT THINGS BALANCE?

APPROXIMATE GRADE LEVEL: K-4

OBJECTIVES: CHILDREN WILL DETERMINE WHICH OBJECTS BALANCE ON A SCALE AND RECORD RESULTS.

MATERIALS: BALANCE SCALE, SMALL OBJECTS, CHART (PATTERN MG-27).

TEACHING NOTES: THIS ACTIVITY INVOLVES COUNTING AND ESTIMATING SKILLS.

MG-11g

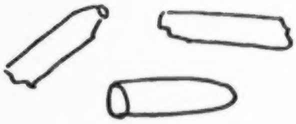
USING THE BALANCE SCALE #1

GATHER SOME SMALL OBJECTS. PLACE OBJECTS IN THE CUP. SEE IF THEY BALANCE OR WHICH IS HEAVIER.

Some Small Objects You Might Use in the Cups:

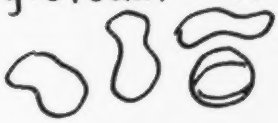
paper clips

coins



crayon pieces

styrofoam bits



rubber bands

MG-11g

USING THE BALANCE SCALE #2

ESTIMATE THE MASS IN GRAMS OF SOME SMALL OBJECTS. USE GRAM WEIGHTS IN ONE CUP AND THE OBJECTS IN THE OTHER CUP OF YOUR BALANCE SCALE. DETERMINE THE MASS OF EACH OBJECT. RECORD YOUR RESULTS.

Objects	Estimate	Actual Mass
2 dimes	2g	3g
plastic spoon	5g	7g
6 paper clips	12 g	12g

USING THE BALANCE SCALE #1

APPROXIMATE GRADE LEVEL: K-6

OBJECTIVES: CHILDREN WILL USE THE BALANCE SCALE AND DETERMINE IF OBJECTS ARE OF EQUAL MASS OR WHICH IS HEAVIER.

MATERIALS: BALANCE SCALE, SMALL OBJECTS.

TEACHING NOTES: THIS ACTIVITY SHOULD BE SIMPLE FOR OLDER YOUNGSTERS, BUT THEY WILL ENJOY USING THE SCALE THEY HAVE MADE.

USING THE BALANCE SCALE #2

APPROXIMATE GRADE LEVEL: K-6

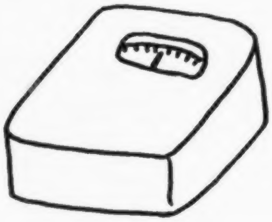
OBJECTIVES: CHILDREN ESTIMATE AND WEIGH SMALL OBJECTS AND RECORD RESULTS.

MATERIALS: BALANCE SCALE, SMALL OBJECTS, GRAM WEIGHTS.

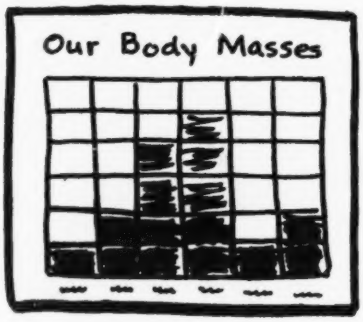
TEACHING NOTES: IF GRAM WEIGHTS ARE NOT AVAILABLE, YOU COULD USE NICKLES FOR 5 GRAM UNITS OR 2 SMALL PAPERCLIPS FOR 1 GRAM.

BODY MASSES

USE THE STEP-ON SCALE. DETERMINE YOUR BODY MASS. MARK IT ON THE GRAPH. COMPARE YOUR BODY WEIGHT TO THAT OF SOME OTHER PEOPLE. USE FRACTIONS.

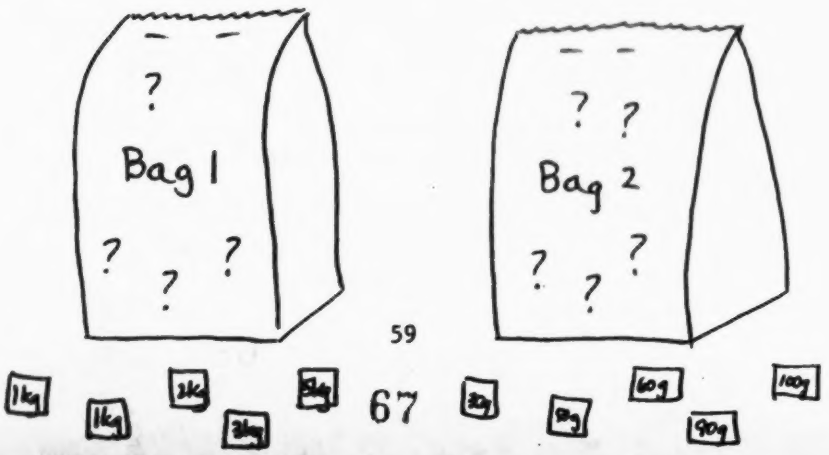


Fred Fullback - 130 kg
Gynny Gymnast 40 kg
Me - 50 kg
"My mass is $\frac{5}{13}$ of Fred's."
"My mass is $\frac{5}{4}$ of Gynny's -- that's $1\frac{1}{4}$ times as great."


Our Body Masses

MYSTERY MASSES

LIFT THE BAGS THAT CONTAIN MYSTERY OBJECTS. ESTIMATE THE MASS OF EACH. WRITE YOUR ESTIMATES ON LITTLE PIECES OF PAPER.


Bag 1
Bag 2
1kg
1kg
2kg
3kg
5kg
30g
50g
60g
90g
100g

BODY MASSES

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL WEIGH THEMSELVES AND MAKE A GRAPH.

MATERIALS: STEP-ON SCALES, LAMINATED BODY MASS GRAPH (PATTERN MG-28).

TEACHING NOTES: CHILDRENS' MASSES ARE CONFIDENTIAL. MARKING THE GRAPH WITH ITS WIDE INTERVALS DOES NOT IDENTIFY ANY PARTICULAR CHILD WITH A MASS. CRAYON MARKS ON THE GRAPH WIPE AWAY FOR SUBSEQUENT USE.

MYSTERY MASSES

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL ORDER OBJECTS ACCORDING TO MASS, ESTIMATE, AND WEIGH OBJECTS.

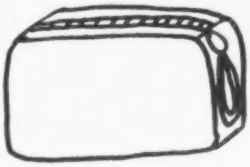
MATERIALS: 3 OBJECTS OF VARYING MASSES HIDDEN IN PAPER BAGS, SCALES, SMALL PIECES OF PAPER.

TEACHING NOTES: HAVE A GROUP OF CHILDREN WORK ON EACH SERIES OF GUESSES. ARRANGE GUESSES IN ORDER. DETERMINE THE RANGE , MODE(S), MEDIAN, AND MEAN OF THE GUESS. SEPARATE PAPERS MAKE DATA EASY TO ARRANGE.

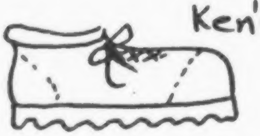
MG-11h

MASS OF MY --- *

FIND 3 OR MORE OBJECTS YOU WOULD LIKE TO WEIGHT. ESTIMATE THE MASS OF EACH. USE AN APPROPRIATE SCALE AND GET THE MASS OF EACH. RECORD YOUR RESULTS ON THE CHART.


Jody's purse

6 pennies


Ken's shoe

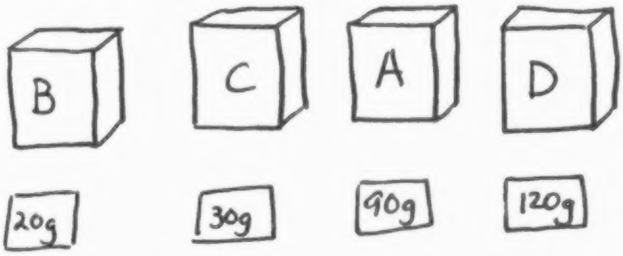
Record the MASS

Object	Estimated Mass	Actual Mass

MG-11h

SAME VOLUME, DIFFERENT MASSES

CHOOSE SOME SIMILAR CONTAINERS. ARRANGE THEM IN ORDER FROM LIGHTEST TO HEAVIEST. ESTIMATE THE MASS OF EACH. NOW WEIGH EACH. RECORD YOUR RESULTS.



Masses of Similar Containers

A		B		C		D	
est.	Act.	est.	act.	est	act.	est	act

MASS OF MY ---

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL ESTIMATE AND WEIGHT AT LEAST 3 OBJECTS AND RECORD RESULTS.

MATERIALS: SCALES, CHART (PATTERN MG-30).

TEACHING NOTES: ENCOURAGE CHILDREN TO BE CREATIVE IN THEIR CHOICES OF OBJECTS TO WEIGHT. SHOES, PURSES, WALLETS, ARE POSSIBLE CHOICES. SHARE RESULTS OR POST THEM ON BULLETIN BOARD.

SAME VOLUME, DIFFERENT MASSES

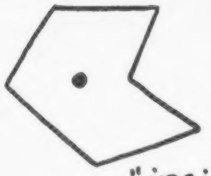
APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL ORDER OBJECTS ACCORDING TO MASS, ESTIMATE, AND WEIGH OBJECTS.

MATERIALS: SEVERAL SETS OF 4 CONTAINERS EACH (EACH IS LETTERED AND FILLED WITH DIFFERENT INGREDIENTS--SAND, WATER, PAPER CLIPS, STYROFOAM, PAPER--TO HAVE A DIFFERENT MASS), CHART (PATTERN MG-29), SCALES.

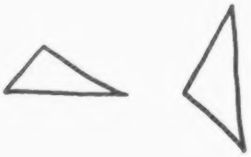
TEACHING NOTES: CHILDREN CAN HELP TO GATHER AND PREPARE THE CONTAINERS.

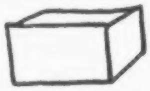
SOME GEOMETRY CONTENT



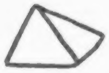
RELATIONAL WORDS

PLANE FIGURES





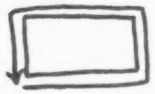
SOLID FIGURES



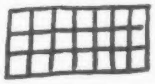
SEGMENTS & LINES

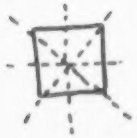
ANGLES

PERIMETER



AREA





SYMMETRY

MG-12b

PRINCIPLES FOR TEACHING GEOMETRY

#1 BASE STUDY ON EXPLORATION

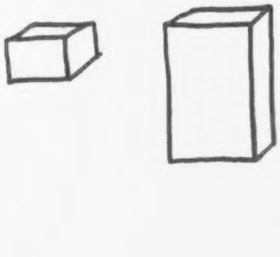
#2 USE VARIOUS MODELS:

real world
classroom manipulatives
child-made models
conventional symbols, names

#3 MASTER VOCABULARY ONLY AFTER EXPERIENCES

#4 RELATE MEASUREMENT & GEOMETRY

#5 COMPARE FIGURES


"6 surfaces"
"right angles"
"taller"
"wider"

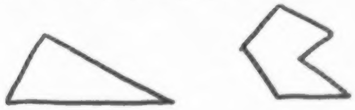
skill development

GATHERING AND ORGANIZING DATA

- LIST ATTRIBUTES OF FIGURES

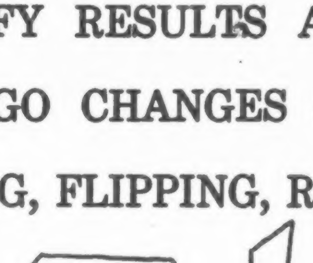
"flat ... 4 angles ... 1 right angle ... 2 equal sides."

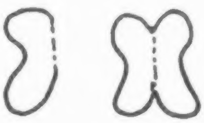
- COMPARE FIGURES



"straight sides"
"3 sides ... 6 sides"
"2 acute angles"

- IDENTIFY RESULTS AS FIGURES UNDERGO CHANGES SUCH AS FOLDING, FLIPPING, ROTATING



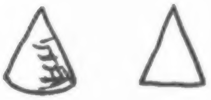


- IDENTIFY REPRESENTATIONS

"isosceles triangle"

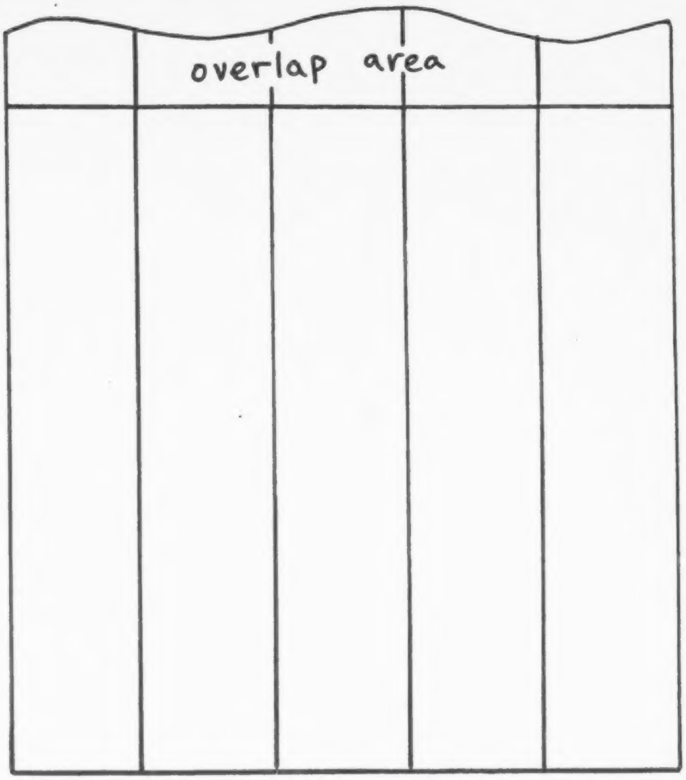
- DESCRIBE RELATIONSHIPS AMONG FIGURES

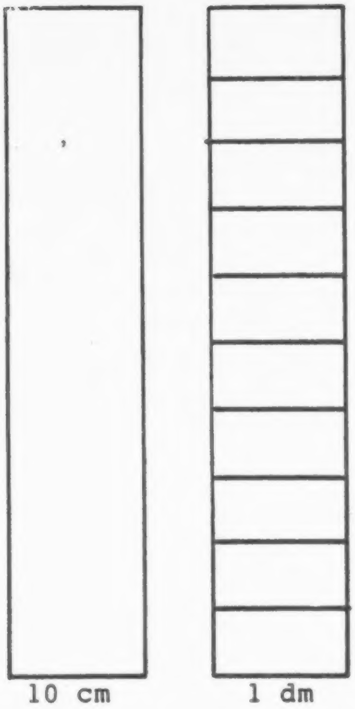
"larger"
"solid ... plane"
"more surfaces"



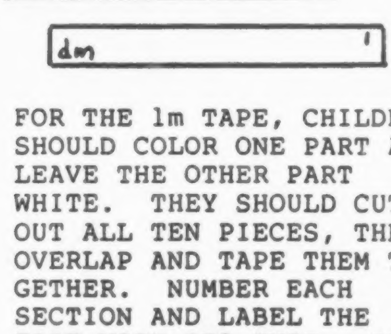
Make your own measuring tape patterns.

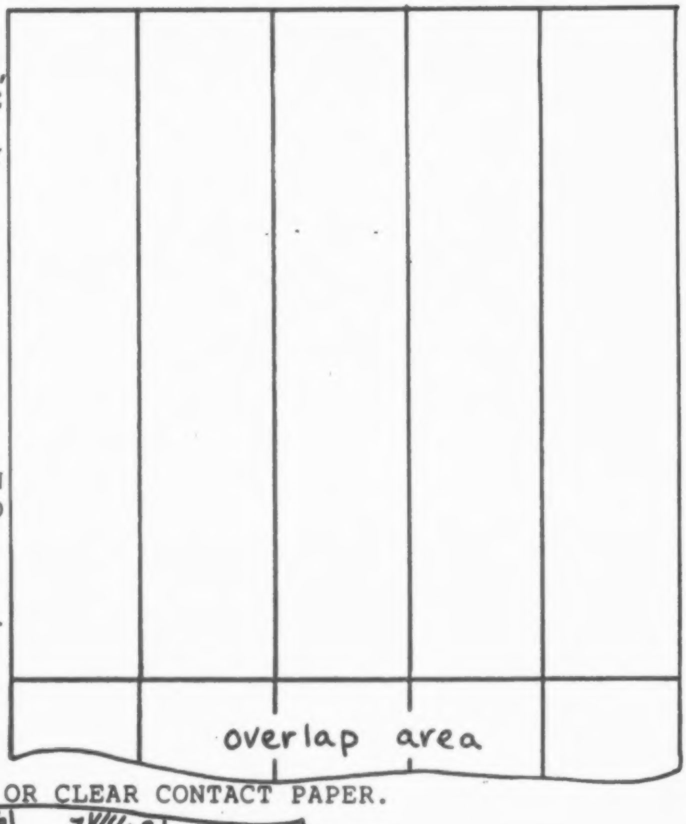
MG-14


overlap area

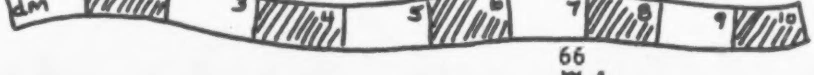


FOR THE 10cm AND 1m TAPES, CHILDREN SHOULD COLOR THE SECTIONS, THEN CAREFULLY CUT OUT THE PIECES. THEY SHOULD NUMBER EACH SECTION AND LABEL EACH TAPE WITH ITS UNIT.


cm
1
2
3
4
5
6
7
8
9
10
dm
1


overlap area

FOR THE 1m TAPE, CHILDREN SHOULD COLOR ONE PART AND LEAVE THE OTHER PART WHITE. THEY SHOULD CUT OUT ALL TEN PIECES, THEN OVERLAP AND TAPE THEM TOGETHER. NUMBER EACH SECTION AND LABEL THE TAPE WITH ITS UNIT. REINFORCE THE TAPE WITH CLEAR PLASTIC TAPE, MASKING TAPE, OR CLEAR CONTACT PAPER.


dm
1
2
3
4
5
6
7
8
9
10

MG-15

PATTERN FOR METRIC AWARENESS SIGN

METRIC AWARENESS

LENGTH OF

____

MEASURED BY

____

Some Measurements In My Classroom

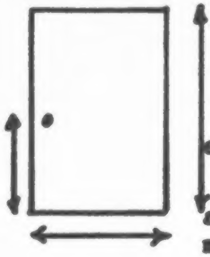
HEIGHT OF SOMEONE ELSE

Who? ____________

estimate ____________

actual measure ____________

FROM THE FLOOR TO THE DOORKNOB



HEIGHT OF THE DOOR

estimate ________

actual measure ________

estimate ________

actual measure ________

ACROSS THE DOOR

estimate ____________

actual measure ____________

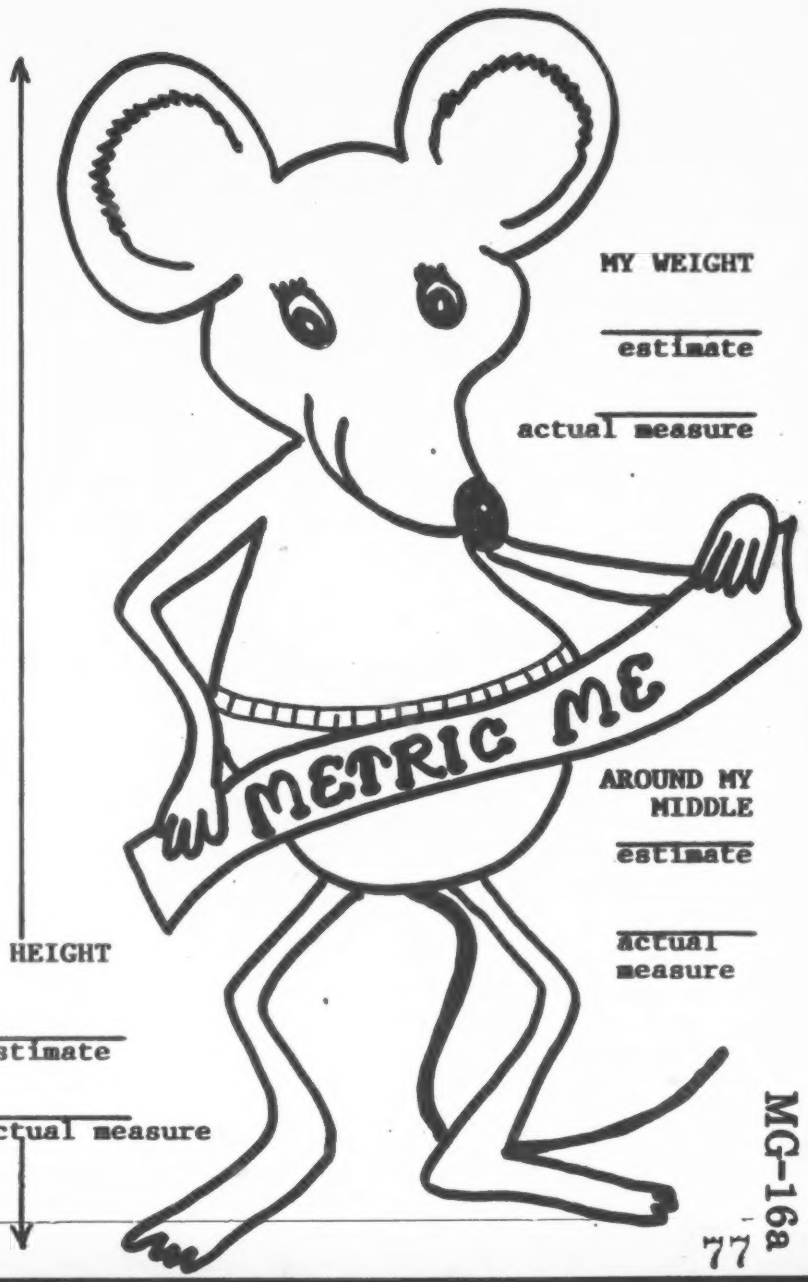
YOUR CHOICE

Draw it.

Make an estimate and measure.

____________ estimate

____________ actual measure



MY WEIGHT

____________ estimate

____________ actual measure

AROUND MY MIDDLE

____________ estimate

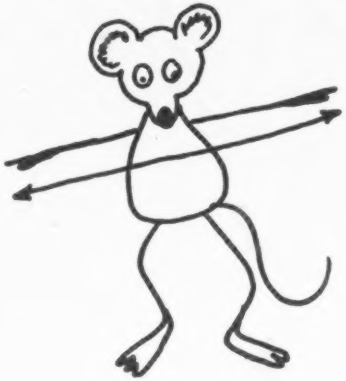
____________ actual measure

MY HEIGHT

____________ estimate

____________ actual measure

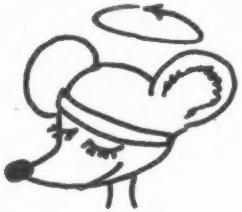
Some Special Measures



MY ARM SPAN

estimate ________

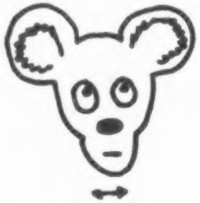
actual measure ________



AROUND BY HEAD

estimate ________

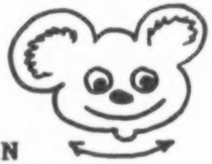
actual measure ________



MY MOUTH WHEN I'M SERIOUS

estimate ________

actual measure ________



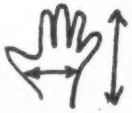
MY MOUTH IN A BIG GRIN

estimate ________ actual measure ________

My Handprint

Trace Your Hand Below.

Make at least 3 measurements on your handprint.



Some Measures In The Classroom

HEIGHT OF A CLASSMATE

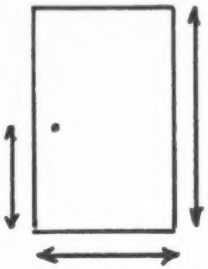
person ______________

estimate ______________

actual measure ______________

A DOOR

	estimate	actual
height	________	________
width	________	________
height from doorknob to floor	________	________



approximate area of door
height x width ______________

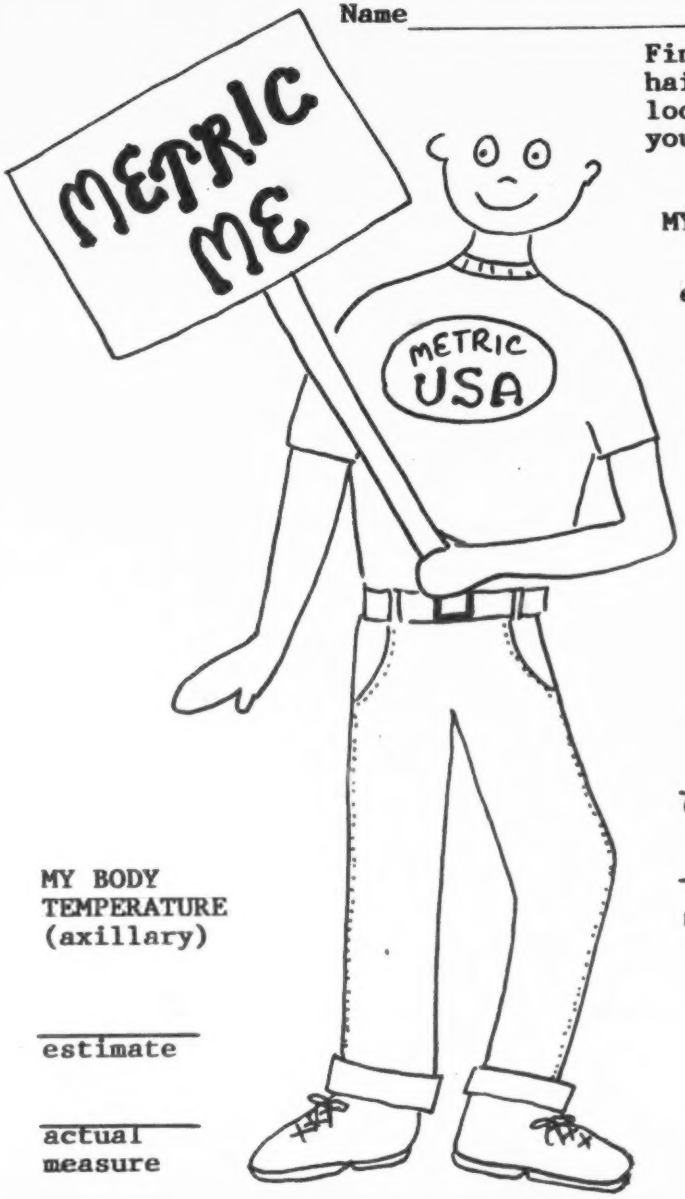
ANOTHER OBJECT (Sketch it.)

	estimate	actual
height	________	________
width	________	________
depth	________	________

approximate volume
height x width x depth ______________

Name ______________________________

Finish the hair to look like yours.



MY HEIGHT

________ estimate

________ actual measure

MY MASS

________ estimate

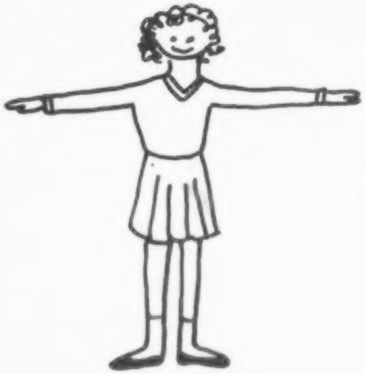
________ actual measure

MY BODY TEMPERATURE (axillary)

________ estimate

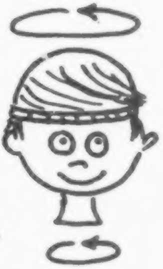
________ actual measure

Some Body Measures



ARM SPAN ______ estimate ______ actual

RATIO OF ARM SPAN TO HEIGHT ______



AROUND HEAD ______ estimate ______ actual

AROUND NECK ______ estimate ______ actual

RATIO OF HEAD TO NECK ______

WIDTH OF MOUTH WHEN SERIOUS ______ estimate ______ actual

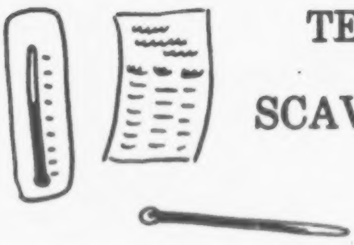
WIDTH OF MOUTH IN A BROAD GRIN ______ estimate ______ actual

RATIO OF SERIOUS TO GRIN ______

Estimate And Measure More Body Parts

BODY PART	ESTIMATE	ACTUAL MEASURE	DIFFERENCE
Across outstretched hand including thumb			
Width of thumb			
Waist			
Length of foot			
Another body measure of your choice ______			

MG-18



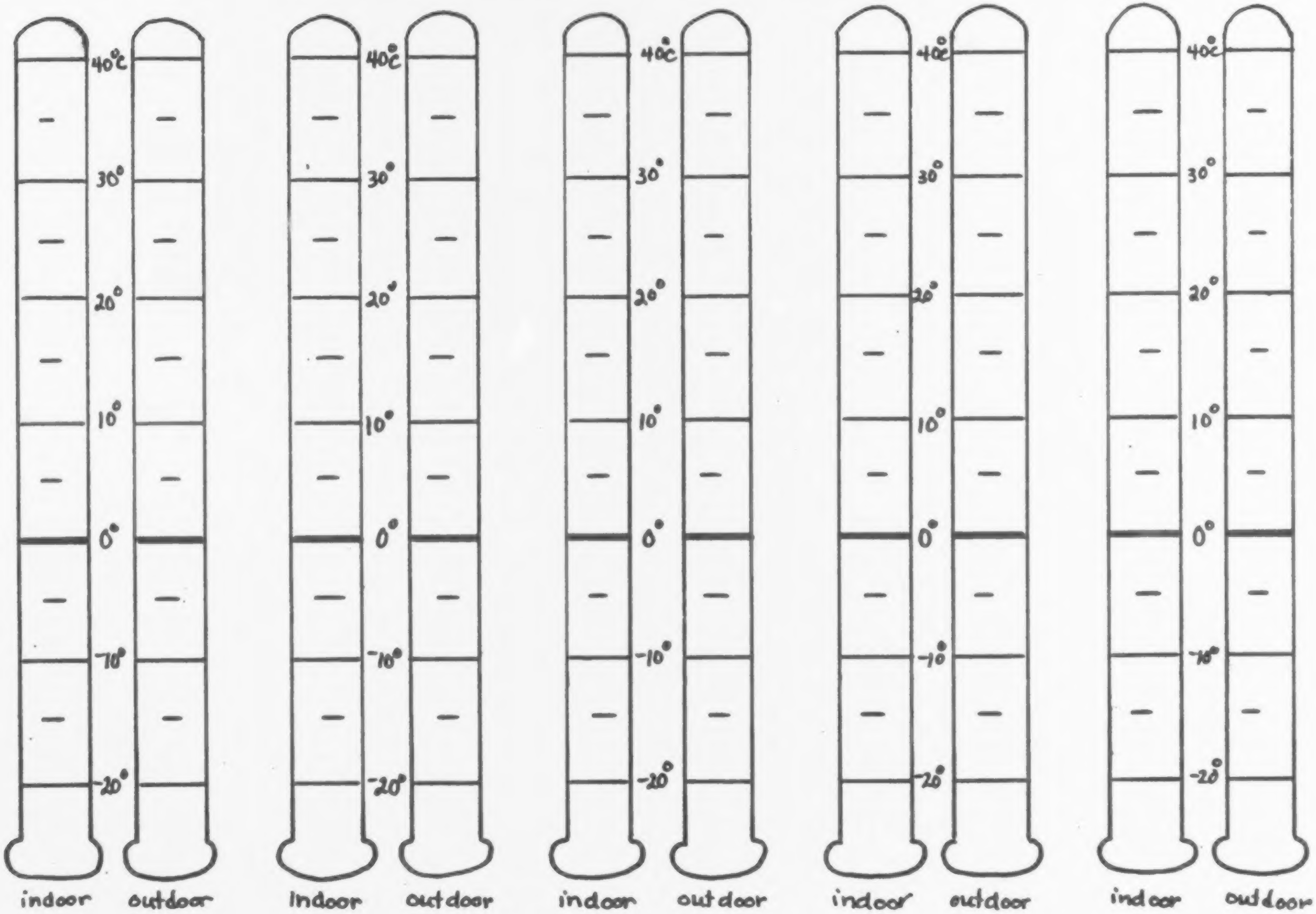
TEMPERATURE SCAVENGER HUNT

FIND THE TEMPERATURE FOR EACH ITEM BELOW. FOR MOST MEASUREMENTS, LEAVE THE THERMOMETER AT LEAST 60 SECONDS. READ IT QUICKLY AFTER 60 SECONDS; IT WILL BEGIN TO RETURN TO ROOM TEMPERATURE ALMOST IMMEDIATELY. USE THE COOKING OR LAB THERMOMETER FOR TEMPERATURES YOU THINK WILL BE ABOVE 50°C.

	ESTIMATE	ACTUAL TEMPERATURE
OUTDOORS	________	________
INDOORS	________	________
DRINKING FOUNTAIN WATER	________	________
TAP WATER		
"HOT"	________	________
"COLD"	________	________
HELD 10 CM ABOVE HOT PLATE	________	________
IN ICE CHEST	________	________
IN CLASSROOM		
HELD 1 DM FROM CEILING	________	________
HELD 1 DM FROM FLOOR	________	________
YOUR CHOICE	________	________

MG-19

INDOOR & OUTDOOR TEMPERATURES

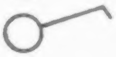
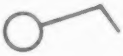
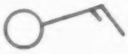
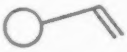
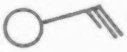
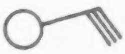
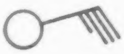
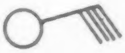
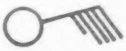
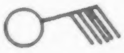
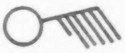
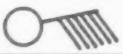

40°C
30°
20°
10°
0°
-10°
-20°
indoor
outdoor

date ________ ________ ________ ________ ________

degrees ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

time of day ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

Beaufort Scale Of Wind Force

Beaufort Number	Wind Effects	Kilometers Per Hour	Weather Bureau Description	Symbol on Weather Map
0	Calm; smoke rises vertically.	less than 2	Calm	
1	Direction of wind shown by smoke drift, but not by wind vanes.	2-5	Light Air	
2	Wind felt on face; leaves rustle; ordinary vane moved by wind.	6-11	Slight Breeze	
3	Leaves and small twigs in constant motion; wind extends light flag.	12-19	Gentle Breeze	
4	Raises dust and loose paper; small branches are moved.	20-29	Moderate Breeze	
5	Small trees in leaf begin to sway; crested wavelets on inland waters.	30-38	Fresh Breeze	
6	Large branches in motion; whistling heard in telegraph wires; umbrellas used with difficulty.	39-50	Strong Breeze	
7	Whole trees in motion; inconvenience felt in walking against wind.	51-60	High Wind	
8	Breaks twigs off trees; generally impedes progress.	61-74	Gale	
9	Slight structual damage occurs (chimney pots and slate removed).	75-86	Strong Gale	
10	Seldom experienced inland, trees uprooted, considerable structural damage occurs	87-100	Whole Gale	
11	Very rarely experienced; accompanied by widespread damage.	101-120	Storm	
12	Great damage	Above 120	Hurricane	

Adapted from Jacobson and Bergman, Science for children (Prentice Hall, 1980), p. 378.

Wind Chill Chart

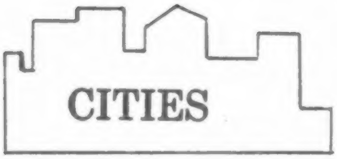
OUTSIDE TEMPERATURE (°C)	6	10	20	30	40	50	60	70	80	90	100
20	20	18	16	14	13	13	12	12	12	12	12
16	16	14	11	9	7	7	6	6	5	5	5
12	12	9	5	3	1	0	-0	-1	-1	-1	-1
8	8	5	0	-3	-5	-6	-7	-7	-8	-8	-8
4	4	0	-5	-8	-11	-12	-13	-14	-14	-14	-14
0	0	-4	-10	-14	-17	-18	-19	-20	-21	-21	-21
-4	-4	-8	-15	-20	-23	-25	-26	-27	-27	-27	-27
-8	-8	-13	-21	-25	-29	-31	-32	-33	-34	-34	-34
-12	-12	-17	-26	-31	-35	-37	-39	-40	-40	-40	-40
-16	-16	-22	-31	-37	-41	-43	-45	-46	-47	-47	-47
-20	-20	-26	-36	-43	-47	-49	-51	-52	-53	-53	-53
-24	-24	-31	-42	-48	-53	-56	-58	-59	-60	-60	-60
-28	-28	-35	-47	-54	-59	-62	-64	-65	-66	-66	-66
-32	-32	-40	-52	-60	-65	-68	-70	-72	-73	-73	-73
-36	-36	-44	-57	-65	-71	-74	-77	-78	-79	-79	-79
-40	-40	-49	-63	-71	-77	-80	-83	-85	-86	-86	-86
-44	-44	-53	-68	-77	-83	-87	-89	-91	-92	-92	-92
-48	-48	-58	-73	-82	-89	-93	-96	-98	-99	-99	-99
-52	-52	-62	-78	-88	-95	-99	-102	-104	-105	-105	-105
-56	-56	-67	-84	-94	-101	-105	-109	-111	-112	-112	-112
-60	-60	-71	-89	-99	-107	-112	-115	-118	-118	-118	-118

WIND SPEED (km/h)

TO FIND THE APPROXIMATE WIND CHILL, READ ACROSS FROM THE OUTDOOR TEMPERATURE AND DOWN FROM THE WIND SPEED IN KILOMETERS PER HOUR. FOR EXAMPLE WHEN THE OUTDOOR TEMPERATURE IS 4°C AND THE WIND SPEED IS 16 KM/H, THE WIND CHILL IS -5°C.

MG-21

TEMPERATURES IN DIFFERENT CITIES

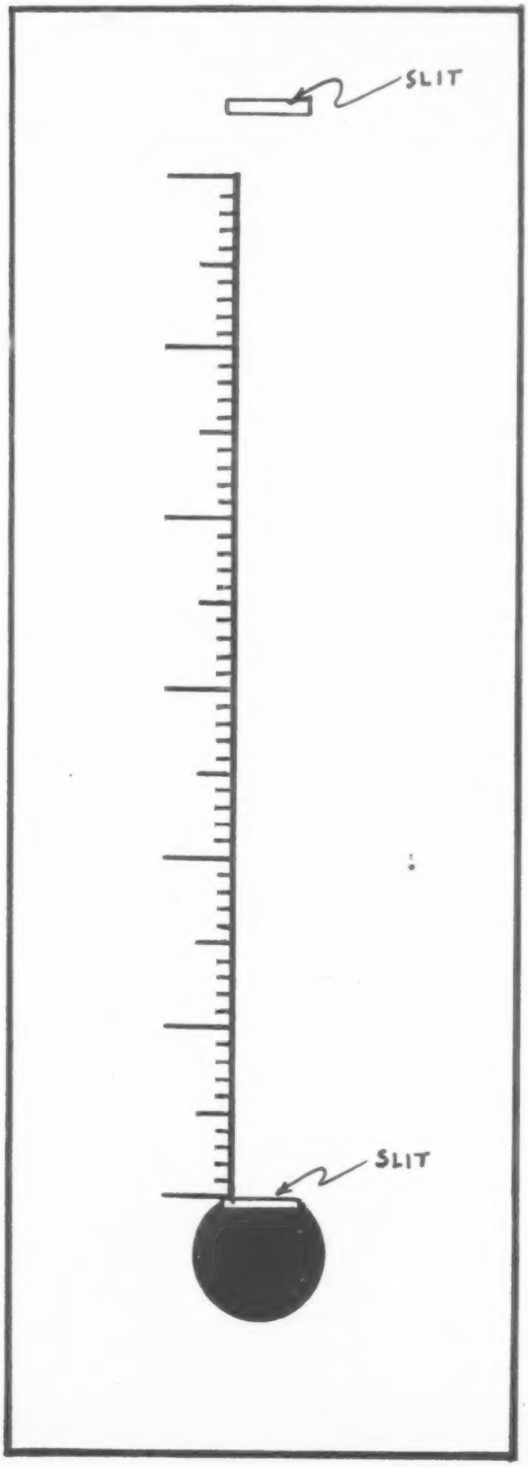


Take the temperature in your own community.
Use the newspaper to get the temperatures for two other cities. Record the temperatures in the chart.

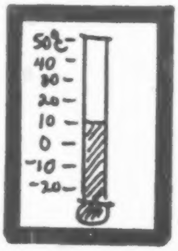
Date	Your Community	City #1	City #2	Comments

MG-22

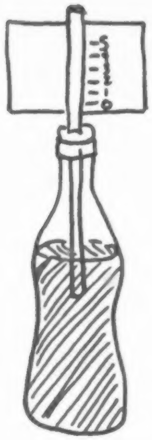
THERMOMETER MODEL



1. NUMBER THE SCALE IN 10° INTERVALS FROM 50° TO -20°C. NUMBER THE 5° INTERVALS.
2. CUT OUT THE NUMBERED SCALE.
3. GLUE THE PATTERN TO A 20 X 30CM PIECE OF POSTER BOARD.
4. CAREFULLY CUT OUT THE 2 SLITS.
5. COLOR HALF A 60CM PIECE OF ELASTIC WITH A PERMANENT RED MARKER. COLOR THE "BULB."
6. THREAD THE ELASTIC THROUGH THE SLITS. STAPLE OR SEW IT TOGETHER AT THE BACK.
7. SLIDE THE ELASTIC TO SHOW VARIOUS TEMPERATURES.



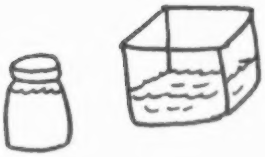
MG-23



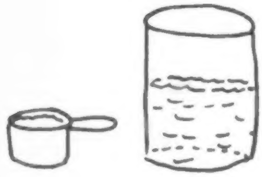
Results Of Using The

HOMEMADE THERMOMETER

WHERE THE THERMOMETER WAS PLACED	SKETCH OF WHAT IT LOOKED LIKE	NUMBER THE WATER REACHED



FILL-'ER-UP

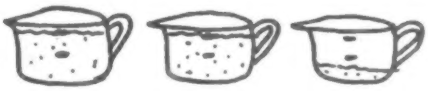


WHICH SMALL CONTAINER?	WHICH LARGE CONTAINER?	ESTIMATE OF NO. OF SMALLS NEEDED TO FILL LARGE CONTINAER	ACTUAL NO. OF SMALLS NEEDED TO FILL LARGE CONTAINER

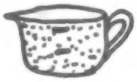
HOT WATER PLAYDOUGH

Measure and mix in a bowl:

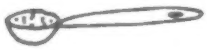
600 ml flour



250 ml salt

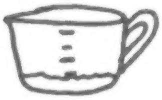


15 ml alum



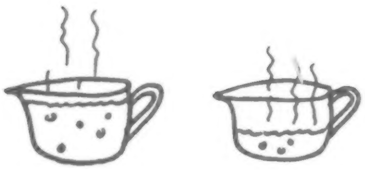
Add:

60 ml oil



Stir in:

375 ml hot water



Knead on table top until smooth. Add a few drops of food coloring if desired. Store in an airtight container.

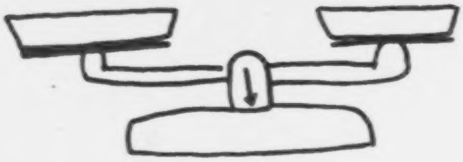
MG-26

VOLUMES -- DRAW A CARD, ESTIMATE, AND MEASURE VOLUME

1 liter	50 mℓ	15 mℓ	100 mℓ	.5 ℓ
700 mℓ	25 mℓ	250 mℓ	400 mℓ	2.5 ℓ
.75 ℓ	300 mℓ	1.5 liters	1.25 ℓ	half a liter
800 mℓ	2 liters	30 mℓ	100 mℓ	.25 ℓ
200 mℓ	fourth of 1 ℓ	900 mℓ	500 mℓ	5 mℓ

MG-27

WHAT THINGS BALANCE?



____________ balances ____________

____________ balances ____________

____________ balances ____________

____________ balances ____________

____________ balances ____________

____________ balances ____________

____________ balances ____________

MG-28

Our *MASSES* In Kilograms

UNDER 20 KG	21-30 KG	31-40 KG	41-50 KG	51-60 KG	61-70 KG	OVER 70 KG

MG-29

MASSES OF IDENTICAL CONTAINERS

ORDER OF CONTAINERS-LIGHTEST TO HEAVIEST	Container A		Container B		Container C		Container D	
	ESTIMATED MASS	ACTUAL MASS	ESTIMATED MASS	ACTUAL MASS	ESTIMATED MASS	ACTUAL MASS	ESTIMATED MASS	ACTUAL MASS

84

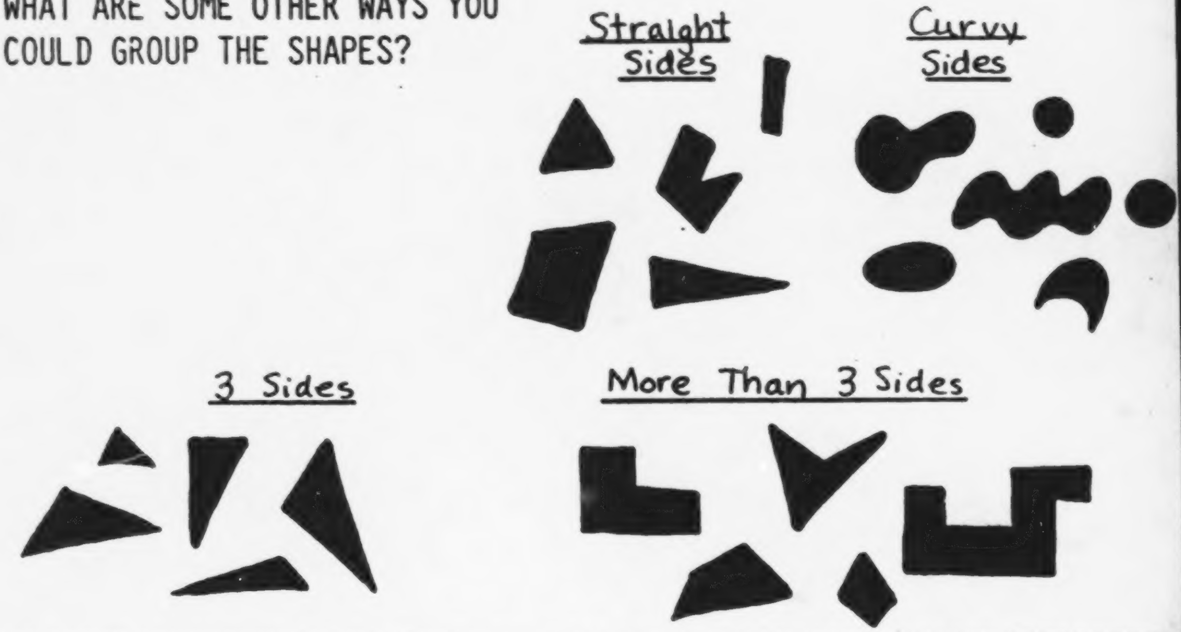
MG-30

WHAT'S THE MASS?

Your Name	Object	Estimated Mass	Actual Mass

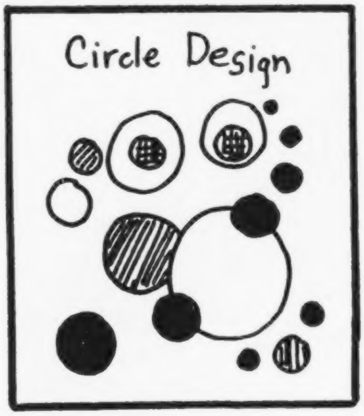
SORTING GEOMETRIC FIGURES

SORT THE SHAPES INTO GROUPS WHERE THE SHAPES ARE ALIKE. WHAT ARE SOME OTHER WAYS YOU COULD GROUP THE SHAPES?


Straight Sides
Curvy Sides
3 Sides
More Than 3 Sides

GEOMETRIC ART

CHOOSE AT LEAST 5 GEOMETRIC SHAPES OF A CERTAIN KIND. ARRANGE THE SHAPES THE WAY YOU WANT THEM TO MAKE A PICTURE. GLUE THEM TO PAPER. NAME YOUR PICTURE IF YOU LIKE.


Circle Design

Triangle Space Girl

SORTING GEOMETRIC FIGURES

APPROXIMATE GRADE LEVEL: K-4

OBJECTIVES: CHILDREN WILL GROUP SHAPES ACCORDING TO VARIOUS CHARACTERISTICS.

MATERIALS: PRECUT PAPER SHAPES, GEOMETRIC SOLIDS, DRAWINGS OF GEOMETRIC SHAPES ON SMALL CARDS (PATTERN MG-32 GIVES A FEW POSSIBILITIES).

TEACHING NOTES: LET CHILDREN SUGGEST WAYS TO SORT THE SHAPES AND NAME THE GROUPS. ALSO YOU SHOULD SUGGEST VARIOUS WAYS TO SORT ACCORDING TO CHARACTERISTICS THE CHILDREN ARE STUDYING. CHILDREN COULD ARRANGE THE SHAPES ON A TABLE OR PIN THEM TO A BULLETIN BOARD.

GEOMETRIC ART

APPROXIMATE GRADE LEVEL: K-2

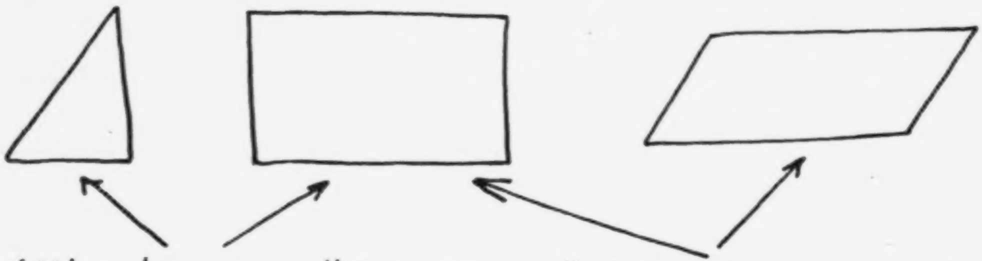
OBJECTIVES: CHILDREN WILL SELECT LIKE SHAPES AND ARRANGE THEM TO FORM PICTURES.

MATERIALS: PRECUT PAPER SHAPES OF DIFFERENT COLORS AND SIZES, PAPER, GLUE, SCISSORS.

TEACHING NOTES: NOTES MAY SPECIFY THE SHAPES TO BE USED SUCH AS "MAKE A CIRCLE DESIGN" OR LET THE CHILDREN TELL ABOUT THEIR CREATIONS. IF THEY CAN, HAVE CHILDREN CUT MORE OF THEIR OWN SHAPES TO USE IN THEIR PICTURES.

ALIKE AND DIFFERENT*

WORK WITH A GROUP OF STUDENTS. CHOOSE 2 SHAPES. TELL ONE WAY THEY ARE ALIKE AND ONE WAY THEY ARE DIFFERENT. LET ANOTHER PERSON CHOOSE A THIRD SHAPE AND TELL HOW IT IS LIKE AND DIFFERENT FROM THE SECOND SHAPE. CONTINUE THE SAME IN THE SAME WAY.



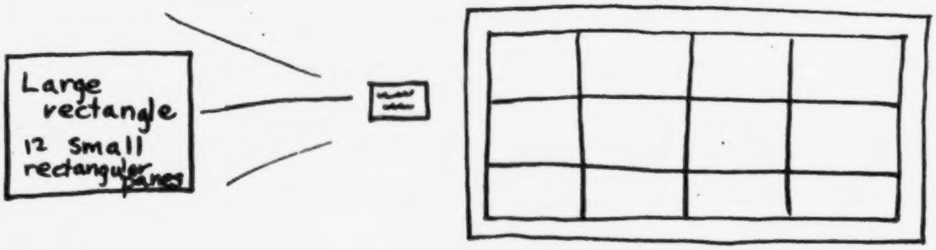
" Alike because they each have a square corner ... different numbers of sides ... "

" Alike because each has 4 sides ... different because one has right angles and one doesn't "

MG-31b

SHAPE WALK

TAKE A WALK AROUND YOUR SCHOOL BUILDING AND SCHOOL GROUNDS. IDENTIFY AS MANY DIFFERENT GEOMETRIC SHAPES AS YOU CAN. INCLUDE SOME SOLID FIGURES--CYLINDERS, PRISMS, SPHERES--AS WELL AS LOTS OF POLYGONS. IF YOU CAN, PUT SIGNS ON DIFFERENT SHAPES FOR OTHERS TO NOTICE AND READ.



ALIKE AND DIFFERENT

APPROXIMATE GRADE LEVEL: K-6

OBJECTIVES: CHILDREN WILL TELL LIKENESS AND DIFFERENCES IN GEOMETRIC SHAPES.

MATERIALS: GEOMETRIC SOLIDS, COMMON OBJECTS, DRAWINGS OF GEOMETIRC SHAPES ON SMALL CARDS (PATTERN SHEET MG-32).

TEACHING NOTES: THIS ACTIVITY IS QUITE OPEN-ENDED: IT ALLOWS FOR MANY INTERPRETATIONS. VARY CONTENT TO FIT LEARNERS.

SHAPE WALK

APPROXIMATE GRADE LEVEL: K-3

OBJECTIVES: CHILDREN WILL IDENTIFY GEOMETRIC SHAPES IN THE ENVIRONMENT.

MATERIALS: IF APPROPRIATE, PAPER, MARKERS, AND TAPE TO FIX SIGNS TO IDENTIFY SHAPES FOR OTHERS TO NOTICE.

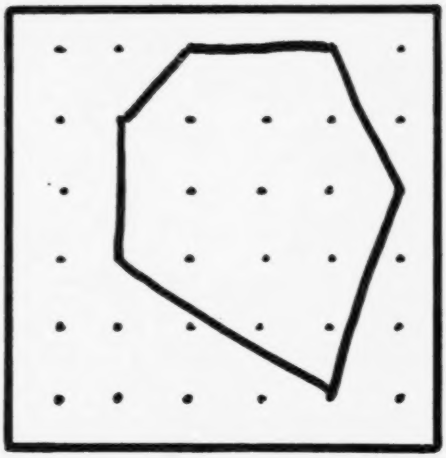
TEACHING NOTES: YOU MIGHT ASSIGN SMALL GROUPS OF CHILDREN CERTAIN SHAPES TO LOOK FOR--ONE GROUP MIGHT FIND "9 SQUARES IN A BLOCK;" ANOTHER MIGHT FIND "TRIANGLES ABOVE OUR HEADS;" ANOTHER GROUP COULD LOOK FOR "RECTANGLES MADE OF GLASS."

EXPLORING WITH GEOBOARDS*

USE THE RUBBER BANDS ON THE GEOBOARDS TO MAKE LOTS OF DIFFERENT GEOMTRIC FIGURES. TELL A PARTNER ABOUT THE FIGURES YOU MAKE. CAN YOU MAKE THESE?

- -TRIANGLES OF TWO DIFFERENT SIZES
- -SQUARES
- -RECTANGLES
- -HEXAGONS
- -ACUTE ANGLES
- -OBTUSE ANGLES
- -RIGHT ANGLES

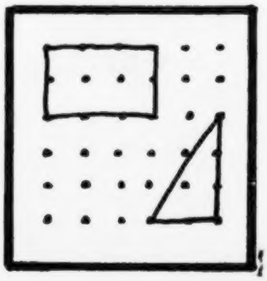
WHAT OTHER DESIGNS CAN YOU MAKE?

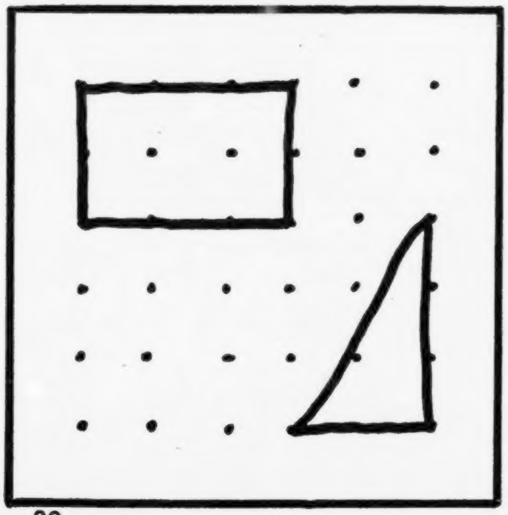


"hexagon"

COPY IT ON THE GEOBOARD*

CHOOSE A PATTERN. DUPLICATE THE PATTERN ON THE GEOBOARD. MAKE SOME GEOBOARD PATTERNS FOR OTHERS TO COPY.





EXPLORING WITH GEOBOARDS

APPROXIMATE GRADE LEVEL: K-6

OBJECTIVES: CHILDREN WILL EXPLORE POSSIBILITIES OF SHOWING DIFFERENT GEOMETRIC SHAPES ON THE GEOBOARDS.

MATERIALS: GEOBOARDS, COLORED RUBBER BANDS.

TEACHING NOTES: USE VOCABULARY APPROPRIATE FOR YOUR GRADE LEVEL. LET CHILDREN CHALLENGE EACH OTHER WITH SHAPES TO MAKE. HAVE CHILDREN NAME AND DESCRIBE THE SHAPES THEY HAVE MADE.

COPY IT ON THE GEOBOARD

APPROXIMATE GRADE LEVEL: K-3

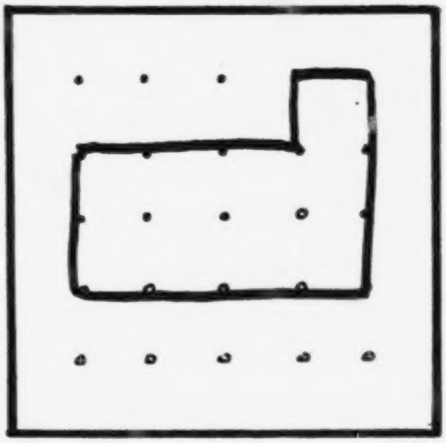
OBJECTIVES: CHILDREN WILL COPY SHAPE PATTERNS ONTO GEOBOARDS NOTING COLORS, SIZES, SHAPES, AND POSITIONS.

MATERIALS: GEOBOARDS, COLORED RUBBER BANDS, GEOBOARD CARDS (PATTERN MG-33 GIVES SOME SUGGESTIONS).

TEACHING NOTES: ASK CHILDREN TO DESCRIBE AND NAME THE SHAPES THEY MAKE. LET CHILDREN SHARE THE PATTERNS THEY DRAW WITH OTHERS.

GEOBOARD AREAS AND PERIMETERS*

WORK WITH A PARTNER. MAKE A SHAPE ON THE GEOBOARD. START WITH DIFFERENT SQUARES AND RECTANGLES. THEN WORK WITH OTHER FIGURES THAT ARE COMBINATIONS OF SQUARES AND RECTANGLES. ASK YOUR PARTNER TO TELL ITS AREA AND PERIMETER. CHECK EACH OTHER'S WORK.

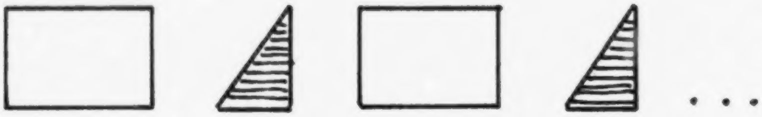


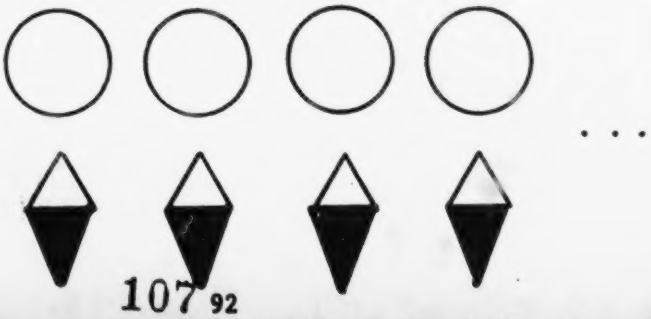
Area = 9
square units

Perimeter: 14 units

BLOCK PATTERNS

START A PATTERN WITH COLORED PARQUETRY BLOCKS. ASK A PARTNER TO CONTINUE THE PATTERN. CHECK YOUR PARTNER. TAKE TURNS STARTING AND CONTINUING MORE PATTERNS.





GEOBOARDS AREAS AND PERIMETERS

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL ARRANGE VARIOUS SHAPES AND DETERMINE THEIR AREAS AND PERIMETERS.

MATERIALS: GEOBOARDS, COLORED RUBBER BANDS, RULERS.

TEACHING NOTES: BEGIN WITH SQUARES AND RECTANGLES, THEN MOVE TO OTHER FIGURES WITH RIGHT ANGLES. EXPRESS AREAS IN SQUARE UNITS. USE RULERS TO DETERMINE PERIMETERS OF TRIANGLES. AREAS OF TRIANGLES WILL PROBABLY HAVE TO BE EXPRESSED IN APPROXIMATE UNITS.

BLOCK PATTERNS

APPROXIMATE GRADE LEVEL: K-2

OBJECTIVES: CHILDREN WILL START, CONTINUE, AND DESCRIBE PATTERNS.

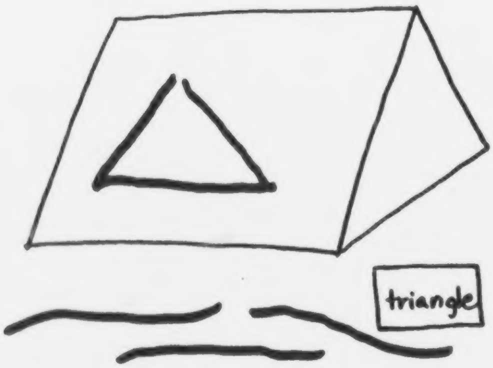
MATERIALS: WOODEN PARQUETRY BLOCKS IN VARIOUS SHAPES AND COLORS.

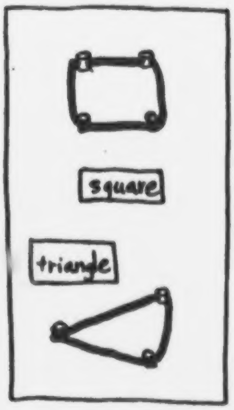
TEACHING NOTES: THE CHILD WHO BEGINS EACH PATTERN SHOULD PROVIDE 2 OR 3 SEQUENCES OF THE PATTERN SO HIS/HER PARTNER CAN FOLLOW IT. ENCOURAGE VERBAL DESCRIPTIONS OF PATTERNS.

MG-31e

YARN CLOSED CURVES*

SELECT A SLIP OF PAPER. USE YARN TO FORM THE FIGURES THE PAPER TELLS YOU. ARRANGE THE YARN ON A FLANNEL BOARD. YOU CAN ALSO USE PUSH PINS ON A CELLOTEX BOARD. PUT IN PUSH PINS FOR THE CORNERS OF FIGURES AND WRAP THE YARN AROUND PUSH PINS TO CREATE A POLYGON.

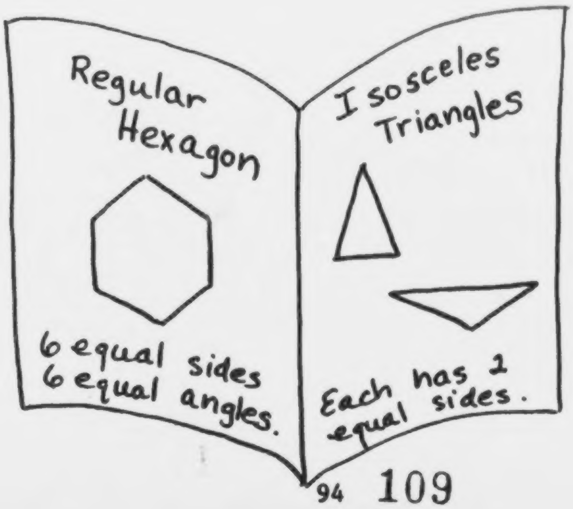



square
triangle

MG-31e

POLYGONS BOOKLET

MAKE A BOOK WHERE YOU DRAW AND TELL ABOUT EACH GEOMETRIC FIGURE YOU ARE STUDYING. WORK HARD TO MAKE YOUR DRAWINGS NEAT AND CLEAR.


Regular Hexagon
6 equal sides
6 equal angles.
Isosceles Triangles
Each has 2 equal sides.

YARN CLOSED CURVES*

APPROXIMATE GRADE LEVEL: K-4

OBJECTVIES: CHILDREN REPRESENT VARIOUS CLOSED CURVES.

MATERIALS: YARN, FLANNEL BOARD, NAMES OF CLOSED CURVES ON SLIPS OF PAPER (OR TELL NAMES TO YOUNGER CHILDREN), PUSH PINS, CELLOTEX BOARD OR BULLETIN BOARD.

TEACHING NOTES: VARY THE NAMES OF CURVES TO FIT THE GRADE LEVEL. ALSO USE NAMES OF ANGLES (ACUTE, OBTUSE, RIGHT).

POLYGONS BOOKLET

APPROXIMATE GRADE LEVEL: 2-6

OBJECTIVES: CHILDREN WILL PICTURE AND DESCRIBE POLYGONS.

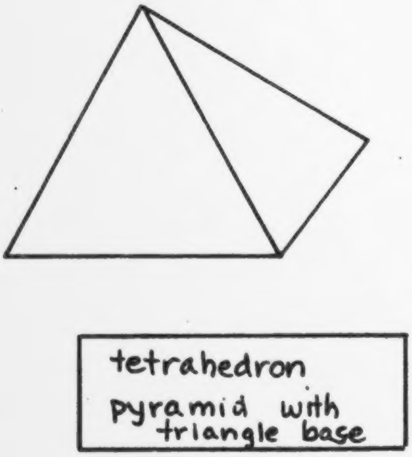
MATERIALS: PAPER, PENCILS, RULERS, STAPLER.

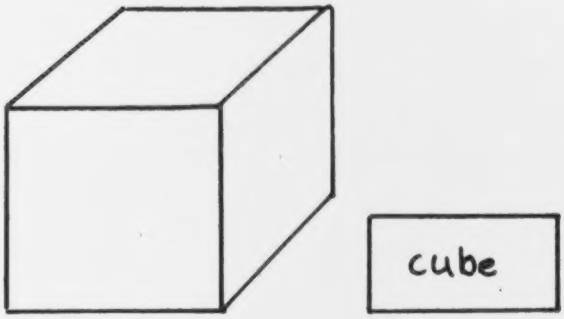
TEACHING NOTES: ASSIGN APPROPRIATE POLYGONS. IF PAPER IS FOLDED IN HALF CROSSWISE, 4 HALF-PAGES RESULT. THIS MAKES A NICE BOOK SIZE. YOU COULD ALSO EXTEND THE BOOKLET TO INCLUDE OTHER GEOMETRIC FIGURES YOU ARE STUDYING.

MG-31f

MODELS OF GEOMETRIC SOLIDS*

CAREFULLY MAKE PAPER MODELS OF GEOMETRIC SOLIDS. CUT THE PATTERN PIECES OUT CAREFULLY. CREASE ON EACH DOTTED LINE. GLUE THE TABS A FEW AT A TIME. PUT THE TABS INSIDE THE SOLID. LABEL EACH MODEL WITH ITS NAME OR MAKE A NAME CARD FOR IT.

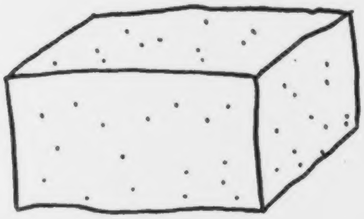


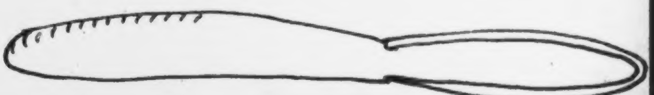


MG-31f

GEOMETRIC SOLIDS OF CLAY*

USE CLAY AND A KNIFE AND YOUR FINGERS TO CUT OUT THE GEOMETRIC SOLIDS. MAKE THE FACES AS SMOOTH AS YOU CAN. LABEL EACH MODEL WITH A NAME CARD. MAKE A CUBE, RECTANGULAR PRISM, SPHERE, AND PYRAMID.





MODELS OF GEOMETRIC SOLIDS

APPROXIMATE GRADE LEVEL: 5-6

OBJECTIVES: CHILDREN WILL MAKE PAPER MODELS OF GEOMETRIC SOLIDS.

MATERIALS: PATTERNS (MG-34), SCISSORS, GLUE, PAPER FOR NAME CARDS.

TEACHING NOTES: REINFORCE THE IDEA THAT SLOW, CAREFUL WORK WILL PRODUCE THE BEST RESULTS IN THIS ACTIVITY. EACH STEP MUST BE DONE ACCURATELY. ALTHOUGH THE CHILDREN MAY HAVE ALREADY MASTERED THE NAMES OF THE SOLIDS, THIS ACTIVITY HELPS THEM UNDERSTAND THE MAKEUP OF EACH SOLID.

GEOMETRIC SOLIDS OF CLAY

APPROXIMATE GRADE LEVEL: 2-6

OBJECTIVES: CHILDREN WILL FORM GEOMETRIC SOLIDS OF CLAY.

MATERIALS: CLAY (OR PLAYDOUGH), PLASTIC KNIVES, COMMERCIALLY MADE MODELS TO COPY, PAPER FOR NAME CARDS, SCISSORS.

TEACHING NOTES: ASSIGN ADDITIONAL SOLIDS IF APPROPRIATE.

112

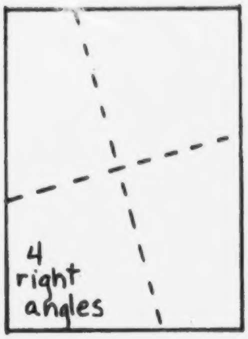
MG-31g

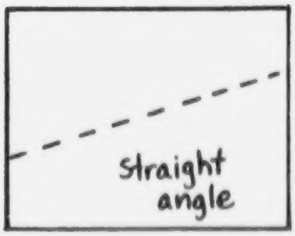
FOLDING ANGLES*

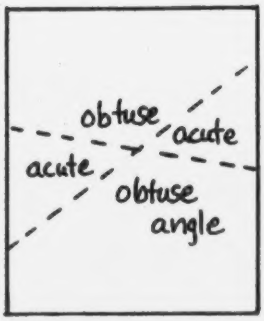
USE PLAIN PAPER. FOLD PIECES OF PAPER INTO THE FOLLOWING ANGLES:

- RIGHT ANGLES
- ACUTE ANGLES
- OBTUSE ANGLES
- STRAIGHT ANGLES

USE YOUR RULER. DRAW OVER EACH FOLD TO DARKEN IN EACH ANGLE. LABEL EACH EXAMPLE.


4
right
angles


straight
angle


obtuse
acute
acute
obtuse
angle

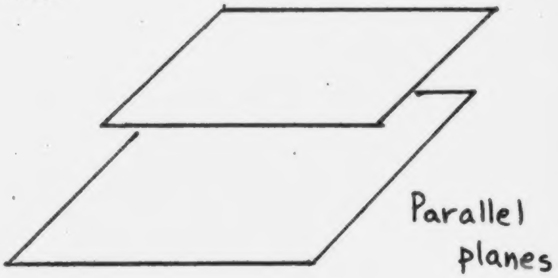
MG-31g

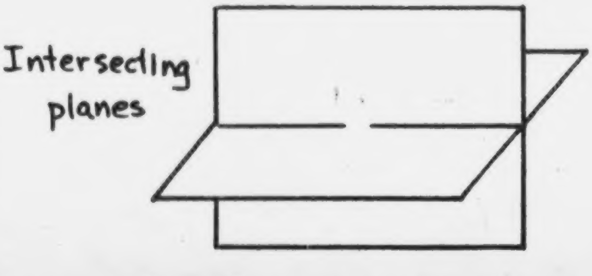
PLANES

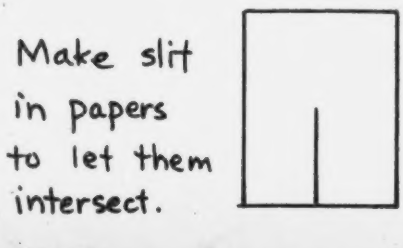
USE PAPERS TO REPRESENT PLANES. HOLD THE PAPERS TO REPRESENT

- PARALLEL PLANES
- INTERSECTING PLANES
- PLANES THAT INTERSECT AT RIGHT ANGLES

FIND EXAMPLES OF DIFFERENT KINDS OF PLANES IN YOU CLASSROOM


Parallel
planes


Intersecting
planes


Make slit
in papers
to let them
intersect.

FOLDING ANGLES

APPROXIMATE GRADE LEVEL:

OBJECTIVES: CHILDREN WILL FOLD PAPER TO REPRESENT ANGLES; THEY WILL DRAW AND LABEL ANGLES.

MATERIALS: PAPER, RULERS.

TEACHING NOTES: THE FOLDED PAPERS CAN BE DISPLAYED OR CAN BE MADE INTO AN ANGLES BOOKLET. NOTICE THAT 4 RIGHT ANGLES ALWAYS RESULT WHEN THE CHILD FOLDS RIGHT ANGLES. MOST PROCEDURES TO FOLD ACUTE ANGLES ALSO RESULT IN OBTUSE ANGLES BEING FORMED.

PLANES

APPROPRIATE GRADE LEVE: 5-6

OBJECTIVES: CHILDREN WILL REPRESENT POSSIBLE RELATIONSHIPS OF PLANES AND FIND EXAMPLES OF THESE RELATIONSHIPS IN THE ENVIRONMENT.

MATERIALS: HALF SHEETS OF PAPER TO REPRESENT PLANES.

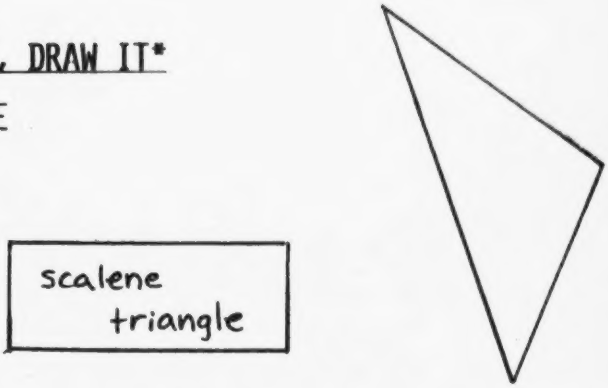
TEACHING NOTES: REMIND CHILDREN THAT PLANES DO NOT END AT THE EDGES LIKE THEIR PAPERS OR WALLS DO. THE PAPERS OR WALLS ARE REPRESENTATIONS OF PLANES.

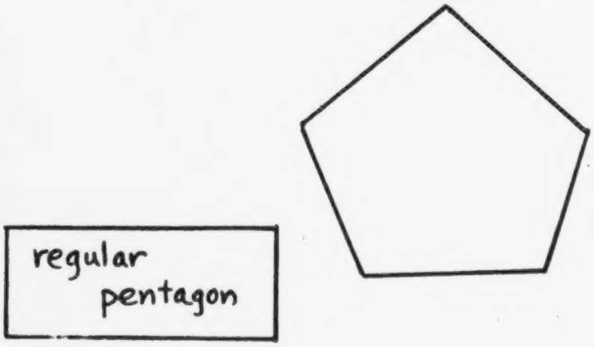
MG-31h

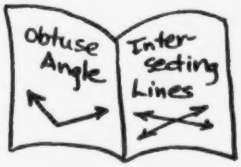
CHOOSE IT, DRAW IT*

PICK A SLIP OF PAPER. DRAW THE FIGURE IT TELLS YOU. LET MEMBERS OF YOUR GROUP CHECK YOU.

PUT YOUR DRAWINGS IN A BOOKLET.


scalene
triangle

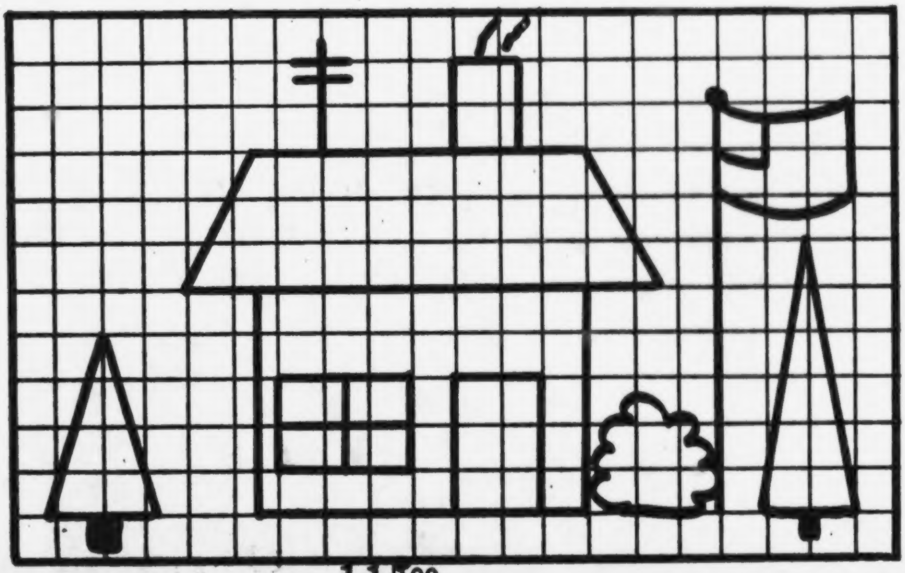

regular
pentagon


Obtuse
Angle
Inter-
secting
Lines

MG-31h

SCALE DRAWINGS

USE GRID PAPER. REPRODUCE THE PICTURE EXACTLY ON DOTTED GRID PAPER.



CHOOSE IT, DRAW IT

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL DRAW REPRESENTATIONS OF LINES, LINE SEGMENTS, AND ANGLES.

MATERIALS: SLIPS OF PAPER (PATTERN MG-36 GIVES SOME POSSIBILITIES), PAPER, PENCILS, RULERS.

TEACHING NOTES: VARY CONTENT ACCORDING TO LEVEL.

SCALE DRAWINGS

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL USE GRID PAPER TO MAKE A SCALE DRAWING.

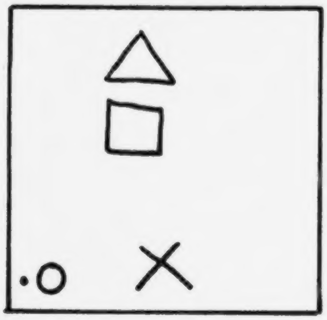
MATERIALS: CHILDRENS' PATTERN (OPPOSITE SIDE), DOT GRID PAPER (PATTERN MG-35).

TEACHING NOTES: HAVE CHILDREN DISCUSS STRATEGIES FOR COMPLETING THE DRAWING. DISCUSS ANGLES, RELATIONSHIPS OF LINES, AND GEOMETRIC SHAPES FOUND IN THE DRAWING.

DIRECTIONS GAME

FOLLOW THE DIRECTIONS YOUR TEACHER GIVES. FOLLOW THESE WRITTEN DIRECTIONS:

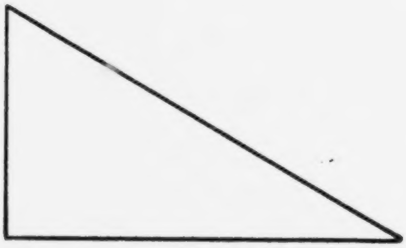
- DRAW A TRIANGLE AT THE TOP OF YOUR PAPER
- DRAW A SQUARE BELOW THE TRIANGLE
- AT THE BOTTOM OF YOUR PAPER, DRAW A CIRCLE
- PUT A DOT AT THE LEFT OF THE CIRCLE
- DRAW AN X IN THE MIDDLE OF THE CIRCLE.

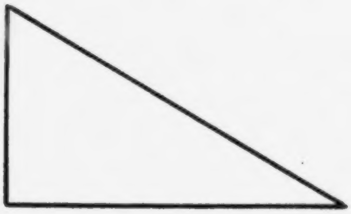


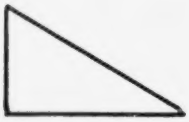
Is this correct?

SORT AND ORDER

SORT THE SHAPES IN PILES THAT ARE ALIKE. ARRANGE SIMILAR SHAPES IN ORDER SMALLEST TO LARGEST.







DIRECTIONS GAME

APPROXIMATE GRADE LEVEL: K-2

OBJECTIVES: CHILDREN WILL FOLLOW DIRECTIONS INVOLVING POSITIONAL AND RELATIONAL WORDS.

MATERIALS: WRITTEN DIRECTIONS FOR SECOND GRADERS (OPPOSITE SIDE).

TEACHING NOTES: CHIDLREN NEED LOTS OF PRACTICE. KEEP SESSIONS SHORT BUT ACTIVE. USE CLASSROOM OBJECTS TO PRACTICE WORDS LIKE OUTSIDE, INSIDE, OVER, UNDER.

SHORT AND ORDER

APPROXIMATE GRADE LEVEL: K-2

OBJECTIVES: CHILDREN WILL SORT AND ORDER GEOMETRIC FIGURES.

MATERIALS: PAPER GEOMETRIC FIGURES (PATTERN MG-37)

TEACHING NOTES: YOU COULD ALSO HAVE CHILDREN SORT SOLID AND FLAT FIGURES. HIDING THE SHAPES (LIKE EASTER EGGS) MAKES THE ACTIVITY FUN. LET THE CHILDREN SORT THE SHAPES MANY WAYS--CURVED VS. STRAIGHT EDGES, RIGHT ANGLES VS. NO ANGLES, ETC.

MG-31i

TANGRAMS*

WORK WITH THE TANGRAM PUZZLE
TO FORM VARIOUS SHAPES.

Can you use all the tangram pieces
and make a shape like a crocodile?

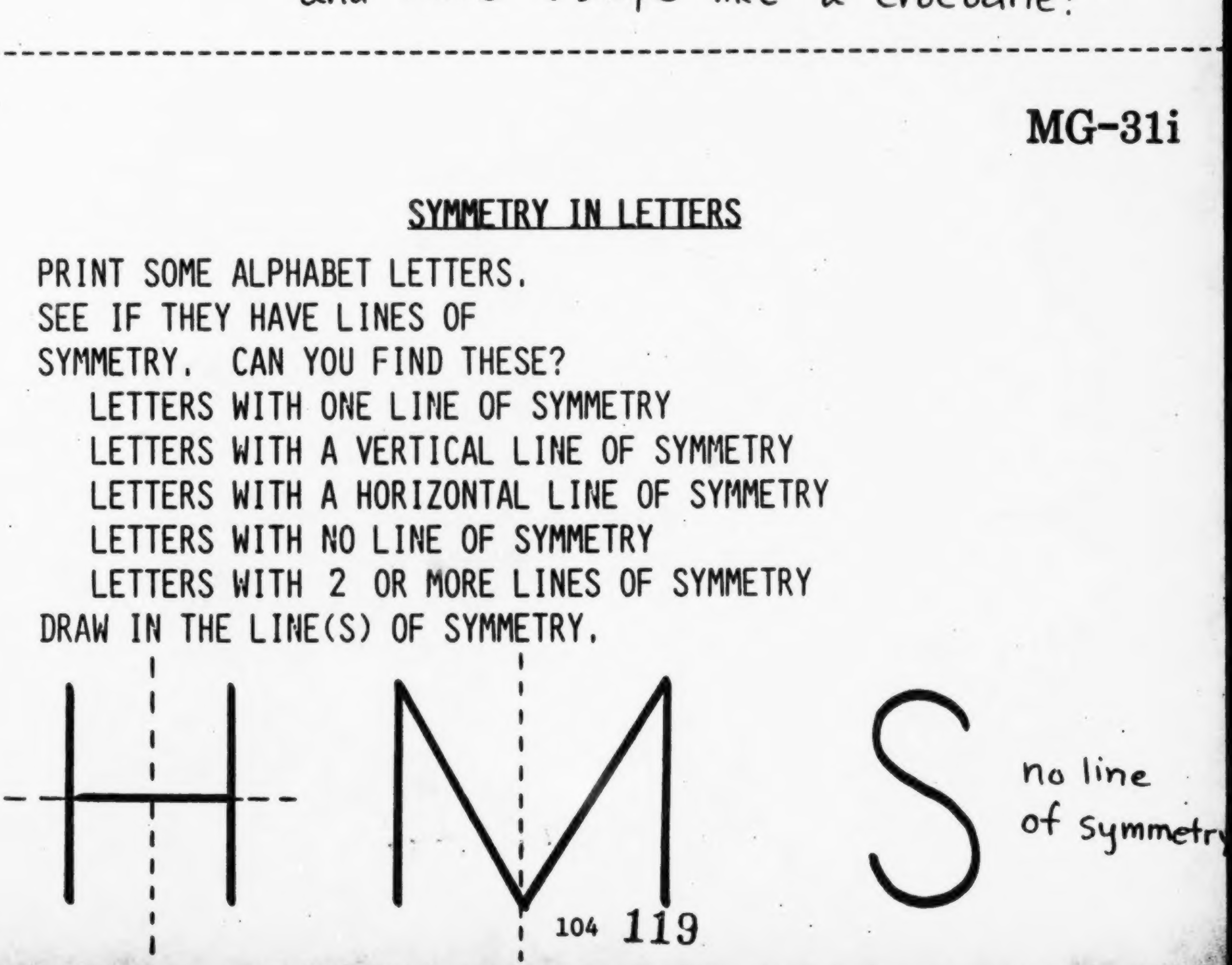
MG-31i

SYMMETRY IN LETTERS

PRINT SOME ALPHABET LETTERS.
SEE IF THEY HAVE LINES OF
SYMMETRY. CAN YOU FIND THESE?

- LETTERS WITH ONE LINE OF SYMMETRY
- LETTERS WITH A VERTICAL LINE OF SYMMETRY
- LETTERS WITH A HORIZONTAL LINE OF SYMMETRY
- LETTERS WITH NO LINE OF SYMMETRY
- LETTERS WITH 2 OR MORE LINES OF SYMMETRY

DRAW IN THE LINE(S) OF SYMMETRY.



TANGRAMS

APPROXIMATE GRADE LEVEL: K-6

OBJECTIVES: CHILDREN WILL WORK WITH TANGRAM PUZZLES.

MATERIALS: TANGRAM PIECES AND PUZZLES (SOME SUGGESTIONS ON PATTERN MG-38).

TEACHING NOTES: OBTAIN OR MAKE SOME MORE TANGRAM PUZZLES. ASSIGN NUMBERS TO THE PIECES. HAVE THE CHILDREN SELECT A FEW PIECES AND SEE WHAT THEY CAN MAKE OF THOSE PIECES. DETERMINE WHAT FRACTION OF THE WHOLE EACH PIECE IS. USE TERMS "CONGRUENT" AND SIMILAR TO DESCRIBE RELATIONSHIPS BETWEEN PIECES. INTRODUCE THE IDEA THAT THE TANGRAM IS A VERY OLE CHINESE PUZZLE. FOR YOUNGER CHILDREN, FILLING IN PREDRAWN SHAPES WITH TANGRAM PIECES IS EASIER THAN SIMPLY MAKING SHAPES SHOWN ON PAGE MG-39B.

SYMMETRY IN LETTERS

APPROXIMATE GRADE LEVEL: 4-6

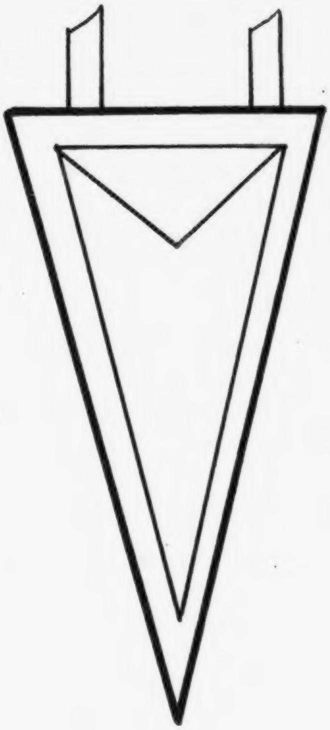
OBJECTIVES: CHILDREN WILL IDENTIFY LINES OF SYMMETRY IN LETTERS.

MATERIALS: PAPER AND PENCILS.

TEACHING NOTES: ALSO LOOK FOR OBJECTS OF SYMMETRY IN GEOMETRIC FIGURES AND CLASSROOM OBJECTS. FOLD PAPER IN HALF (FORMING A LINE OF SYMMETRY) AND CUT OUT SYMMETRICAL SHAPES.

MG-31j

TRANSFORMATIONS #1



MAKE A COPY OF THE PENNANT.
WRITE YOUR SCHOOL'S NAME ON IT.
TRANSFORM THE PENNANT BY:
- FLIPPING IT OVER.
- ROTATING OR TURNING IT:
  - ¼ TURN CLOCKWISE
  - ½ TURN COUNTER CLOCKWISE

DRAW YOUR RESULTS. MAKE ANOTHER SHAPE AND DRAW ITS TRANSFORMATIONS.

MG-31j

TRANSFORMATIONS #2

USE YOUR COPY OF THE PENNANT.
TRANSFORM IT BY:
- SLIDING IT 4 cm TO THE RIGHT
- SLIDING IT 10 cm TO THE LEFT
- SLIDING IT TO THE UPPER RIGHT OF YOUR PAPER.

WHAT HAPPENS IF YOU:
- TURN ¼ TURN CLOCKWISE AND ¼ TURN CLOCKWISE AGAIN TWO MORE TIMES.

DRAW YOUR RESULTS.

R --------→ R

R slides 6 cm to the right

R turns $\frac{1}{4}$ turn counterclockwise:

TRANSFORMATIONS #1

APPROXIMATE GRADE LEVEL: 2-6

OBJECTIVES: CHILDREN WILL DRAW PICTURES OF FLIPPING AND TURNING TRANSFORMATIONS.

MATERIALS: PENCILS, PAPER.

TEACHING NOTES: ENCOURAGE CHILDREN TO FLIP AND TURN OTHER OBJECTS AND MAKE SIMILAR PROBLEMS FOR EACH OTHER.

TRANSFORMATIONS #2

APPROXIMATE GRADE LEVEL: 2-6

OBJECTIVES: CHILDREN WILL DRAW PICTURS OF SLIDING AND TURNING TRANSFORMATIONS.

MATERIALS: PENCILS, PAPER.

TEACHING NOTES: ENCOURAGE CHILDREN TO SLIDE AND TURN OTHER OBJECTS AND MAKE SIMILAR PROBLEMS FOR EACH OTHER.

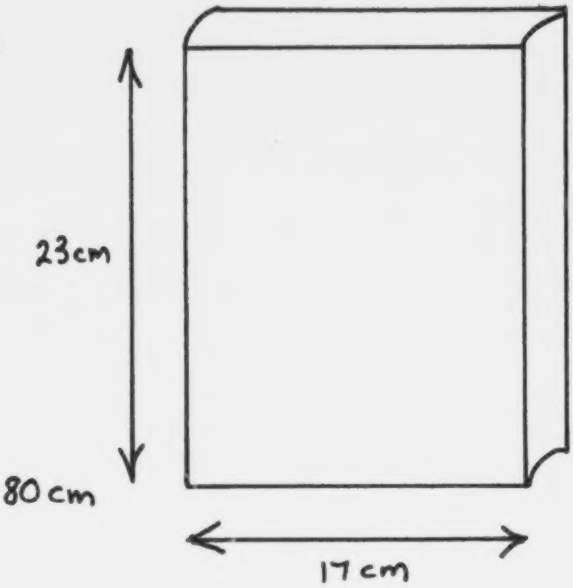
MG-31k

WHAT'S THE PERIMETER?

WHAT'S THE PERIMETER OF EACH OF THESE:

- THE COVER OF YOUR TEXTBOOK
- THE TOP OF YOUR DESK
- A SHEET OF NOTEBOOK PAPER

ESTIMATE EACH PERIMETER. MEASURE AROUND THE SHAPE. ADD UP YOUR MEASUREMENTS TO GET THE PERIMETER.



perimeter = 23 + 17 + 23 + 17 = 80 cm

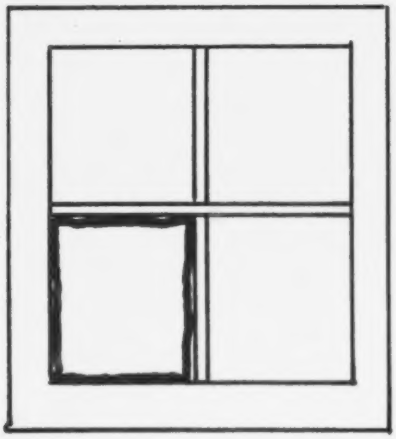
MG-31k

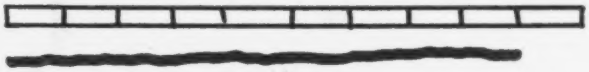
YARN PERIMETERS*

HOLD YARN AROUND EACH OF THE FOLLOWING TO GET THE PERIMETER.

- A WINDOWPANE
- A NOTECARD
- THE COVER OF A REFERENCE BOOK.

NOW STRETCH OUT THE YARN ON A METER STICK AND READ THE PERIMETER.





Stretched - out yarn's length = 90 cm

Perimeter = 90 cm

WHAT'S THE PERIMETER?

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL DETERMINE PERIMETERS OF VARIOUS RECTANGLES.

MATERIALS: RULERS.

TEACHING NOTES: LEAD CHILDREN TO DEVELOP A "SHORTCUT" OR FORMULA FOR FIGURING AREA.

YARN PERIMETERS

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL DETERMINE PERIMETERS OF VARIOUS RECTANGLES.

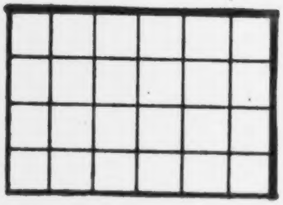
MATERIALS: PIECES OF YARN, METER STICKS, RULERS.

TEACHING NOTES: THIS ACTIVITY EMPHASIZES THAT PERIMETER IS A LINEAR MEASURE. ENCOURAGE THE CHILDREN TO FIGURE OUT OTHER PERIMETERS.

MG-311

PERIMETER AND AREA

USE GRID PAPER. MAKE RECTANGLES WITH AREAS OF 24 cm. MAKE ALL THE RECTANGLES YOU CAN. COLOR IN EACH AREA WITH ONE COLOR. OUTLINE THE PERIMETER OF EACH. IN ANOTHER COLOR. GLUE YOUR RECTANGLES TO PAPER AND WRITE THE AREA AND PERIMETER BELOW EACH ONE.



Area = $24 cm^2$

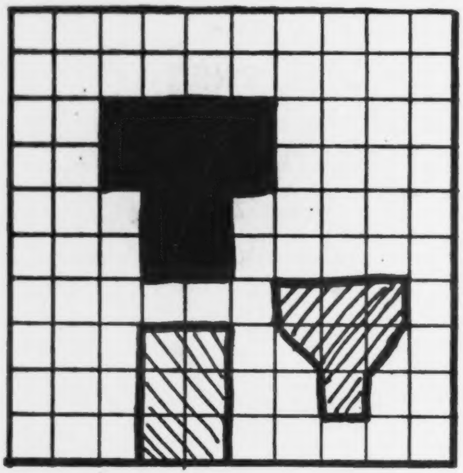
Perimeter = 20 cm

What other perimeters can there be?

MG-311

AREA PICTURES*

MAKE A PICTURE ON THE GRID PAPER. USE AT LEAST 3 COLORS. MAKE STATEMENTS ABOUT WHAT FRACTION OF THE TOTAL AREA EACH COLOR REPRESENTS.



black = $\frac{12}{100} = \frac{4}{25}$

green = $\frac{6}{100} = \frac{3}{50}$

red = $\frac{6}{100} = \frac{3}{50}$

white = $\frac{76}{100} = \frac{19}{25}$

PERIMETER AND AREA

APPROXIMATE GRADE LEVEL: 4-6

OBJECTIVES: CHILDREN WILL MAKE VARIOUS RECTANGLES WITH A GIVEN AREA AND DETERMINE THE PERIMETER OF EACH.

MATERIALS: GRID PAPER (PATTERN MG-39), CRAYONS, PAPER.

TEACHING NOTES: COLORING THE SURFACE AND PERIMETER STRESSES THAT AREA IS TWO-DIMENSIONAL WHILE PERIMETER IS LINEAR. ALSO USE 36 cm^2 40cm^2 or 72 cm^2 FOR SIMILAR EXERCISES.

AREA PICTURES

APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL COLOR AREAS AND STATE FRACTIONS OF A TOTAL AREA.

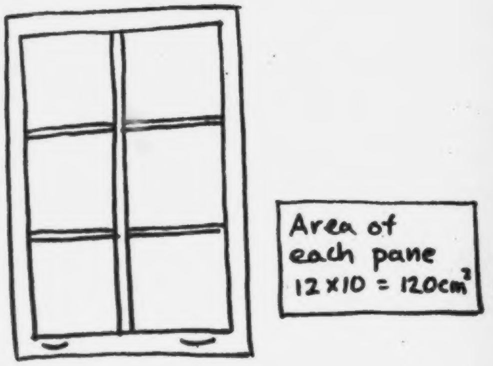
MATERIALS: GRID PAPER (PATTERN MG-39), CRAYONS, PAPER.

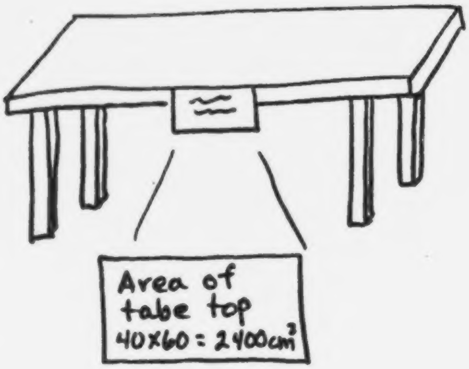
TEACHING NOTES: FIRST OR SECOND GRADERS CAN DO THE EXERCISE AND SIMPLY COUNT THE SQUARES OF EACH COLOR. COUNTING COLORED SURFACES INCREASES UNDERSTANDING OF AREA AS A TWO-DIMENSIONAL MEASUREMENT.

MG-31m

WHAT'S THE AREA?

FIND THE AREA OF SOME SURFACES IN YOUR CLASSROOM. TAPE A SIGN TO EACH AREA TO TELL OTHERS WHAT YOU FOUND.

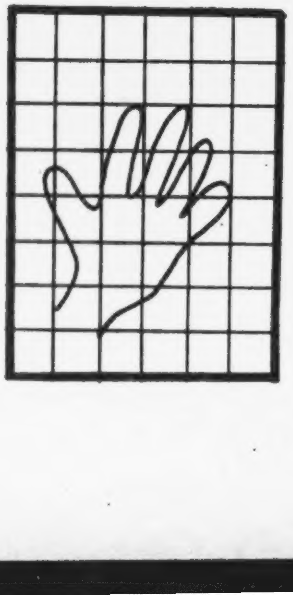

Area of
each pane
12 x 10 = 120cm²


Area of
tabe top
40x60 = 2400cm²

MG-31m.

AREA OF MY HAND*

TRACE AROUND YOUR HAND ON GRID PAPER. FIGURE OUT THE APPROXIMATE AREA OF YOUR HANDPRINT. YOU CAN COUNT SOME WHOLE SQUARES AS PART OF THE AREA. FIGURE OUT A WAY TO ACCOUNT FOR THE PARTS OF SQUARES. LABEL YOUR HANDPRINT WITH ITS APPROXIMATE AREA.



Area of hand =

about $120 cm^2$

WHAT'S THE AREA?

APPROXIMATE GRADE LEVEL: 5-6

OBJECTIVES: CHILDREN WILL DETERMINE AREAS OF CLASSROOM OBJECTS AND LABEL THE AREAS.

MATERIALS: RULERS, METER STICKS, PAPER, TAPE, PENCILS.

TEACHING NOTES: THIS ACTIVITY COULD ALSO BE DONE FOR PERIMETER.

AREA OF MY HAND

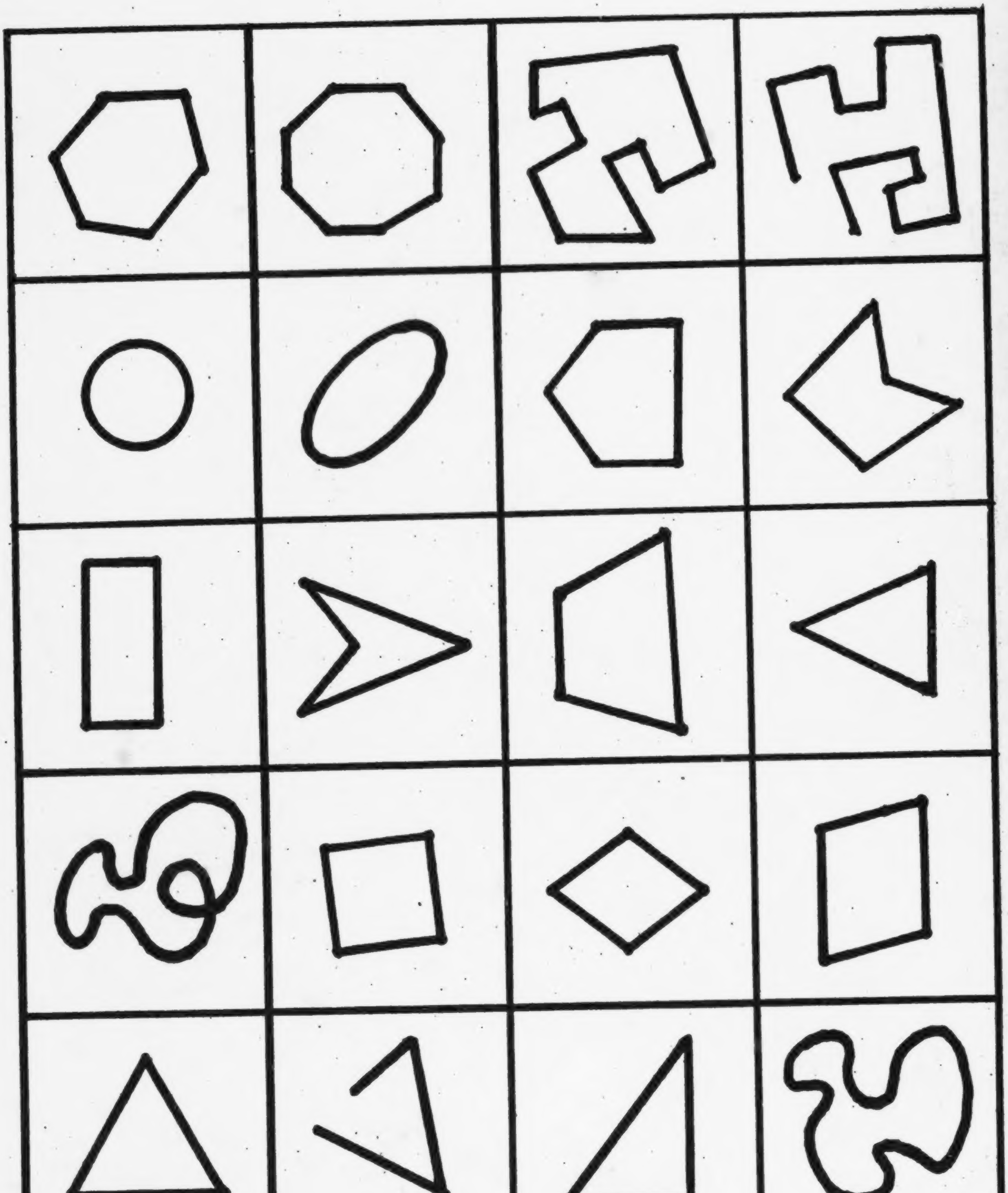
APPROXIMATE GRADE LEVEL: 3-6

OBJECTIVES: CHILDREN WILL DETERMINE THE APPROXIMATE AREA OF THEIR HANDPRINTS.

MATERIALS: GRID PAPER, PENCILS.

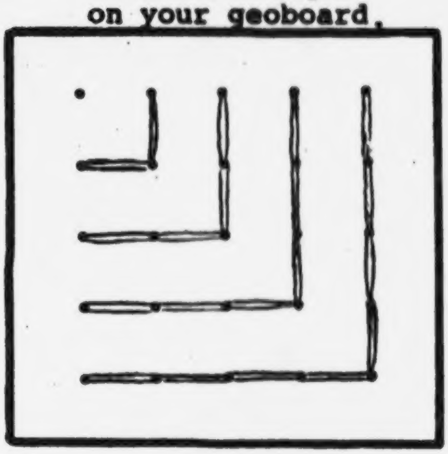
TEACHING NOTES: LET CHILDREN SHARE DIFFERENT STRATEGIES FOR DETERMINING THE AREA. STRESS THAT ANSWERS ARE APPROXIMATE.

Shapes To Sort

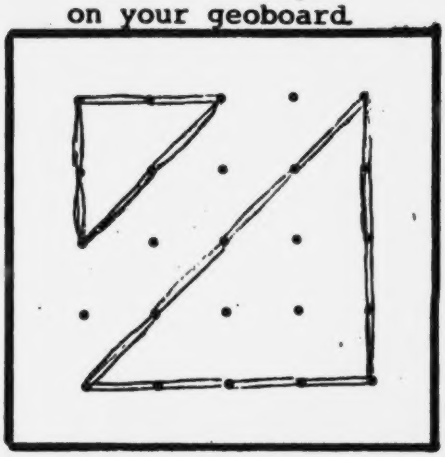


GEOBOARD TASKS

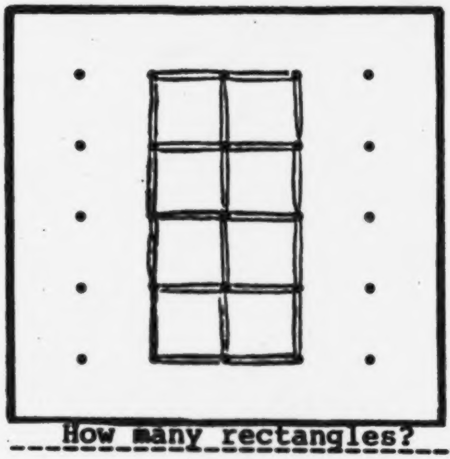
Make the shapes on your geoboard.



Make the shapes on your geoboard.

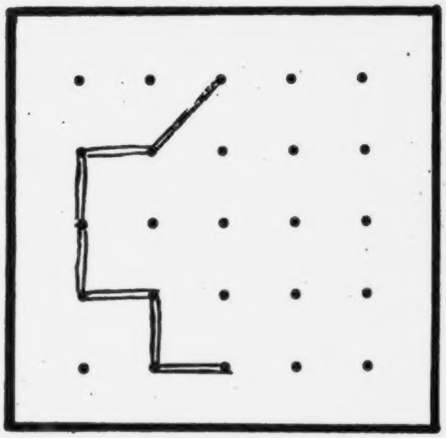


Copy the shapes.
How many squares are there?

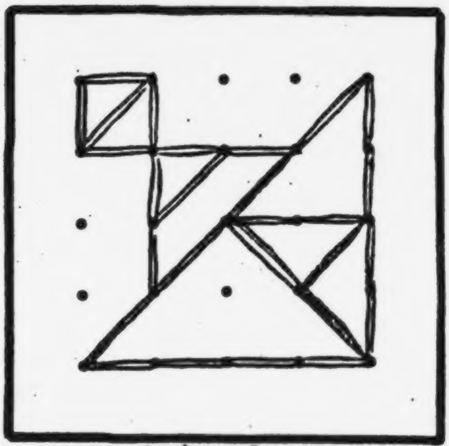


How many rectangles?

Make another half to the design to look exactly like this half.

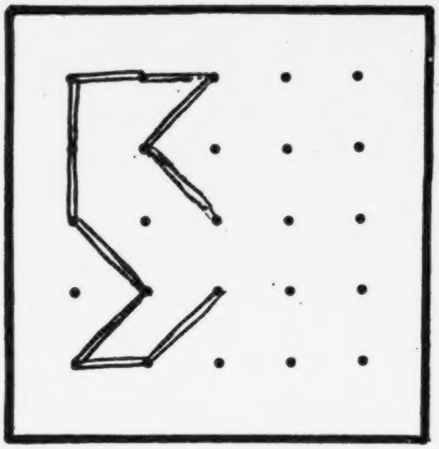


Copy the shapes.



How many triangles are here?

Make another half to the design to look exactly like this half.



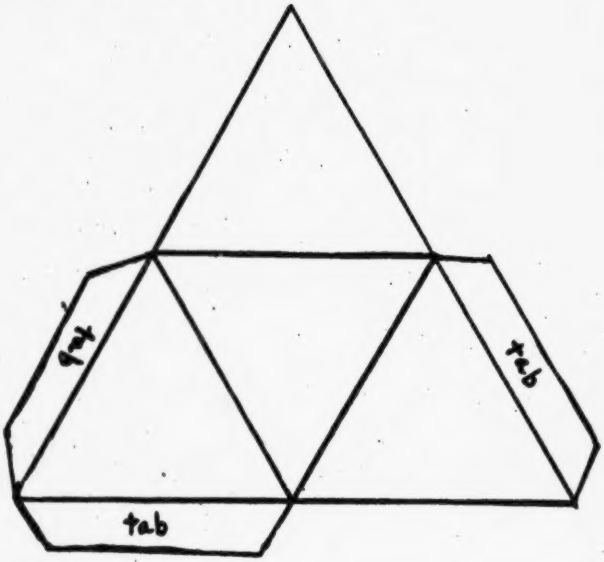
Types Of Polygons (and the circle)

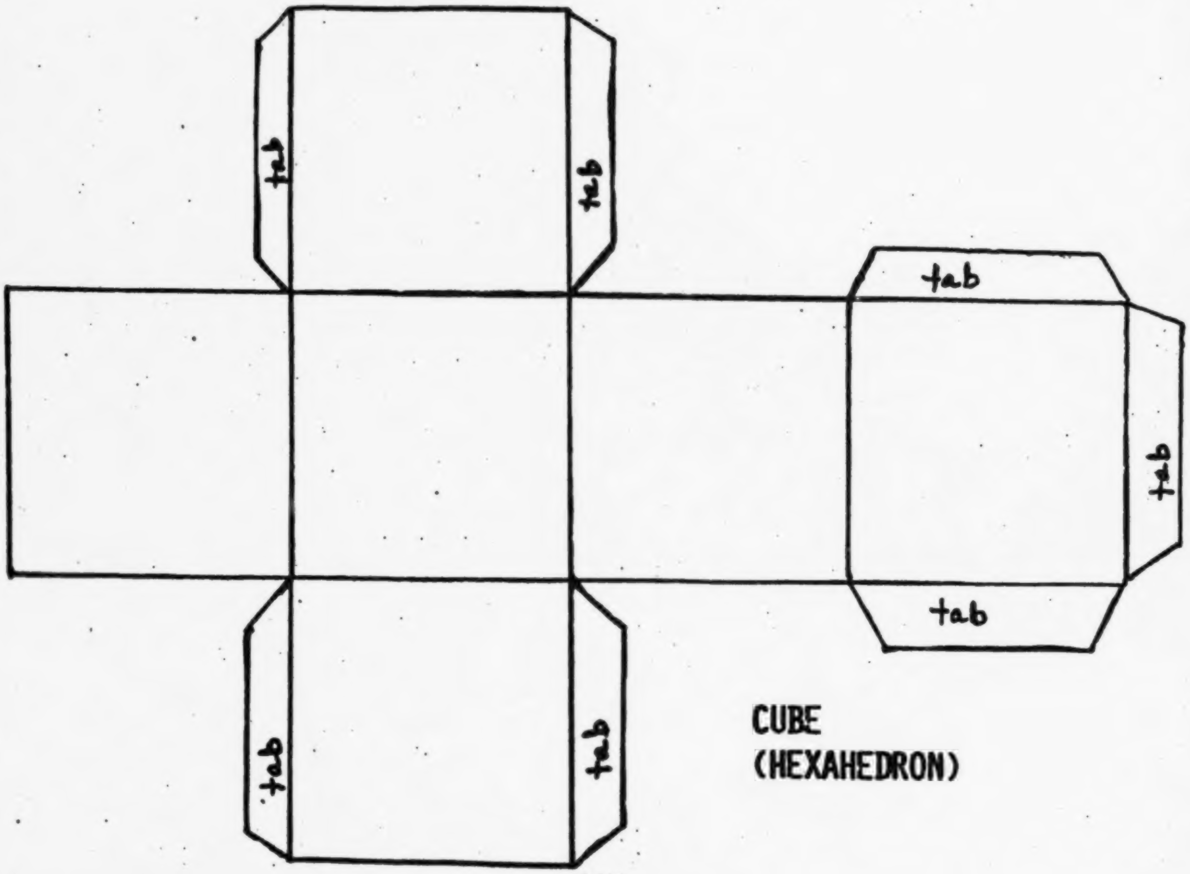
MG-34

TRIANGLE	RECTANGLE	REGULAR PENTAGON	CIRCLE
EQUILATERAL TRIANGLE	PARALLELOGRAM	HEXAGON (NOT REGULAR)	CIRCLE A WITH RADIUS AB
SCALENE TRIANGLE	RHOMBUS	HEPTAGON (NOT REGULAR)	DIAMETER OF A CIRCLE
RIGHT TRIANGLE	QUADRILATERAL	REGULAR HEPTAGON	NONAGON
ISOSCELES TRIANGLE	TRAPEZOID	OCTAGON (NOT REGULAR)	DECAGON
SQUARE	PENTAGON (NOT REGULAR)	REGULAR OCTAGON	OVAL

Patterns For Geometric Solids

PYRAMID WITH
TRIANGLULAR BASE
(TETRAHEDRON)

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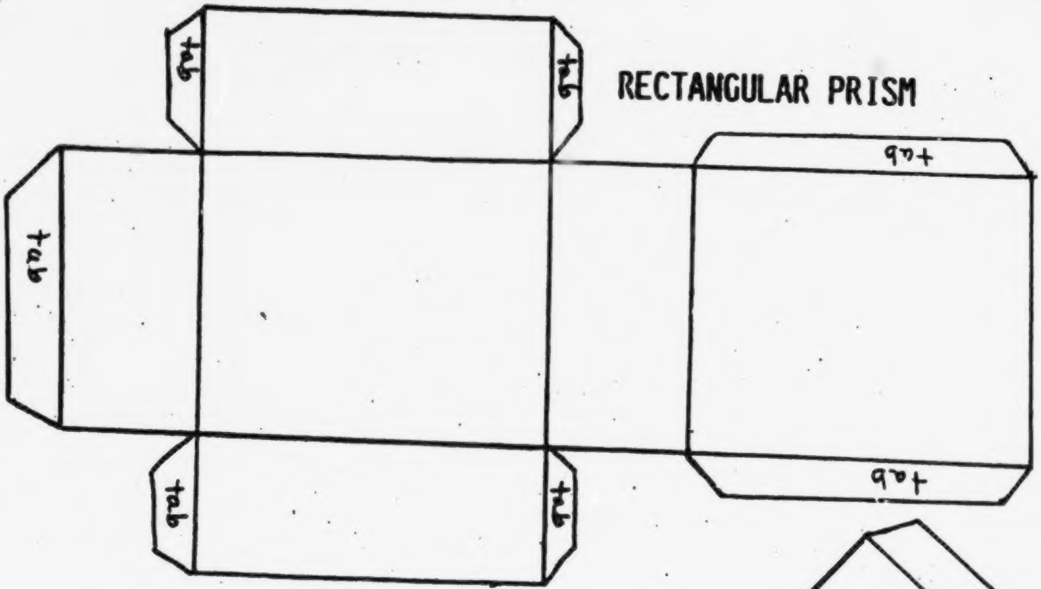
CUBE
(HEXAHEDRON)

RECTANGULAR PRISM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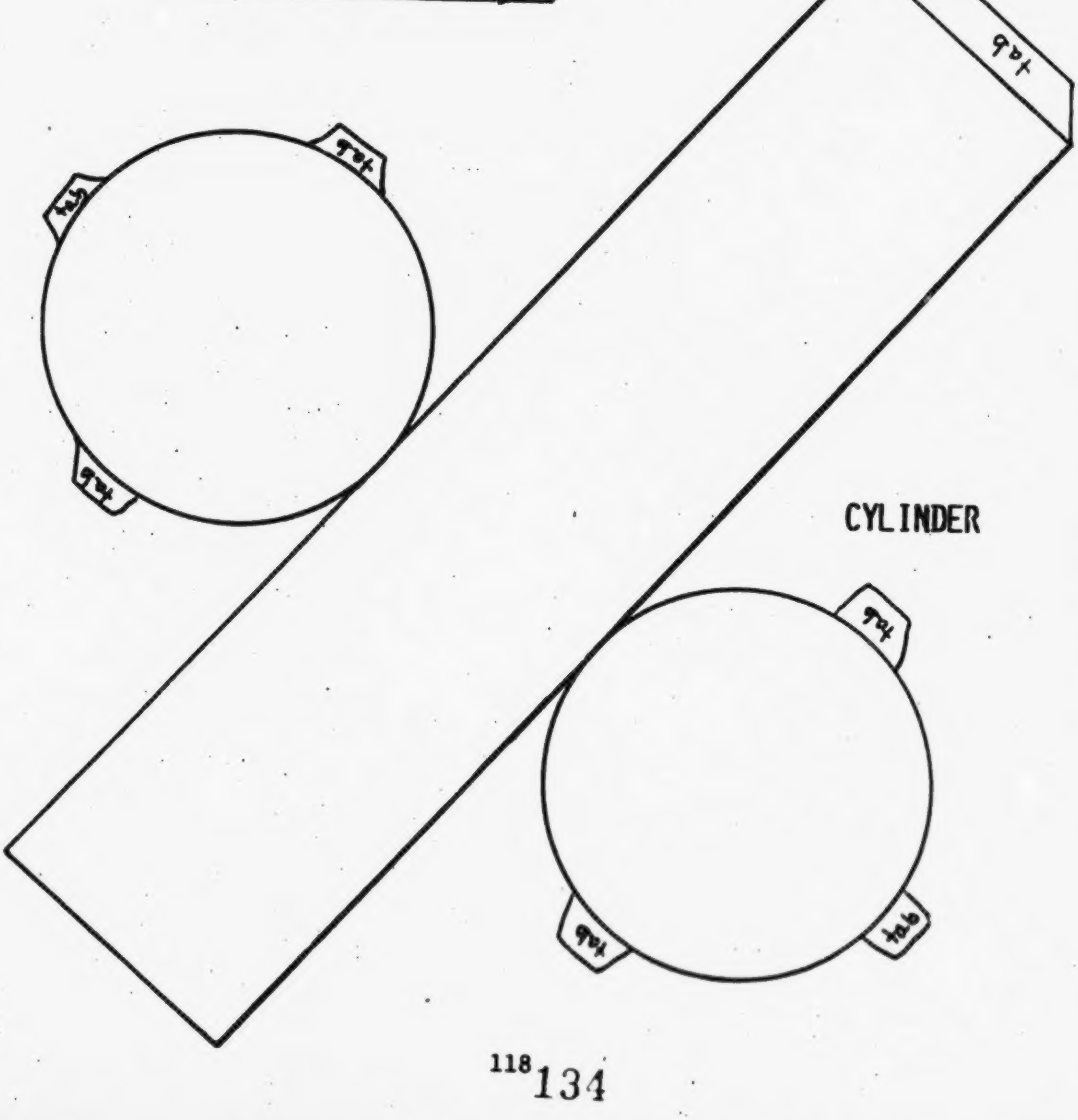
CYLINDER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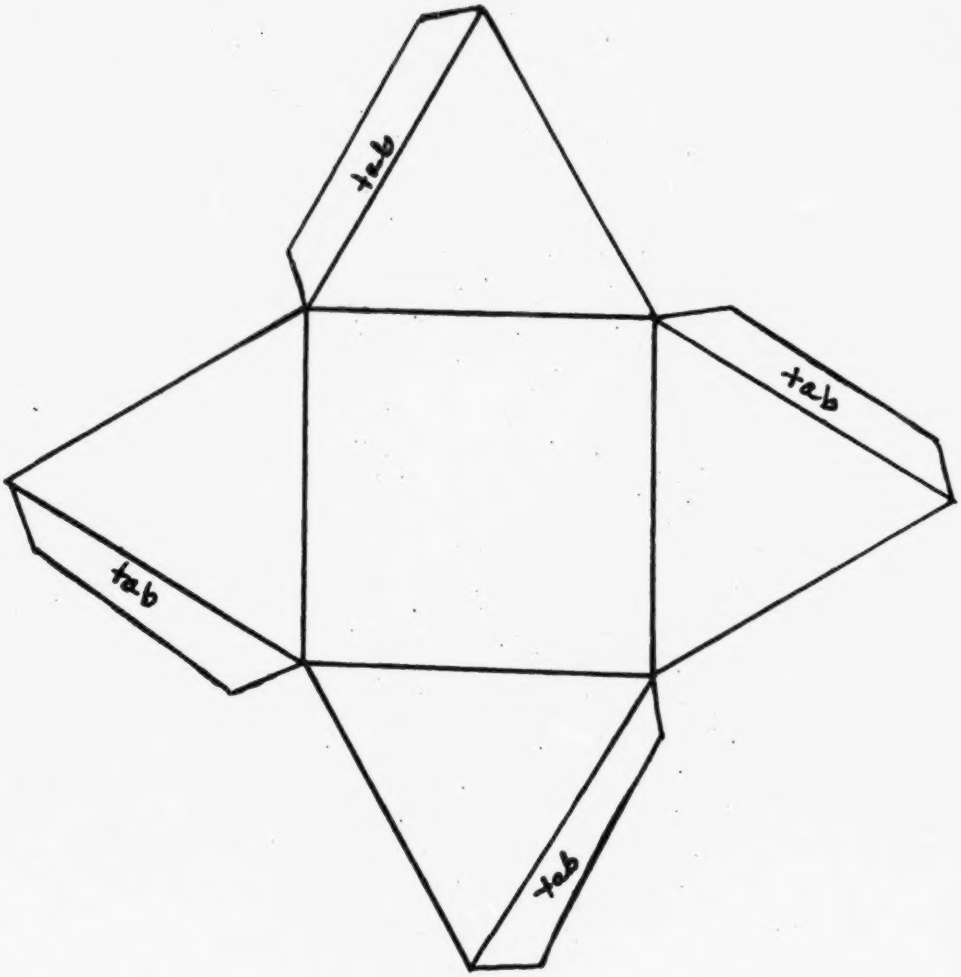
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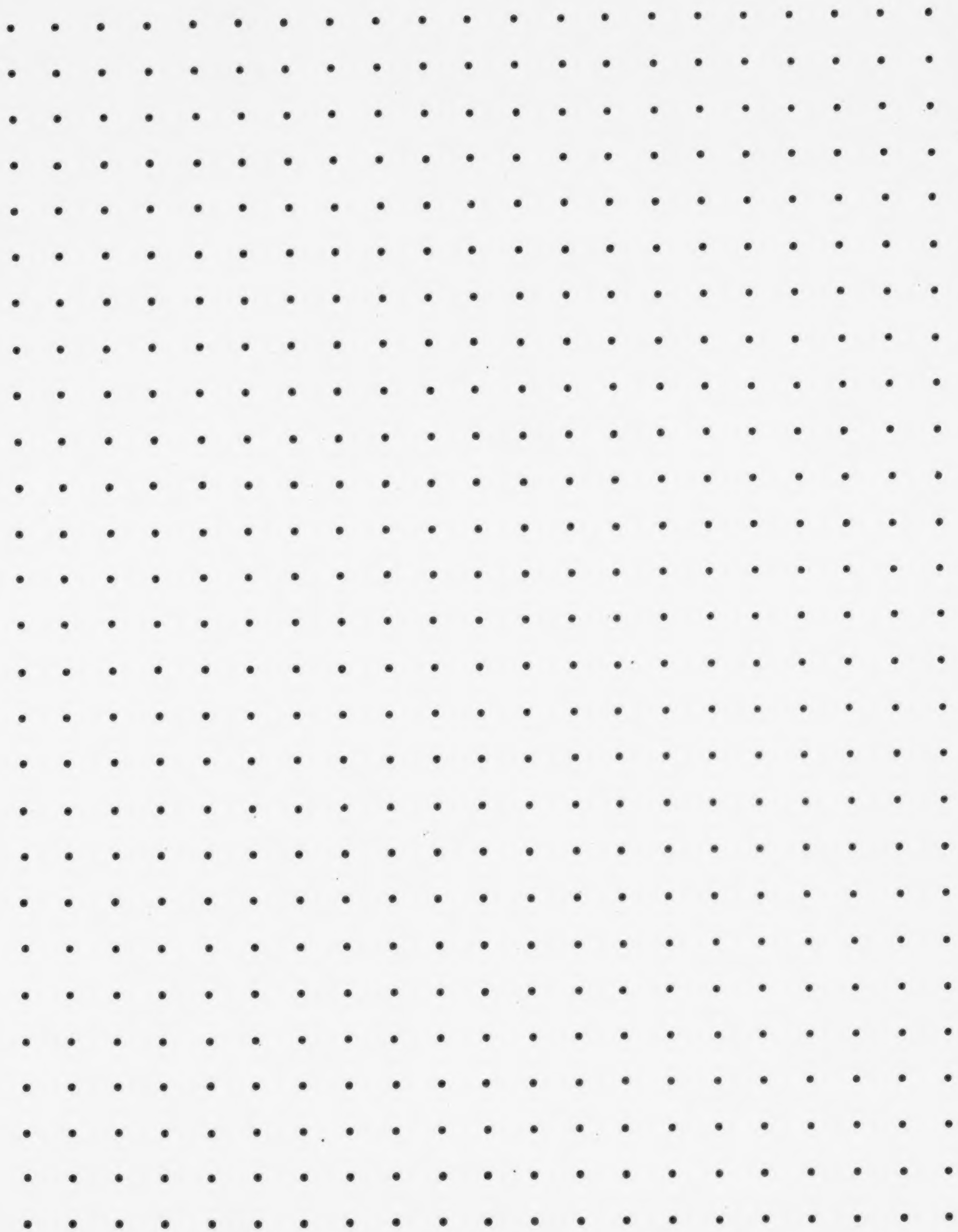
PYRAMID WITH SQUARE BASE

Dotpaper



Types Of Line Segments And Angles

MG-36

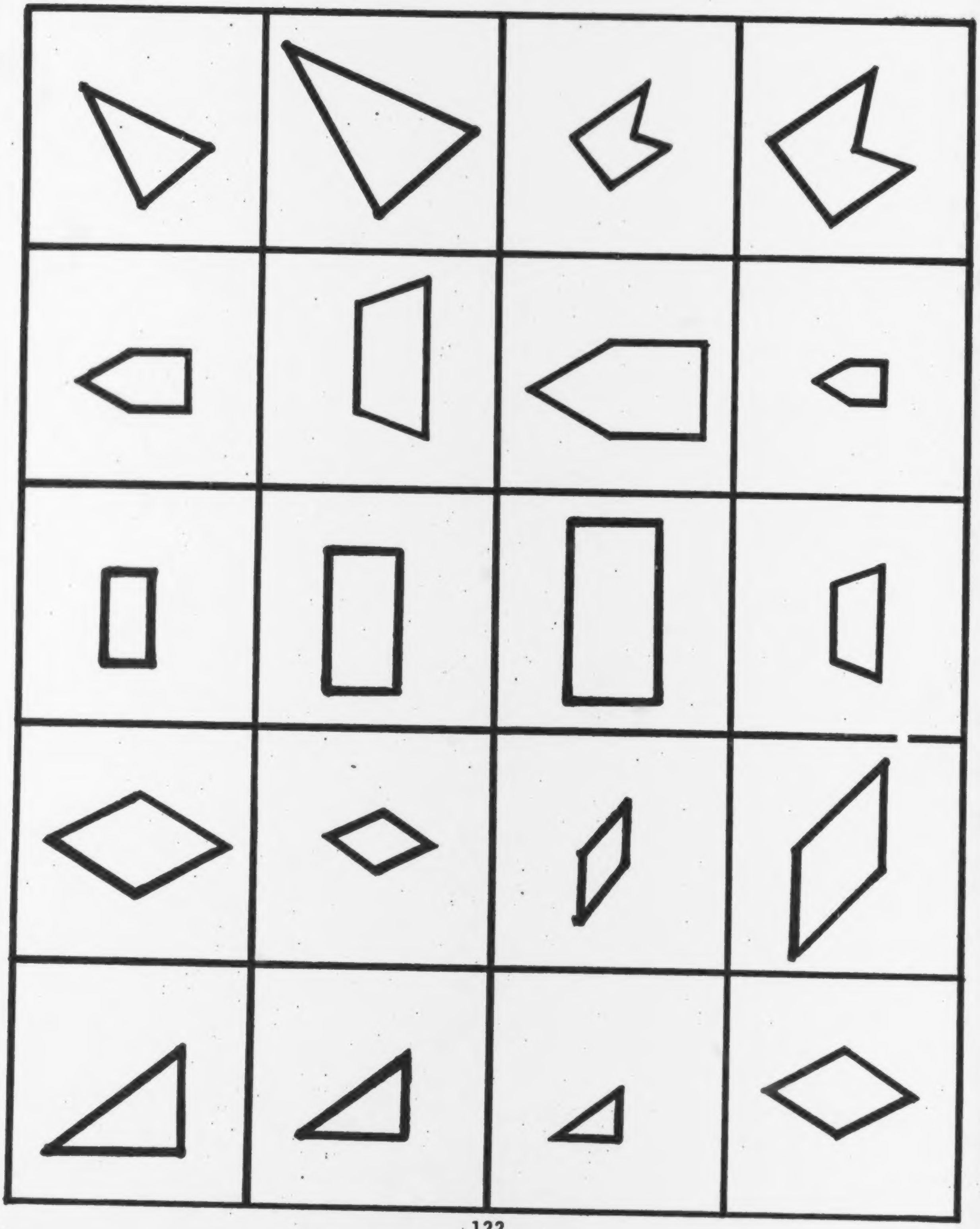
PERPENDICULAR LINES	$\overleftrightarrow{MN}$	RIGHT ANGLE	∠WXY ABOUT 60°
PERPENDICULAR LINES	$\overline{OP}$	OBTUSE ANGLE	∠RST ABOUT 45°
PARALLEL LINES	$\overline{QR} \perp \overline{ST}$	ACUTE ∠ABC	∠UVW ABOUT 30°
PARALLEL LINES	$\overleftrightarrow{UV} \parallel \overleftrightarrow{WX}$	STRAIGHT ∠DEF	∠FGH ABOUT 120°
SEGMENT AB PARALLEL TO SEGMENT CD	ACUTE ANGLE	RIGHT ∠GHI	∠IJK ABOUT 150°
SEGMENT EF INTERSECTING SEGMENT GH	STRAIGHT ANGLE	OBTUSE ∠KLM	∠LMN ABOUT 15°

138

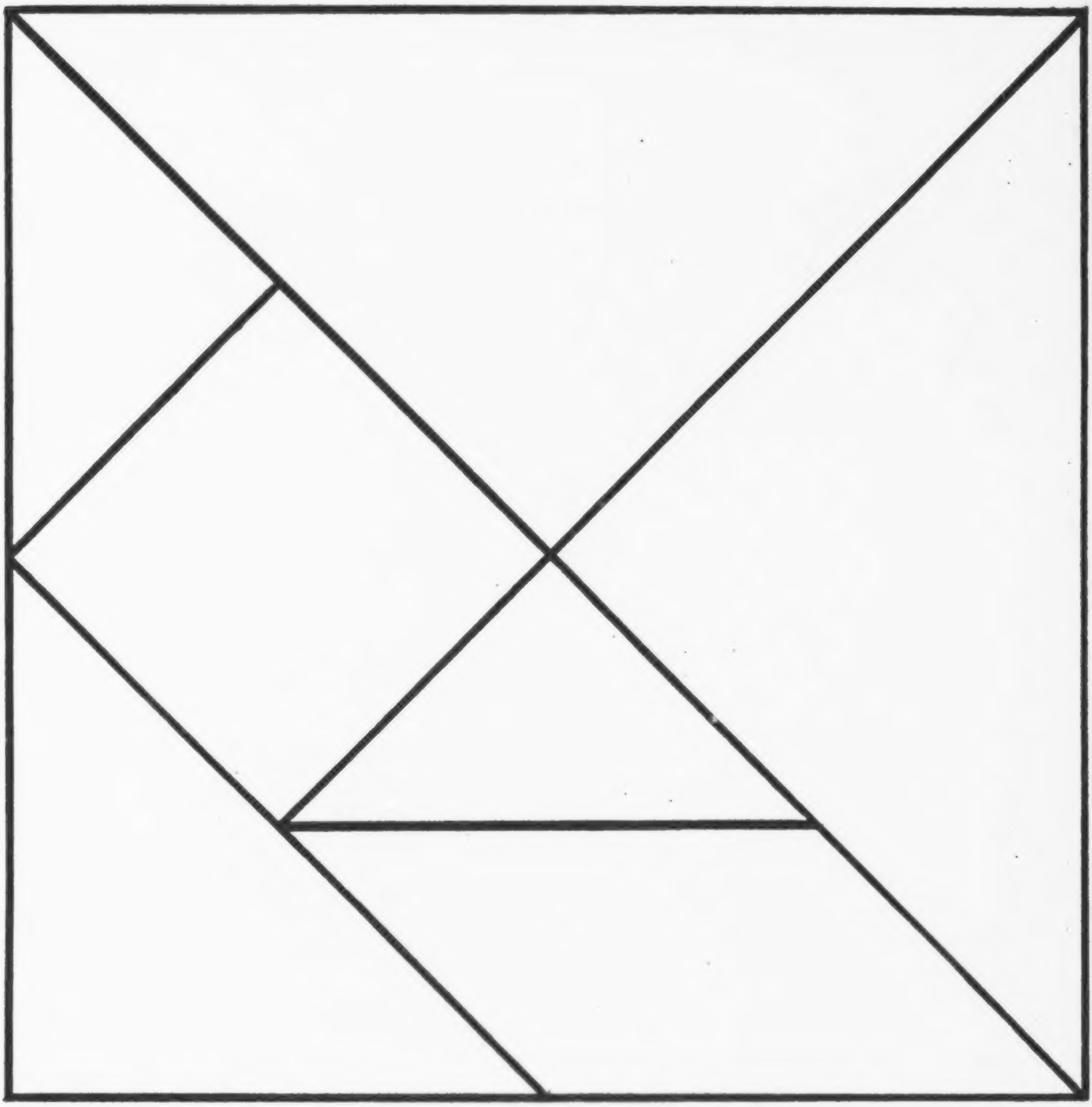
137

MG-37

Similar Polygons

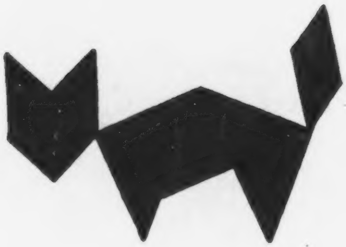


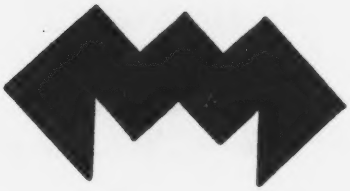
TANGRAM PATTERN

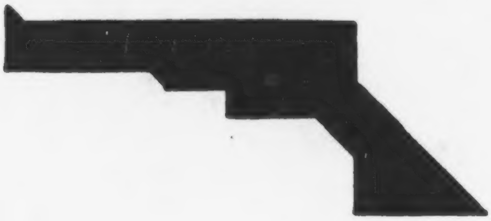


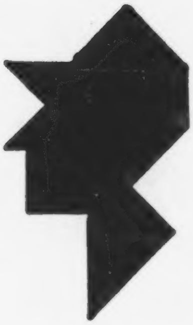
MG-38b

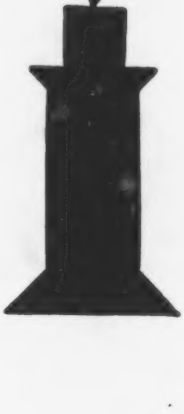
Use the tangram pieces to make shapes like these.

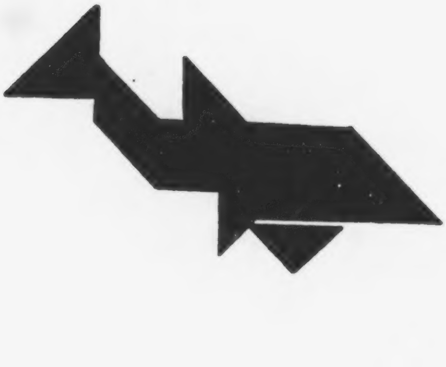












MG-39

Centimeter Square Grid

MEASUREMENT AND GEOMETRY ACTIVITIES
FOR HOME USE

1. Find objects of approximately the lengths of units you are studying (1 cm, 1 dm, 1 m, 1 in., 1 ft., 1 yd.). Bring objects to school for sharing.

2. Measure given objects, perhaps using child-made measuring tapes. Children might measure the heights of their couches, lengths of beds, heights of doors, or heights from floor to door knobs. In the classroom, children can compile data and find range, median, and average.

3. Trace handprints and footprints of family members. Cut these out. Make a life-sized graph of them in the classroom. Measure and order them in the classroom.

4. Bring in boxes, labels, and other containers of products with measurements on them. Arrange these in categories such as "Sold by Length," "Sold by Volume," "Sold by Mass," "Sold by Other Units."

5. Use a home thermometer (or use one loaned from school) to measure various temperatures such as hot drinks, cold drinks, bath water, temperature by a heating or cooling unit, refrigerator or freezer temperature.

6. Make a list of household objects that represent geometric ideas such as polygons, circles, kinds of angles, or geometric solids.

7. Determine the perimeter and area of a bedroom. Make a scale drawing of the room and some pieces of furniture.

8. Look for geometric shapes in patterns of clothing. Sketch various patterns.

9. Determine the volume of liquid consumed in a 24-hour period, perhaps using a child-calibrated cup to measure volumes.

10. Add ideas of your own. Consult students for suggestions for more ideas.

STAFF DEVELOPMENT FOLLOW-THROUGH PLAN
1987 - 1988

ACTIVITY:

Mathematics Workshop: Measurement

GOAL OF ACTIVITY:

Participants will receive information on measurement and principles for teaching measurement.

PROJECTED OUTCOME OF ACTIVITY:

Participants will help their students develop measurement concepts and skills through an activity based approach.

DESCRIPTION OF ACTIVITY:

1. The workshop leader will discuss measurement and reasons for teaching measurement through activities.
2. Participants will conduct measurement activities using similar techniques and materials to those which could be used by the students in their classrooms. A learning center approach will be used.

ASSIGNMENT(S):

Each participant will carry out at least one activity in each of these: volume, temperature, mass or weight, and length.

FOLLOW-UP OF ASSIGNMENT(S):

Each participant will teach a measurement lesson in which his/her students participate in at least one of the activities learned in the workshop.

DATE____________________TIME____________________

PLACE____

STAFF DEVELOPMENT CREDIT: TRAINING__________SUPPORT__________

*This workshop could be conducted during two sessions lasting 1½ hours each.

FRACTIONS

Dr. Mary Alyce Mize

Dr. Gordon Johnston

FRACTIONS

OVERVIEW AND RATIONALE

In this workshop participants will receive information on the meaning of fractions, the vocabulary, symbols, and skills needed to understand fraction concepts, equivalent fractions with renaming, the four basic operations and how to teach these to children. Participants will be actively involved in sharing ideas and experiences with fractions, as well as working with materials and practicing procedures they can use later in the elementary classroom.

The content for this workshop was selected as a result of needs identified from standard tests administered in schools across the state of Mississippi. The procedure chosen is based on the belief that learning is an active process and effective learning experiences depend on each learner's active participation. Elementary school children are at a concrete level of cognitive development, a stage in which they learn best by seeing and doing. Therefore, for all age and grade levels, new ideas and skills will be introduced and developed through the use of concrete object models for fraction ideas. The belief that children should understand and be able to handle an idea with a physical object before they are asked to deal with it orally or in writing is basic to the approach recommended in this workshop.

It is further believed that students learn best in an environment free of the fear of failure, and since children differ in their intellectual abilities, their performance abilities, their interests, their rates of learning and their ways of learning, a variety of approaches using different procedures and different materials will be suggested. The fraction ideas presented will not be rigidly assigned to a particular age or grade level, but will be identified according to three major classifications; those related to the meaning of fractions, equivalent fractions and four basic operations with fractions. The experiences suggested are primarily activities, games, projects and practice exercises designed for individuals or for small groups of children. Mary Ann Haubner's article, "Manipulatives for a Better Understanding" (FM-1) provides a clear and concise examination of the research evidence supporting this approach.

BEHAVIORAL OBJECTIVES

At the end of the workshop each participant should be able to:

1. Deepen his/her knowledge of fractions and the 4 basic operations with fractions.
2. Give a rationale for active learning experiences for the development of fraction concepts and skills.
3. Identify prerequisite and developemntal fraction concepts and skills and appropriate activities for children to learn them.
4. Develop active learning experiences for classroom use in the teaching/learning of fractions.

WORKSHOP LEADER BEHAVIORS

The leader should:

1. Develop the several meanings of fractions, give examples and encourage participants to give examples and situations involving various meanings of fractions.
2. Discuss the importance of active learning experiences for the development of fraction concepts and skills.
3. Identify prerequisite and developmental fraction concepts and skills and how to provide children opportunities to learn them.
4. Involve participants in group activities to act out, discuss, develop and share ideas for active learning of fraction concepts and skills.

PREPARING FOR THE WORKSHOP

The leader should:

1. Read carefully the section on workshop content and activities.
2. Prepare and gather the following materials for use in the workshop.

*Transparencies: Materials identified FT-1 through FT-26 are black line masters suitable for preparation of transparencies to be used by the workshop leader.

*Handouts: Materials identified FA-1 through FA-15 are task cards and directions for developing teacher-made activities for hands-on experiences.

*Patterns for Fraction Projects and Practice Materials: Materials identified F-1 through F-27 are patterns for fraction projects and practice materials.

*Teacher Materials:
Haubner article - FM-1
Manipulative Aids for Experiences with Fractions and Decimals - FM-2
Some Words We Need to Know - FM-3
Addresses for Math Materials - FM-4
References - Fractions - FM-5
Skills for Math Education - FM-6

*Collect materials and prepare exhibit of various manipulative materials recommended for use in elementary classrooms in activities involving fractions. These may include:

Materials to collect:

- Adding Maching Tape
- Beads
- Butter Carton and models of butter sticks
- Calendar
- Colored Rods (Cuisenaire)
- Counters
- Cup marked in fractions
- Egg cartons with artificial eggs
- Geoboards
- Flanned Board
- Fraction Blocks
- Fraction Board
- Fractioneer
- Number line
- Nickles and dimes
- Paper for folding and cutting
- Pattern Blocks
- Postage stamps
- Squared paper

Materials to make:

Instructions for construction and use of these materials are provided.

- Fraction Board and cards
- Deck of Fraction cards
- Deck of Fraction Numeral cards
- Fraction Strips
- Fraction Charts
- Fraction Slide Rule
- Set of Fraction Dominoes
- Fraction Disks and/or Squares
- Fraction Cubes

WORKSHOP CONTENT AND ACTIVITIES

1) I think most of us like candy bars. Maybe even too much. What I would like to do today is ask you some questions about candy bars that are going to appear on the screen. (FT-3) First of all, I will tell you there are 2 candy bars. I will have one pictured on the right and one on the left. If you could have any one of them which one would you choose?.... Does it matter which one you choose at this point? (Show picture a) Were you happy with your choice? Why or why not? I think all of you will agree it is important to know the size of the bar or unit in deciding which one to choose.

Now, both candy bars are the same size. I have cut each bar into 3 pieces. From which candy bar would you like to choose a piece (the one on the left or right)? (Show picture b) Did it matter which bar you chose a piece from? Yes, it did. It depends on whether the bar is cut into equal-sized pieces which is also an important concern with dealing with fractions.

This time, the candy bars are the same size and each one is cut into equal-size pieces but each bar has some missing pieces. Does it matter which bar you choose? (Show picture c) This illustrates the importance of knowing the number of equal-sized pieces that are missing.

Now, the candy bars are the same size, no pieces are missing, and each one is cut into equal-sized pieces. Does it matter which bar you choose to get a piece from? (Show picture). This time it was important to know the number of pieces the candy bar has been cut into. The size of the piece is determined by the number of pieces the bar has been cut into. The more pieces, the smaller the piece.

In review when working with fractional amounts, it is important:
to know the size of unit or whole object being used.
to know each piece is the same size.
to know the number of pieces to be considered.
to know the number of pieces in the whole unit.

Children's study of fractions begins very early in their lives. They have had to share with others by cutting/breaking a cookie in half. They know when doing this sharing the pieces should be the same size especially if the friend gets to choose his/her piece first.

When children begin to formally study fractional numbers, teachers need to plan many concrete experiences in order to develop understanding of the numbers. We also know the concrete experiences lay the foundation for later work at the semi-concrete and symbolic levels. In other words, we should help children learn about fractions using the Teaching Learning Sequence. (Show FT-4. Discussion of transparencies FT-5, 6, and 7 will be useful to reinforce the importance of concrete experiences.)

(Show FT-8) Fractional numbers can be expressed as common fractions, decimal fractions, and percents. Any fractional number can be expressed in these ways, i.e., ¼, .25, 25%.

In today's presentation, we are going to focus on common fractions. They can be used to represent a number of things. (Show FT-9)

1) Unit subdivided into equal-size parts (regions). One piece of paper folded into 2 equal parts illustrates this. If we shade in 1 piece, we can discuss the meaning of the numerator and denominator of the fraction. The total number of pieces the paper region is divided into is the denominator. The number of shaded pieces or pieces we are interested in is the numerator. So in summary we can say 1 of 2 pieces is shaded and write it as "½." (do paper folding)

2) Set subdivided into equal-size groups (counters). This idea of fractions can be shown with 9 chips arranged into 3 equal-sized groups. We now can discuss how many are in 1 of three groups. This can be written mathematically as 1/3 of 9 = 3. We could continue to work with these counters to show the amounts in 2 of 3 groups or even 3 of 3 groups. (Show groups of transparent counters on the overhead projector.)

3) Common fractions as ratios. We can also think of fractions as ratios when we want to make comparisons between groups of things. An example would be the number of books per child in a classroom. We could show this in a 1 to 6 relationship, meaning 1 child has 6 textbooks. We can write this as 1/6 or 1:6.

4) Common fractions indicate division. We know common fractions indicate division, so 3/4 can also be written as $3 \div 4 = ?$. This can be shown with a physical model illustrating 3 feet of cloth divided into 4 equal-sized parts. (Show with model.)

5) Finally, we know common fractions are rational numbers.

So far, what we have been discussing lays the foundation for the rest of today's presentation, which will be in three broad areas. They are:

*Show FT-10, Meaning of Fractions, which deals with the identification and comparison of fractions.

*Show FT-11, Working with Equivalent Fractions, which includes reducing fractions.

*Show FT-12, Operations with Fractions, which includes addition, subtraction, multiplication and division of fractions.

2) Meaning of Fractions

Identification. As a first step in learning about fractions, we must learn how to identify fractions. There are several ways of handling this.

1) Concrete level: At the concrete level, we can use regions (variety of shapes - circles, squares, rectangles FT-13) in the introductory lessons. Here, you would give children paper to fold according to your instructions, being sure to monitor their work so that folding results in equal-size parts. (FT-14) An effort would be made to point out the number of parts into which the shape has been divided.

Sometimes children, especially young children, are unable to accurately fold and cut the regions or paper models, so you will have to premark the models. (Show FT-15 and 16) The clock handout should help you do the marking precisely.

During this early work, which usually takes place in first grade, the numbers for the denominator and numerator should be associated with the regions only after the children thoroughly understand the concept of fractions. When you introduce these values you do not give them their technical name, you can simply call them the top number and the bottom number. The bottom number should be explained as the total number of parts the shape has been divided into; the top number is the number of parts we are interested in.

Structured materials like Cuisenaire Rods and Pattern Blocks can also be used at the concrete level to help children learn to recognize fractions. (Illustrate various fractions with Cuisenaire Rods or Pattern Blocks for the overhead projector.)

(Show FT-17) Number lines can also be used at this level if the children mark the fractional parts of lines as they work, rather than seeing ready-made lines. If the lines are ready-made, the number line is considered semi-concrete rather than concrete.

Sets of objects can be used to help children identify fractions, even though this is somewhat harder for children than the use of regions. (Illustrate with 10 chips to find 1/2 and 1/5 of the chips. An egg carton with artificial eggs may also be used to illustrate 1/2, 1/3, 1/4, etc. of the dozen eggs.)

2) Semi-concrete level: Later, we can work at the semi-concrete level to learn about identification of fractions by using workbook pages (FT-18) with pictures or play games with pictorial representations. Children now look at the pictures and choose the regions showing specific fractional amounts. (Show FT-19)

3) Abstract level: Finally, at the abstract level children just see the fractional amount written mathematically. (Show FT-20) At this level (because of the earlier concrete and semi-concrete work), children know what the fraction means and can read it.

Comparison. As children work with the identification of fractions, it is a simple matter to expand this knowledge to the comparison of fractions.

1) Concrete level: At the concrete level, the folded regions allow the child to decide first-hand which amount is smaller or larger than another. (Illustrate with strips or sheets of paper as in 1 on page 5.)

Concrete structural materials such as Cuisenaire Rods or Pattern Blocks can also be used. After establishing the unit, the children can use the different rod trains/fractional pieces to divide the rod into fractional amounts that can be compared. (Illustrate with Cuisenaire Rods for the overhead projector.)

Number lines can also be used for the comparison of numbers. (Show FT-24.)

2) Semi-concrete level: At the semi-concrete level, the child simply compares pictures of fractional amounts in workbooks and games or works with the Fraction Bars Program. (F-17) (Show FT-21 and 22.)

3) Abstract level: At the abstract level, the child looks at two symbolic fractions and decides which one is bigger, smaller, etc. When working with unit fractions with unlike denominators (1/2, 1/8) and fractions with like denominators (2/4, 3/4) at the abstract level, the comparison of fractions can be done without the aid of algorithmic procedures, since these comparisons are built on understanding fractions. (FT-23)

3) Hands-on Activities

Experiences with Cuisenaire Rods: It is important for workshop participants to experience some of the ideas that have been idscussed. Each participant will need Cuisenaire Rods (or Happy Face pieces cut to represent 10 different lengths: F-16), "MAT" (F_18) and the manual USING THE CUISENAIRE RODS, A Photo Text Guide, by J. Davidson.

(If participants indicate that they do not have Cuisenaire Rods available in their schools, remind them that the Education Reform Act requires that money be spent for necessary instructional materials. They should inquire as to the proper procedure for requesting purchase of instructional aids. Cuisenaire Rods, Pattern Blocks and Fraction Bars are excellent manipulative materials and should be purchased if at all possible.)

Have participants pour rods onto the MAT and "get acquainted" with the rods by arranging and rearranging them as shown on Cuisenaire pages 8, 9, and 10. Encourage discussion of the relationships between certain rods. The leader should demonstrate various arrangements and relationships with Cuisenaire Rods for the overhead projector.

After participants have had ample time to discover many options for using the rods to identify and compare fractions, they should be directed to centers where materials have been set up for individual and small group activities, games, or practice exercises. These could include:

1) Pattern Match, Fraction Match (FA-1)
2) Fraction Board Games #1, #2, and #3 (FA-1 and 2)
3) The Hidden "one" #1, #2, and #3 (FA-5)
4) Fractions - Skill 1 - Learning Activities (FA-8)

4) Equivalent Fractions - Renaming

Equivalent fractions will be introduced in this section. Prerequisite skills are an understanding of the meaning of fractional numerals and unit fractions as well as mastery of basic multiplication and division of whole number facts.

Included in this section are:

*Identifying and producing equivalent fractions
*Renaming fractional numerals
. . .reduction
. . .raising the order
*Comparing simple fractions

Mastery of these skills is a prerequisite to performing the four basic arithmetic operations on fractional numerals.

1) Identifying and Producing Equivalent Fractions

Equivalent fractions are fractions that name the same number (have the same value). We can demonstrate this concept by folding paper. Have the participants fold a rectangular sheet of paper into 2 equal parts. Have them shade one of the 2 parts. Point out that 1/2 of the paper is shaded. Ask them to fold the paper in half again. Now there are 4 equal parts, and 2/4 of the paper is shaded. Since 1/2 and 2/4 name the same amount, they are equivalent fractions. (Fraction Strips F-23 may also be useful here.)

A "real-life" example may be seen in a model of a pound of butter or margerine. The box represents 1 whole pound of butter. When the box is opened, it can be separated into 4 equal parts (sticks) of butter. Each stick is 1/4 of the pound. Two sticks of butter represent 2/4 or 1/2 of the whole pound. Participants should be encouraged to contribute other examples of equivalent fractions from the "real" world (eggs, crayons in a box, etc.)

Allow participants time to produce and identify equivalent fractions using the Cuisenaire Rods. Guide pages 46, 47, 48 and 49 will be helpful as a guide for the use of the rods. The workshop leader should participate with transparent rods and appropriate questions to stimulate interest and understanding of the use of rods to illustrate equivalent fractions. (Pattern blocks can also be used.)

2) Renaming Fractional Numerals

Rods, Fraction Disks (F-19), Fraction Squares (F-1), Pattern Blocks, pies and other concrete materials of one kind of another, will show quite soon that it is all well and good to try out one equivalent fraction after another to see which "size of piece" will fit, but this is a rather slow process if we do not already know which pieces to choose.

The vertical number line provides an excellent situation for moving from the concrete identification of equivalent fractions to the semi-concrete and abstract statements of equality. The number line can be developed on a transparency (FT-24) by the teacher and pupils working together. The teacher will want to use similar number lines showing halves, thirds, sixths and twelfths, or showing halves, fifths and tenths; but each may be handled in somewhat the same way.

When fractions that have the same value have been "built" on the vertical number line, attention should be called to one set of equivalent fractions like 1/2, 2/4 and 4/8. How are they alike? Ask what is done to the numerator and denominator of the fraction 1/2 to get the fractions 2/4 and 4/8? Develop the ide: that if you multiply the numerator and denominator of a fraction by the same number, the resulting fraction is equivalent. That is, it does not change the value of the fraction. It should also be noted that dividing the numerator and denominator by the same number also does not change the value of the fraction and is called simplifying a fraction.

Children may notice that the number line shows that if the parts are half as big, it takes twice as many parts to equal the original amount. We can also say that when parts are half as big, we use twice as big a denominator. When we need twice as many parts, we use twice as big a numerator.

A "real life" example may be seen in a comparison of coins. If the coins are thought of as representing fractional parts of a dollar, children know that 4 dimes are worth the same as 8 nickels. This is because each nickel is worth half as much as a dime and it takes twice as many of them to equal the same value. Each dime is 1/10 of a dollar (in value); each nickel is 1/20 of a dollar (in value): 4/10=8/20. The second numerator is twice as large as the first. The second denominator is twice as large as the first. The 2 fractions are equivalent, because the second one shows twice as many parts, but each part is half as valuable. Other examples from the "real" world should be encouraged (eggs in a carton, etc.).

The generalization for renaming fractions may be expressed:

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{n} \times \text{numerator}}{\text{n} \times \text{denominator}} = \frac{\text{numerator} \div \text{n}}{\text{denominator} \div \text{n}}$$

3) Comparing Simple Fractions

Participants should pour out Cuisenaire rods on MATS and arrange the rods in a stair as illustrated on Page 46 of the Cuisenaire reprinted materials. As they answer the questions, the leader should guide the activity toward these observations:

1) If 2 fractions have the same denominators, then the one with the greater numerator has the greater value.

2) If 2 fractions have the same numerators, then the one with the greater denominator has the lesser value.

Give participants time to discover several comparisons. Obviously one can't walk around for the rest of her life with rods and fraction pieces in her pockets. At some point the concrete materials are put aside and we take up pencil and paper. When this happens the fractions to be compared may be 3/8 and 4/10. To decide which of the two fraction numerals is greater, these are the steps children could use:

1) Find the least common multiple of the denominators. This number will be called the least common denominator (LCD).

LCM (8,10)=40

2) Find equivalent fractions having the least common denominator.

$$\frac{3 \times \square}{8 \times \square} = \frac{?}{40}$$

□ must be 5, since 8x5=40.

$$\frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

$$\frac{4 \times \square}{10 \times \square} = \frac{?}{40}$$

□ must be 4, since 10x4 =40

$$\frac{4 \times 4}{10 \times 4} = \frac{16}{40}$$

3) Now, order the numerators.
16 is greater than 15, so 16/40 is greater than 15/40. This means that 4/10 is greater than 3/8.

The procedure for finding equivalent fractions may also be explained as a number multiplied or divided by one (1). The following explanations may be helpful when illustrated or drawn on the overhead or chalkboard:

Any fraction with the same numeral in the numerator and denominator is equal to one (1). (Illustrate this concept with rods or pattern blocks.)

Example: $\frac{6}{6} = 1, \quad \frac{3}{3} = 1, \quad \frac{9}{9} = 1$

Now, use this information to find equivalent fractions without manipulatives.

Example: $\frac{1}{4} \times \underset{\substack{\downarrow \\ \text{same as 1}}}{\frac{3}{3}} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$ (same as $\frac{1}{4}$)

An equivalent fraction can often be found by dividing by 1. This is how a fraction can be reduced to its lowest term. Lowest terms means using the smallest possible numbers in the numerator and denominator!

Example: $\frac{4}{12} \div \underset{\substack{\downarrow \\ \text{same as 1}}}{\frac{4}{4}} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$ (same as $\frac{4}{12}$)

The fractions that we have compared so far have all been proper fractions - that is, fractions that are less than 1. Examples of proper fractions are 1/3, 3/4, 5/7, and 7/8. The numerator of a proper fraction is less than the denominator. Comparing fractions sometimes involves improper fractions - that is, fractions that are equal to or greater than 1. Examples of improper fractions are 2/2, 3/3, 7/5, and 11/8. The numerator of an improper fraction is equal to or greater than the denominator. To rename improper fractions as a whole number or a mixed number, these examples are helpful:

1) Divide the numerator by the denominator

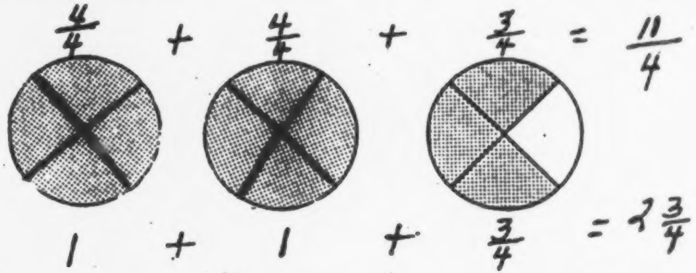
$$\frac{11}{4} = 11 \div 4 = \begin{array}{r} 2 \\ 4\overline{)11} \\ 8 \\ \hline 3 \end{array} =$$

2) Write the remainder as a fraction in simplest form. (The remainder is the numerator of the fraction; the divisor is the denominator.)

$$\begin{array}{r} 2\frac{3}{4} \\ 4\overline{)11} \\ \underline{8} \\ 3 \end{array}$$

The resulting mixed number has a whole number part and a fractional part. The "2" is the whole number part and the "3/4" is the fractional part. This mixed number is read "2 and three fourths."

A further explanation can be generated by drawing the following example on the overhead or the chalkboard.


$\frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$
$1 + 1 + \frac{3}{4} = 2\frac{3}{4}$

11/4 (improper fraction) and 2 3/4 (mixed number) clearly name the same number. They both have the same denominator but different ways of naming the number of shaded parts.

Explain that:

$$2\frac{3}{4} = 2 + \frac{3}{4} = 1 + 1 + \frac{3}{4}$$

also:

$$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$$

Participants may be encouraged to suggest a variety of ways to further explain this concept. Using $1\frac{3}{4}$, $6\frac{1}{5}$, $3\frac{1}{3}$, how might you illustrate the relationship between mixed numbers and improper fractions with Cuisenaire rods or "real life" examples.

The intent of these activities has been to focus attention on meanings and concepts before getting involved with operations on fractions. Many of the serious difficulties experienced by some children come from having been taught only the operations and algorithms without really understanding what fractions mean or why the procedures work.

After participants have had time to share ideas, questions and concerns, they whould be given the opportunity to examine and "try out" materials for individual and small group activities, games and practice exercises. These may include:

1) Folder games F-20 and 21
2) Fraction Squares Game F-22
3) Fraction Strips Activities F-23
4) Fractions on a Geoboard FA-2 and 3
5) Ordering Fraction Strips FA-3
6) Equivalent Fractions Activity FA-4

5) Addition and Subtraction of Fractions

In this section we will introduce some procedures for "easing into" the learning of operations with common fractions. The success that students have will depend on certain background understandings and abilities. These include:

1) clear, correct concepts of the operations with whole numbers
2) understanding of the basic properties which apply (commutation, association, distributivity)
3) understanding the meanings of numbers represented by various fraction forms
4) concepts of factors and multiples used in renaming fractions to complete computations

Since addition and subtraction of fraction numbers are so interrelated, they will be developed together.

Included in this section are:

* Addition and subtraction of like fractions
* Addition and subtraction of unlike fractions
    .. a given denominator as common denominator
    .. the product of denominators as common denominator
    .. least common denominator
* Addition and subtraction with mixed numbers

1) Addition of Like Fractions

The "pound of butter" model that was used to illustrate equivalence can be helpful in the introduction of addition of like fractions. The workshop leader may take the participants through a simulated problem situation that is so easy that students can be lead into addition of fractions by emphasizing understanding and minimizing "newness."

Teacher: I have here some sticks that look like butter sticks. How much would this carton weigh if it were real butter?

Pupils: One pound.

Teacher: How much would each stick weigh?

Pupils: One-fourth of a pound.
quarter-pound

Teacher: Suppose your mother wants to know if she has enough butter for a special cake she would like to bake. She has two sticks of butter in the refrigerator and there is one full stick of butter on the butter plate. How many sticks of butter does she have?

Pupils: Three sticks. (very easy...)

Teacher: I know that is simple to you, but I want to start with something very simple and lead up to something else. Let's write what we have done on the chalkboard.

2(sticks)+ 1(stick) = 3 (sticks)

Teacher: We said that these sticks of butter could be called by other names.

2 quarters + 1 quarter = 3 quarters
2 fourths + 1 fourth = 3 fourths

All of these sentences say the same thing. Now I going to write it still another way. Watch while I write it a shorter way. This is one way we write the addition of fraction numbers.

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

This example can guide the teacher as she presents the fraction as a new kind of unit. The denominator of the fraction is seen as the name of the fraction unit. The numerators tell the number of such units being added and the number of such units in the sum. The workshop leader may encourage participants to provide other examples similar to the "butter" model. These may include:

Inches - twelfths of a foot
Days - sevenths of a week
Eggs - twelfths of a dozen

It is important in the early stages of addition with fractions that algorithms be written to include **each** step in the procedure:

$$\frac{2}{5} + \frac{3}{5} = \frac{2+3}{5} = \frac{5}{5} = 1$$

$$\frac{4}{12} + \frac{5}{12} = \frac{4+5}{12} = \frac{9}{12}$$

The writing of "2+3" and "4+5" over the single denominator emphasizes the fact that addition takes place with the numerators, not the denominators. This step also paves the way for later understanding of $\frac{a+b}{c}$ in algebra.

2) Subtraction of Like Fractions

The procedure for introducing subtraction with like fractional numbers follows the same pattern as for addition. You will probably wish to present them together, going directly from the "butter" addition situation to a subtraction situation.

Teacher: If I have three sticks of butter and use one stick for spreading on toast, how many sticks of butter will be left?

As with addition, there is nothing new about subtraction of fractions except the terminology and the use of names and notations. Participants may wish to add other situations from "real life" for this introductory level. You may cut a paper plate into eighths. Give one participant 7/8 of the "pie". Have another take away 3/8. What part of the pie does the first person have left?

It should be made clear to students that in addition and subtraction of like fractions, the fractions are being used as measures - measuring units for whole groups or whole things, so the denominator identifies the fraction unit being used (sticks of butter or pieces of pie). First, one must check to see that the names (denominators) for each term are the same, and then add or subtract the numbers (numerators).

$$\frac{\text{numerator}_1}{\text{denominator}} + \frac{\text{numerator}_2}{\text{denominator}} = \frac{\text{numerator}_1 + \text{numerator}_2}{\text{denominator}}$$

$$\frac{\text{numerator}_1}{\text{denominator}} - \frac{\text{numerator}_2}{\text{denominator}} = \frac{\text{numerator}_1 - \text{numerator}_2}{\text{denominator}}$$

Hands-on Activities

The workshop leader is encouraged to use Pattern Block activities for the exploration and discovery of addition and subtraction concepts involving fractions. If Cuisenaire Rods are used, the Cuisenaire pages 16,17, 49, 51 will be helpful. The use of Pattern Blocks for operations with fractions will provide older children with a new and interesting manipulative . It is important for teachers to feel comfortable with them and understand a variety of ways in which they may be used. They are easier to use than

Have participants spend some time "getting acquainted" with Pattern Blocks by arranging and rearranging them in a variety of ways. If time is available, some unstructured experiences with Pattern Block Activity Cards will be useful.

The workshop leader should use Pattern Blocks for the Overhead Projector as participants are solving problems with blocks. Some of the activity sheets for addition and subtraction of fractions using Pattern Blocks may be desirable as transparancies. Participants should be encouraged to use materials numbered FM-8 to discover the value of using Pattern Blocks for developing understandings of operations with fractions.

Activities, games and practice sheets should be available for participants to examine and use. These may include:

1) Addition of Fractions, FA-3
2) Fractions on a Geo-Board, #4 and #5 FA-3
3) Fraction Slide Rule Activity, #1 and #2, FA-4,5
4) Add the Fractions, FA-4

3) Addition and Subtraction of Unlike Fractions

When unlike fraction numbers are to be added or subtracted, a preliminary step is necessary. This step is not a part of the addition or subtraction, but it makes addition and subtraction possible.

Since only like terms may be added or subtracted, unlike terms must be renamed before the adding or subtracting can take place. With fractions, it is the denominators which must be renamed to make them alike. Students will be quick to point out that a change in denominators causes a change in the units used to state a certain quantity - that means that when the denominator changes so will the numerator.

The renaming step before adding and subtracting fractions varies in difficulty depending on the type of change necessary. These types are classified as 1) those in which one of the denominators is the common denominator, 2) those in which the product of the denominators is used as the common denominator, 3) those in which some other number is the least common denominator.

The workshop leader should encourage participants to work through each example being careful not to "skip" any steps. They may also wish to make up some examples of problems that children would enjoy working with that would give them practice with a variety of of demonstration techniques.

$$1/2 + 1/3 = 3/6 + 2/6 = \frac{3+2}{6} = \frac{5}{6}$$

$$2/3 - 1/4 = \frac{8-3}{12} = \frac{5}{12}$$

Pattern Blocks can be very helpful to illustrate computation with fractions. Practice sheet F-24 can provide guidance for use of the Blocks, especially pages 62, 76, 94, 98 and 99.

The third type of change needed before adding and subtracting is found in problems where the product of the denominators is so large that it is hard to compute. The answers that result in these cases also must be changed to their simplest form. The use of the least common denominator is much better in such instances. Since this is the only new feature to solving such problems, the only point to be developed is to provide children with an efficient procedure.

Many people recommend the following procedure:

1) Is one of the denominators a multiple of the other? If it is, use it as the common denominator.
2) If suggestion 1 does not work, multiply the larger denominator by 2. Is the product divisible by the smaller denominator? If it is, use it.
3) If suggestion 2 does not work, continue to multiply the larger denominator by 3, 4, etc., until a product is secured that is a multiple of the smaller denominator. Use the smallest such product.

One word of caution. Since finding common denominators and reducing to lowest terms are opposite processes, with the former involving multiplication and the latter division, it will be confusing to the students. Stress that the lowest terms are needed in answers, while the common denominator is needed in working addition/subtraction of fraction problems!

The game "Fraction Roulette" may be used to help students have fun learning about fractions. (FA-8)

Fraction Strips (F-23) provide a method for identifying equivalent fractions for addition and subtraction of fractions with unlike denominators.

4) Addition and Subtraction with Mixed Numbers

Computation with mixed numbers is an extension of the algorithms used earlier. Children may need to review the steps taken to write a mixed number as a fraction as well as fractions as mixed numbers.

Remember:

To write a mixed number as a fraction:

- Multiply the whole number by the denominator
- Add the numerator to the product
- Write the sum over the denominator

Any fraction in which the numerator is equal to or greater than the denominator can be written as a whole number or a mixed number using division.

The vertical number line (FT-24) or a number line drawn on the chalkboard is helpful to illustrate these procedures. Fraction Blocks, Fraction charts (F-8 & 9) Cuisenaire Rods and Pattern Blocks can help with this review. Practice sheet F-24, Fraction Circles (F-19), Squared Paper (F-25) as well as many other materials and activities will be good review. (The leader should demonstrate these procedures.)

The workshop leader should give participants the opportunity to use squared paper in the following way:

- Outline five non-intersecting rectangles, each 1 unit high and 2 units wide. Shade $4\frac{1}{2}$ rectangles. Count halves, and complete: $4\frac{1}{2} = \frac{?}{2}$.
- Outline 6 non-intersecting rectangles, each 1 unit high and 3 units wide. Shade $5\frac{2}{3}$ rectangles. Count thirds, and complete: $5\frac{2}{3} = 3\frac{?}{3}$.

*** Outline 8 non-intersecting rectangles, each 2 units high and 3 units wide. Shade $7\frac{1}{6}$ rectangles. Count sixths, and complete: $7\frac{1}{6} = \frac{?}{6}$.**

The procedures for computing sums and differences with mixed numbers includes the following:

*** Compute the sum**
Find $2\frac{2}{5} + 1\frac{4}{5}$

1) Add the fractions

$$\begin{array}{r} 2\frac{2}{5} \\ +\ 1\frac{4}{5} \\ \hline \frac{6}{5} \end{array}$$

Add the whole numbers

$$\begin{array}{r} 2\frac{2}{5} \\ +\ 1\frac{4}{5} \\ \hline 3\frac{6}{5} \end{array}$$

Write in lowest terms

$$\begin{array}{r} 2\frac{2}{5} \\ +\ 1\frac{4}{5} \\ \hline 3\frac{6}{5} = 4\frac{1}{5} \end{array}$$

$3\frac{6}{5}$
↓
$$\begin{array}{r} 1\frac{1}{5} \\ 5\overline{)6} \end{array}$$
$3+1\frac{1}{5}=4\frac{1}{5}$

*Compute the sum:
Find $2\frac{1}{2} + 3\frac{2}{3}$

2) Find a common denominator

$$\begin{array}{r} 2\frac{1}{2} \\ +\ 3\frac{2}{3} \\ \hline \end{array}$$

Write equal fractions

$$\begin{array}{r} 2\frac{1}{2} = 2\frac{3}{6} \\ +\ 3\frac{2}{3} = 3\frac{4}{6} \\ \hline \end{array}$$

Add the fractions
Add whole numbers
Write in lowest terms

$$\begin{array}{r} 2\frac{1}{2} \ = \ 2\frac{3}{6} \\ +\ 2\frac{2}{3} \ = \ 3\frac{4}{6} \\ \hline 5\frac{7}{6} = 6\frac{1}{6} \end{array}$$

*Compute the difference
Find $3\frac{9}{10} - 2\frac{1}{2}$

3) Rewrite fractions with common denominator

Subtract fractions
Subtract whole numbers
Write in lowest terms

$$\begin{array}{r} 3\frac{9}{10} \ = \ 3\frac{9}{10} \\ -\ 2\frac{1}{2} \ = -\ 2\frac{5}{10} \\ \hline 1\frac{4}{10} = 1\frac{2}{5} \end{array}$$

*Compute the difference
Find $7 - 3\frac{4}{5}$

4) First rename. Then subtract.

$$\begin{array}{r} 7 = 6\frac{5}{5} \longleftarrow \\ -\ 3\frac{4}{5} = 3\frac{4}{5} \\ \hline 3\frac{1}{5} \end{array}$$

Rename 7.
$7 = 6+1$
$= 6 + \frac{5}{5}$
$= 6\frac{5}{5}$

Although everyday use of addition and subtraction of fractions is not very extensive when compared with addition and subtraction of whole numbers, it may be very important to perform these operations correctly when they apply. The most common uses involve standard measures of various kinds - in cooking; in buying, selling, and using materials such as cloth or lumber; in nursing and various industrial situations.

To keep "sharp" and accurate in addition and subtraction of fractions, youngsters should practice their skills. Many excellent puzzle and practice sheets, games, and activities are available. Some excellent source books for practice materials include:

- Invitation to Mathematics, Blackline Masters
  Scott Foresman

- Mathimagination, Book D
  Creative Publications
- Working With Fractions
  Dale Seymour Publishers
- No Frill Math Drill
  Instructo
- Understanding Fractions
  Dale Seymour Publishers
- Versa-Tiles Intermediate Math Lab
  Book 6, Fractions I
  Book 7, Fractions II
  Educational Teaching Aids
- Basic Skills Bonanza: Adding and Subtracting Fractions
  Gamco Ind., Inc.

6) Multiplication and Division of Fractions

Multiplication and division of fractions will be introduced in this section. To help children understand these ideas, the use of physical representations will be stressed. The approaches presented will be focused on the older elementary school students.

Included in this section are:

Multiplication
- * Identification and generation of patterns in multiplication with fractional numerals.
- * The additive interpretation for multiplication with fractions.
- * The area interpretation for multiplication with fractions.

Division
- * Two alternative methods for dividing fractions,

1) Generating Patterns in Multiplication with Fractions

Participants should arrange and rearrange Pattern Blocks according to instructions on pages 101 through 107, . . . The workshop leader can stimulate interest in the activities by use of the Pattern Blocks for the Overhead Projector. (Pattern Block materials are selected from Fractions With Pattern Blocks by , Mathew Zullie .)

Participants should be encouraged to describe the pattern used to multiply fractions by multiplying the numerators and multiplying the denominators.

Example:

$$\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20} = \frac{3}{10}$$

2) The Additive Interpretation for Multiplication With Fractions

Addition of fractions depends on the concept of fractions as describing fraction units. Since one of the basic concepts of multiplication is that of repeated additions, our first problem in multiplication of proper fractions grows easily out of already developed ideas, that is, multiplication of a whole number and a fraction.

The workshop leader should record the steps in this procedure on the overhead projector or the chalkboard.

Problem: Jean practices her piano lesson $\frac{2}{3}$ of an hour every day except Sunday. How many hours does she practice each week?

A "time line" may be constructed using rods of uniform size. Let each rod equal $\frac{1}{3}$ of an hour. The problem can readily be solved by addition - laying two blocks, end to end, for each $\frac{2}{3}$ hour. Six sets of 2 thirds each should be in place. Adding "2 thirds" 6 times is obviously the same as multiplying 6 X 2 thirds.

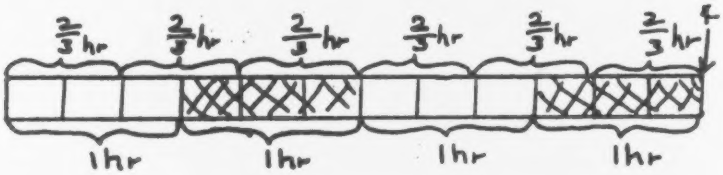
The progression of steps is as follows:

1) 2 thirds + 2 thirds + 2 thirds + 2 thirds + 2 thirds + 2 thirds = 12 thirds

2) $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{12}{3}$

3) 6 X 2 thirds = 12 thirds

4) $6 \times \frac{2}{3} = \frac{12}{3}$



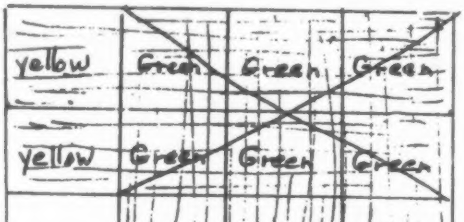
Additional problems should use other concrete and semi-concrete materials (pies, flannel-board fractional parts, ruler fractions, etc.) until the children understand the generalization that when a whole number is multiplied times a proper fraction, the whole number is multiplied times the numerator of the fraction. If necessary, that product is reduced or changed to a whole or mixed number.

3) The area interpretation for Multiplication with Fractions

Multiplication of two fraction numbers lends itself well to an area interpretation. In this example, the major new concept to be developed is how to handle two given denominators and come out with a third denominator in the product.

One way that this interpretation can be developed is to ask each participant to act out the procedure. The workshop leader will set up the problem in the following way:

1) Take a rectangular sheet of paper.
2) Fold the paper lengthwise so as to make thirds. (the leader should demonstrate)
3) Open up the paper and draw heavy lines where the folds are.
4) Color 2 of the thirds of your paper with a yellow crayon.
5) Now fold the whole sheet of paper in the other direction to make fourths.
6) Open up the paper and draw heavy lines where the new folds are.
7) Do not pay any attention to the yellow part. Look at the whole sheet of paper and find 3 of the fourths you just made by folding the paper. Color 3 of the fourths with a blue crayon. Color blue over yellow if you have to.
8) Now I have a problem question for you to answer. What part of the whole sheet of paper is green?



$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

Participants may be encouraged to offer suggestions for key questions which would stimulate students understanding of multiplication of a fraction times a fraction.

The value of waiting until after the situation has been thoroughly discussed to develop the algorithm lies in the opportunity for children to arrive at the understanding as an outgrowth of "what we did". The importance of taking plenty of time with one good learning situation cannot be overestimated.

Following a careful experience such as the one just described, students may be ready for more abstract situations. Some "shortcuts" may include:

1) Find $\frac{1}{2} \times \frac{8}{9} =$

Use this shortcut:
First divide the numerator
and the denominator by a common factor

8÷2=4

$$\frac{1}{2} \times \frac{8}{9} = \frac{1 \times \cancel{8}^{4}}{\cancel{2}_{1} \times 9} = \frac{1\times 8}{2\times 9} \div 1 = \frac{1\times 8}{2\times 9} \div \frac{2}{2} =$$

2÷2=1

$$\frac{1\times 8 \div 2}{2\times 9 \div 2} = \frac{1\times 4}{1\times 9} = \frac{4}{9}$$

2) Find $2\frac{1}{2} \times 2\frac{2}{3}$

Use this shortcut:
Write each mixed number as an improper
fraction.
Multiply the fractions.

$$\underset{\times}{\overset{+}{2\frac{1}{2}}} \qquad \underset{\times}{\overset{+}{2\frac{2}{3}}} \qquad \frac{5}{2} \times \frac{8}{3} = \frac{5\times 8}{2\times 3} = \frac{40}{6}, \text{ or } 6\frac{2}{3}$$

$$\frac{5}{2} \times \frac{8}{3}$$

Division of Fractions

Objective

To introduce two alternative methods for dividing fractions.

Materials

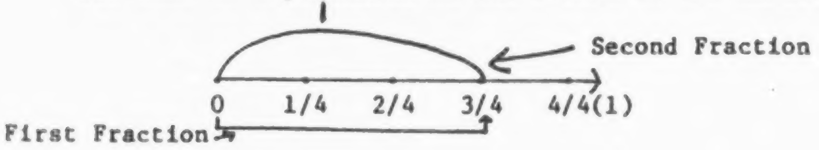
Paper, pencil, and a ruler

Description

Two related alternative methods for dividing fractions. These should be used prior to and as an introduction to the invert and multiply method. To use either alternative method, the fractions must be expressed with the same denominator.

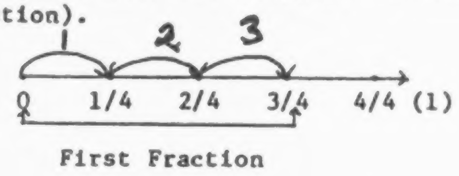
Method One

Example: 3/4 ÷ 3/4 means "How many sets of 3/4 are in 3/4?" The number line can be used to find the answer. Since both fractions have a denominator of 4, the number line should be divided into one-fourths. The first fraction is 3/4, so that is the size of a set that is represented on the bottom of the number line.



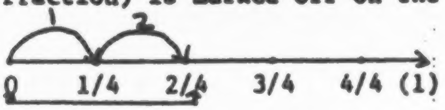
Looking at the number line, I see there is 1 set of 3/4 in 3/4. Therefore, 3/4 ÷ 3/4 = 1.

Example: 3/4 ÷ 1/4 means "How many sets of 1/4 are in 3/4?" The fractions have the same denominator (4) so the number line will be divided into one-fourths. I will need a set of 3/4 (first fraction).



Looking at the top of the number line, I count 3 sets of 1/4. Therefore, there are 3 sets of 1/4 in 3/4 or 3/4 ÷ 1/4 = 3.

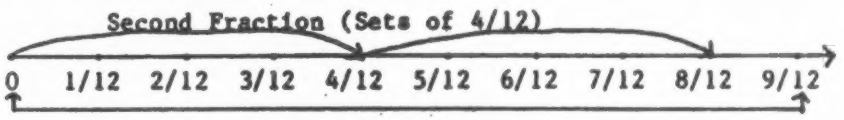
Example: 1/2 ÷ 1/4 means "How many sets of 1/4 are in 1/2? The denominators are not the same. Using the L.C.D. of 4, the first fraction is renamed as 2/4. The problem now is 2/4 ÷ 1/4. The number line is divided into one-fourths. A set of 2/4 (first fraction) is marked off on the bottom of the number line.



First Fraction

Sets of 1/4 are marked off on the top of the number line until the set of 2/4 is used up. The sets of 1/4 are then counted (2). Therefore, 1/2 ÷ 1/4 = 2.

Example: 3/4 ÷ 1/3 means "How many sets of 1/3 are in 3/4?" Using the L.C.D. of 12 and renaming both fractions, the problem is 9/12 ÷ 4/12 or "How many sets of 4/12 are in 9/12?" Since the L.C.D. was 12, the number line will be divided into one-twelfths. The first fraction (9/12) is marked off on the bottom of the number line.



First Fraction

Notice we have 2 whole sets of 4/12 and 1/12 left over. The 1/12 is 1/4 of a set of 4/12, so we have 2 1/4 sets of 4/12. 3/4 ÷ 1/3 = 2 1/4.

Method Two

Method Two consists of these steps:

1. Make sure fractions have the same denominator.
2. Divide the first denominator by the second denominator. (This will always give an answer of 1 and can be forgotten.)
3. Divide the first numerator by the second numerator. This will be the answer.

Example: 3/4 ÷ 3/4

Step 1: In this example, the denominators are the same.
Step 2: 4 ÷ 4 = 1.
Step 3: 3/3 = 1, so 1 is the answer.

Example: 3/4 ÷ 1/4 means "How many 1/4s are in 3/4? Here again, the denominators are the same.
Step 2: 4 ÷ 4 = 1.
Step 3: 3 ÷ 1 = 3/1 = 3, so there are 3 sets of 1/4 in 3/4.

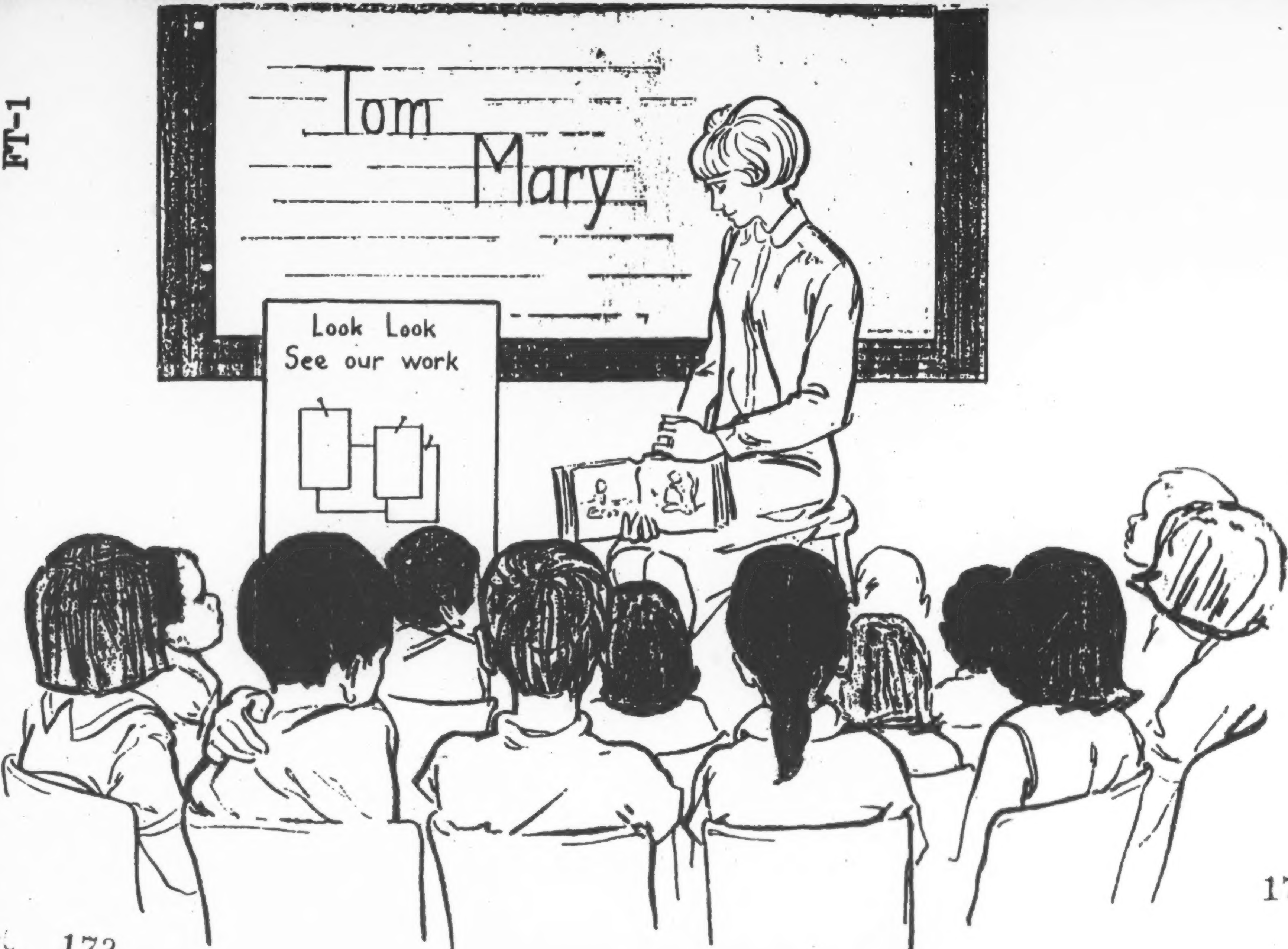
Example: 1/2 ÷ 1/4 means "How many sets of 1/4 are in 1/2?
Step 1: The denominators are not the same using the least common denominator (4) and renaming 1/2, the problem becomes 2/4 ÷ 1/4.
Step 2: 4 ÷ 4 = 1.
Step 3: 2 ÷ 1 = 2/1 = 2. Therefore, there are 2 sets of 1/4 in 1/2.

Example: 3/4 ÷ 1/3 means "How many sets of 1/3 are there in 3/4?
Step 1: Using the least common denominator (12) and renaming both fractions, the problem becomes: 9/12 ÷ 4/12.
Step 2: 12 ÷ 12 = 1.
Step 3: 9 ÷ 4 = 9/4 or 2 1/4.

There are 1/ sets of 1/3 in 3/4.

Notes: Using these techniques, mixed numbers should be changed to improper fractions.

FT-1

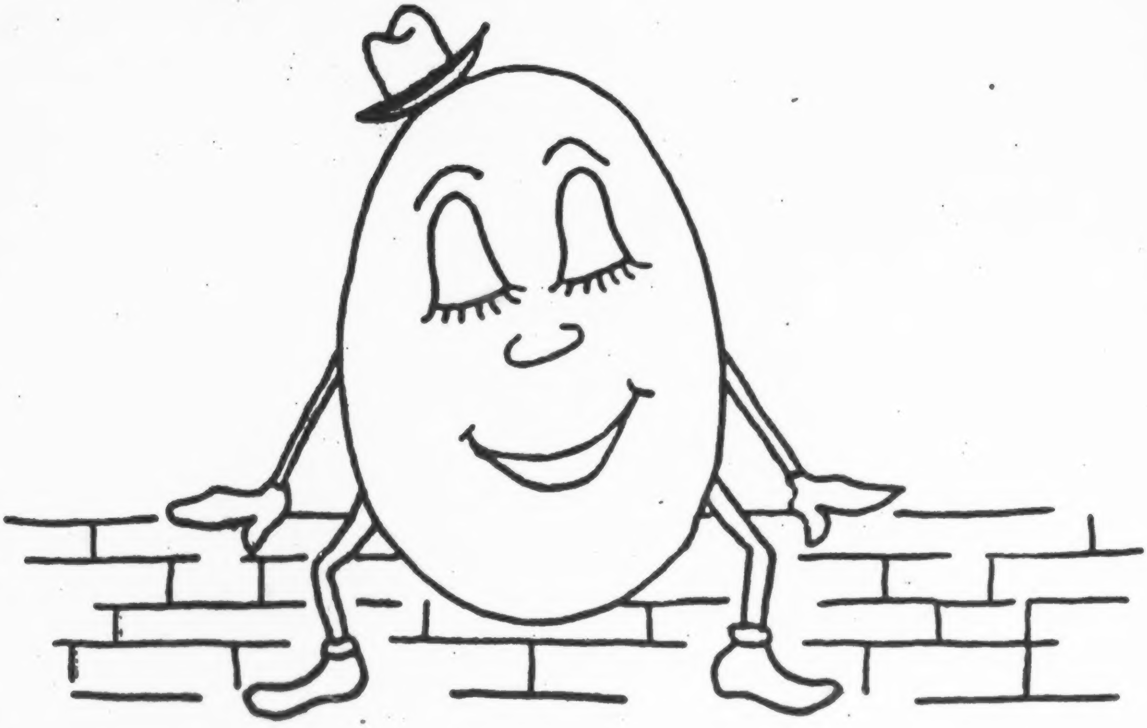

Tom
Mary
Look Look
See our work

173

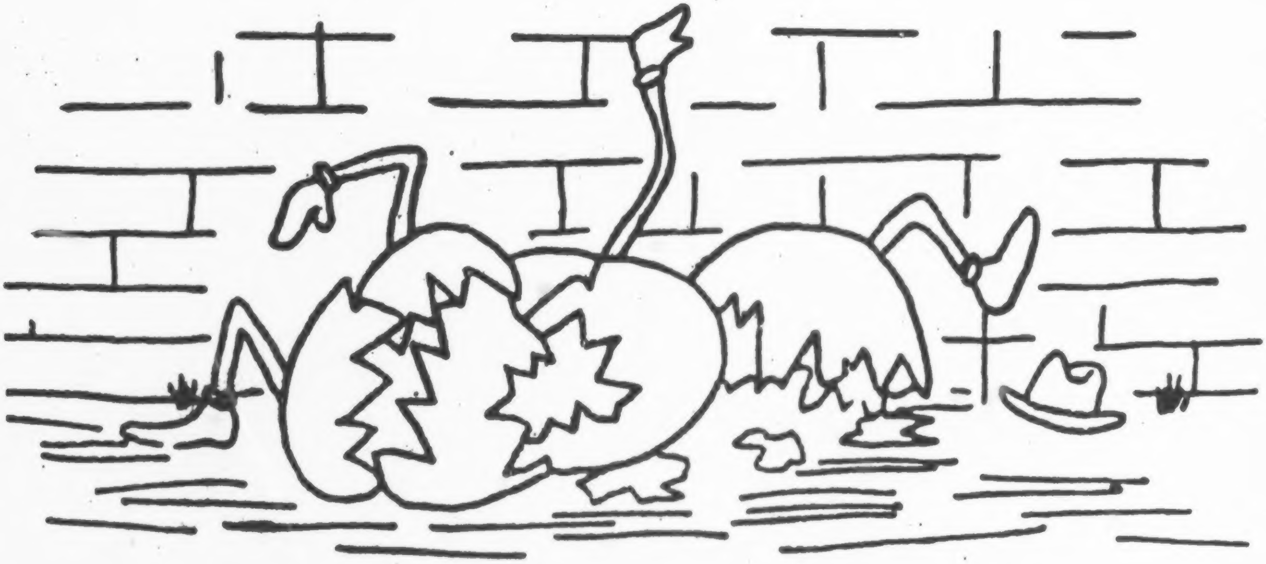
174

FT-2

HUMPTY DUMPTY SAT ON A WALL



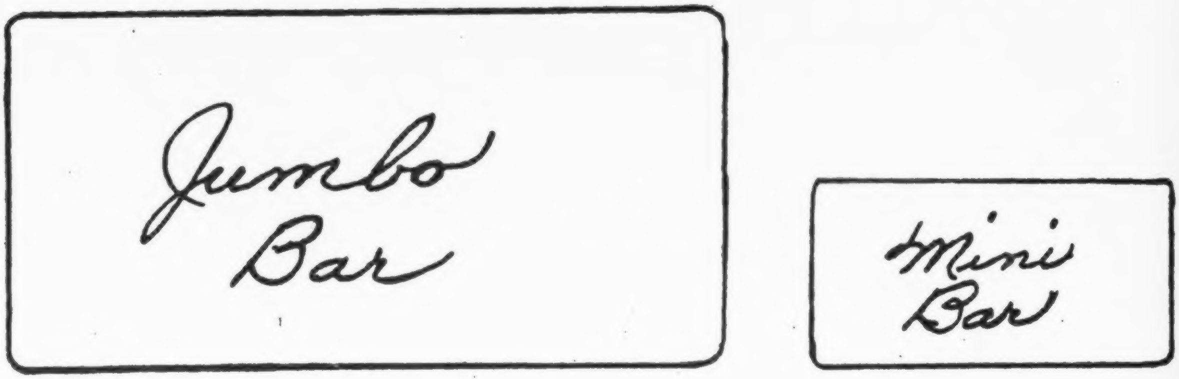
HUMPTY DUMPTY HAD A GREAT FALL



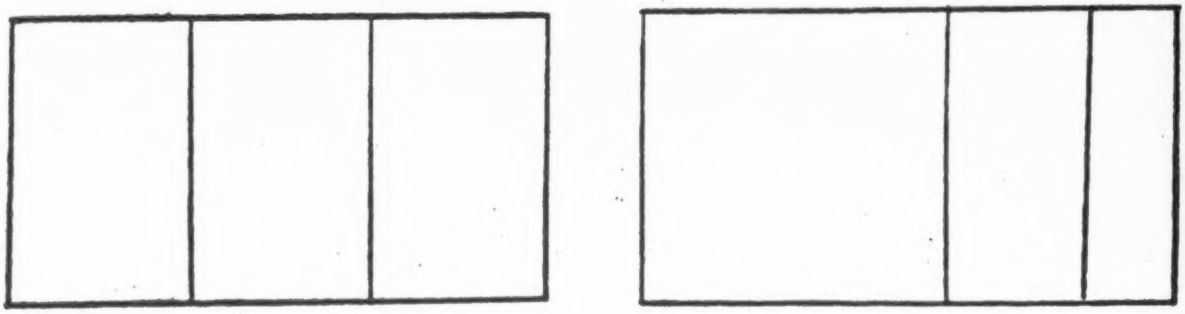
WHAT A FRACTIONAL EXPERIENCE!

FT-3

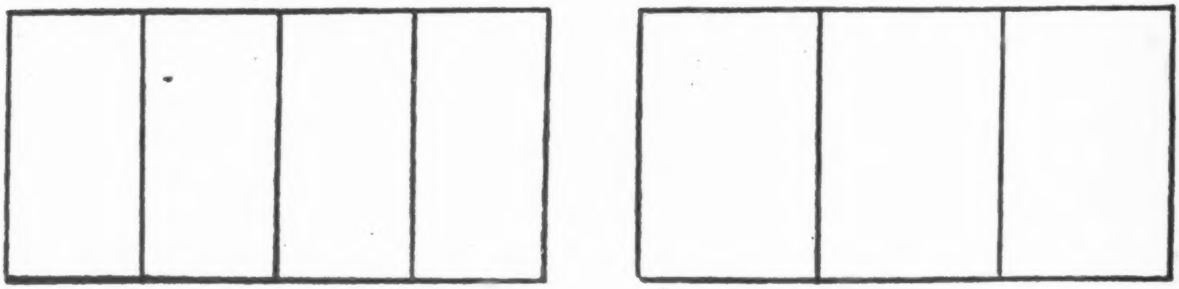
A


Jumbo Bar
Mini Bar

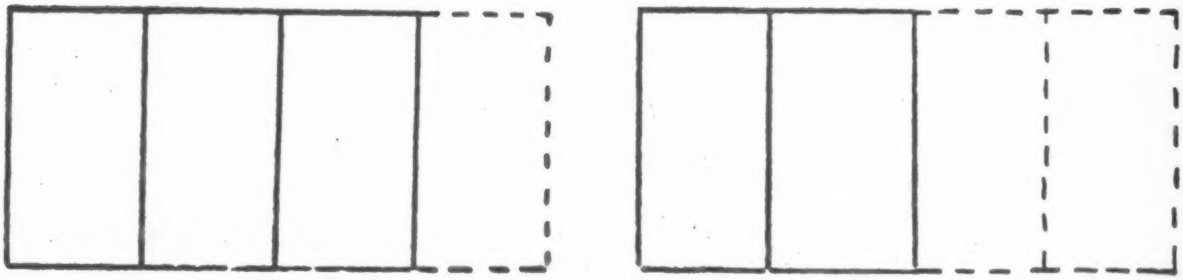
B



D



C



TEACHING LEARNING SEQUENCE

- ☆ CONCRETE: REAL OR REPRESENTATIONAL OBJECTS
- ☆ SEMI-CONCRETE: PICTURES OF OBJECTS
- ☆ ABSTRACT: SYMBOLS

RESEARCH

LESSONS USING MANIPULATIVE MATERIALS HAVE A HIGHER PROBABILITY OF PRODUCING GREATER MATHEMATICAL ACHIEVEMENT THAN DO NON-MANIPULATIVE LESSONS.

SYMBOLIC TREATMENTS ARE NOT AS EFFECTIVE AS THOSE IN WHICH MANIPULATIVE MATERIALS ARE USED.

THE USE OF MANIPULATIVES APPEARS TO BE EFFECTIVE WITH CHILDREN AT ALL ACHIEVEMENT LEVELS, ABILITY LEVELS AND SOCIOECONOMIC LEVELS.

Suydam, M. N. and Higgins, J. L. (1977). Activity - Based Learning in Elementary School Mathematics: Recommendations from Research. Columbus, Ohio: ERIC.

HOW WE LEARN

SENSE	%
TASTE	1
TOUCH	1 1/2
SMELL	3 1/2
HEARING	1
SIGHT	83

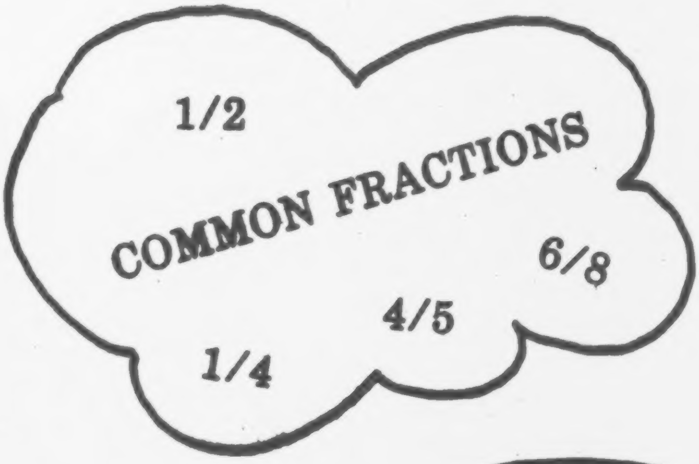
FT-7

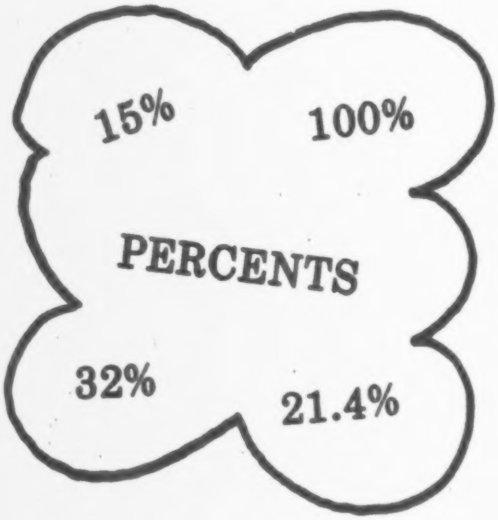
I HEAR, AND I FORGET.

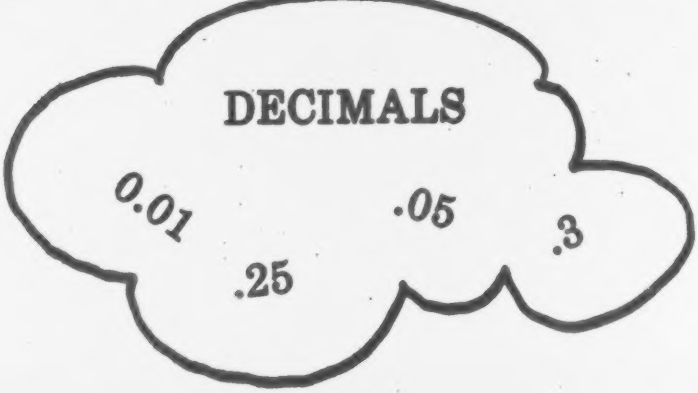
I SEE, AND I REMEMBER.

I DO, AND I UNDERSTAND.

FRACTIONAL NUMBERS


1/2
COMMON FRACTIONS
6/8
4/5
1/4

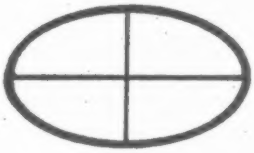

15%
100%
PERCENTS
32%
21.4%


DECIMALS
0.01
.05
.3
.25

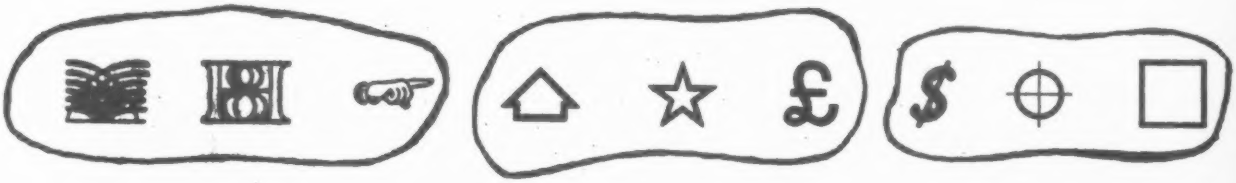
FT-9

COMMON FRACTIONS

Unit Subdivided Into Equal-Sized Parts



Set Subdivided Into Equal-Sized Groups



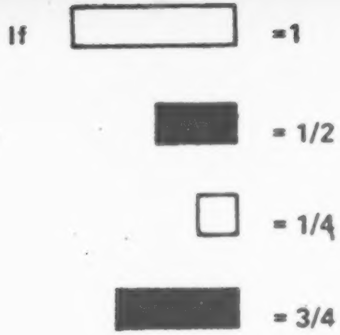
Ratios 1/6 or 1:6

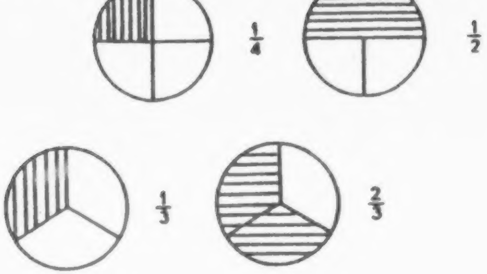
Indicated Division 3/4 or 3-4

Rational Numbers

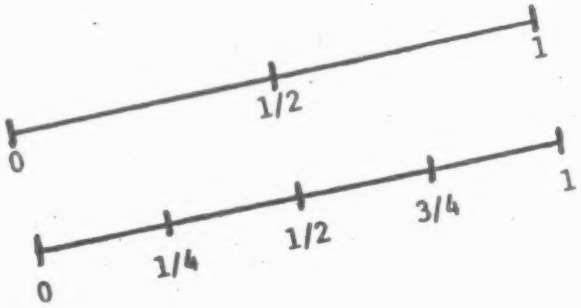
(a quotient of two integers)

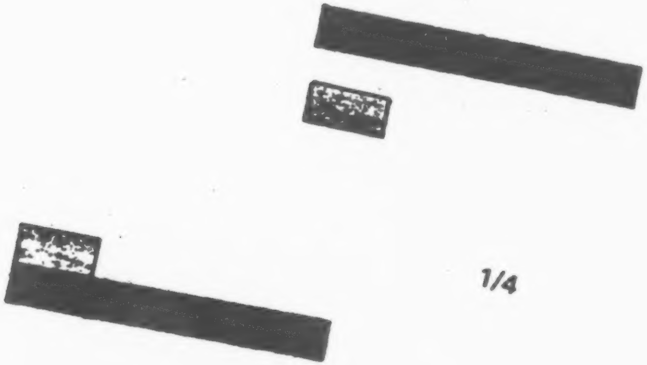
FT-10

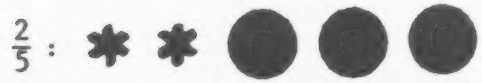



$\frac{1}{4}$
$\frac{1}{2}$
$\frac{1}{3}$
$\frac{2}{3}$

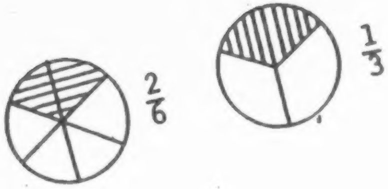
Meaning of Fractions

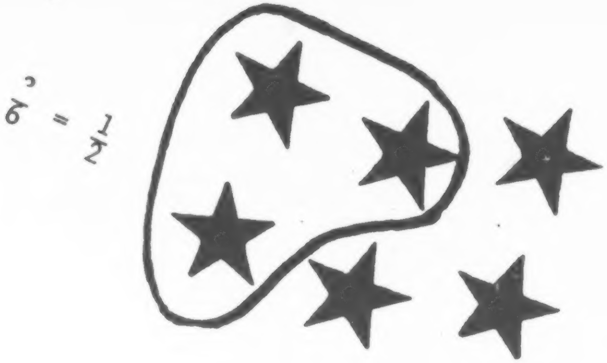

0
1/2
1
0
1/4
1/2
3/4
1





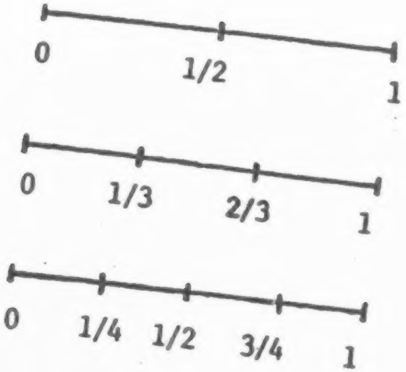
FT-11

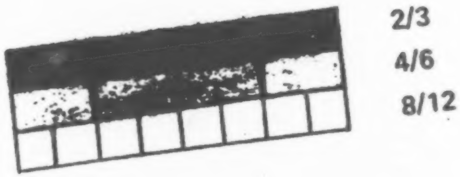




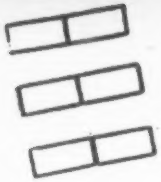
Equivalent Fractions

38

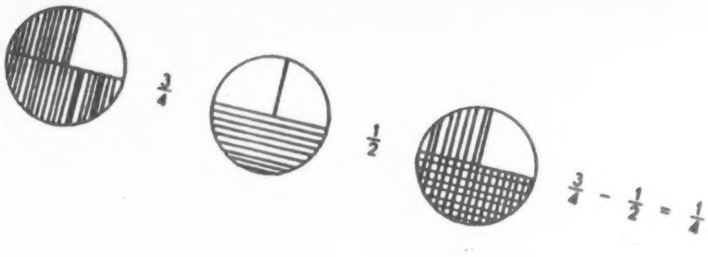

0
1/2
1
0
1/3
2/3
1
0
1/4
1/2
3/4
1



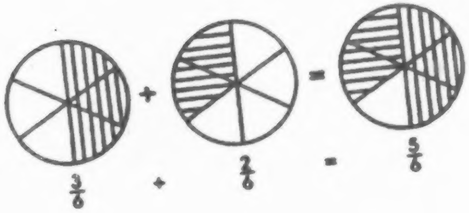
FT-12



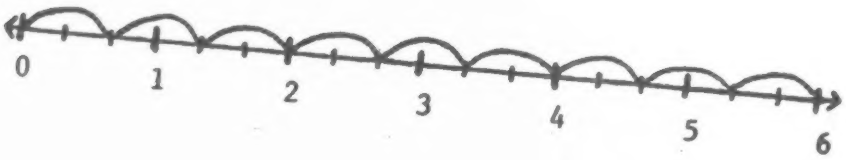
$3 \times \frac{2}{5} = ?$

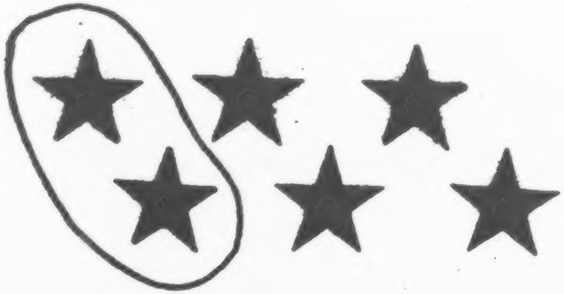

$\frac{3}{4}$
$\frac{1}{2}$
$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

Fraction Operations


$\frac{3}{6}$ + $\frac{2}{6}$ = $\frac{5}{6}$

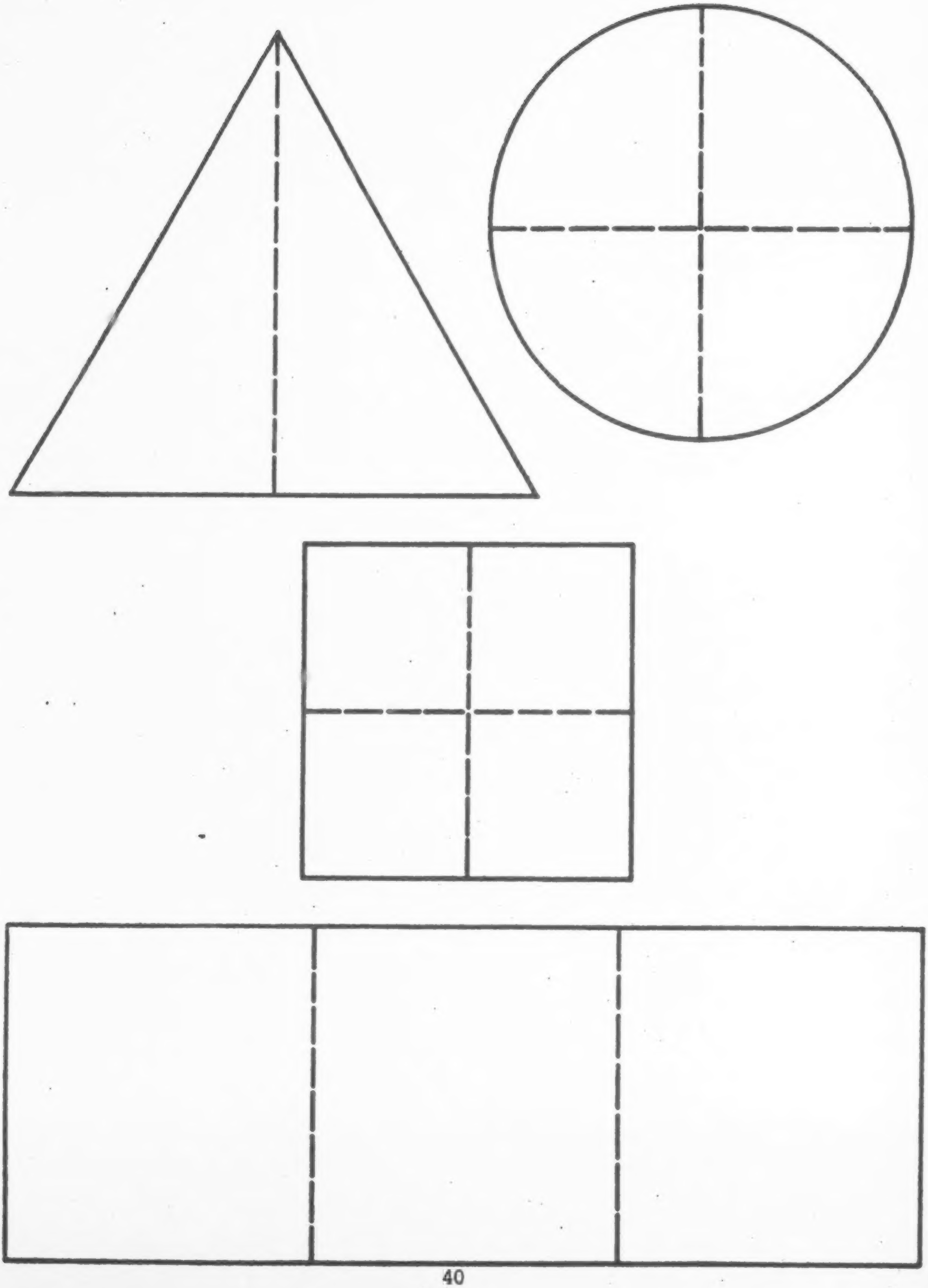
$6 \div \frac{2}{3} = ?$





$\frac{1}{3}$ of $6 = ?$

FT-13



Shape Up

Cut out the triangle.

Fold it to make halves.

Color $\frac{1}{2}$.

Cut out the circle.

Fold it to make fourths.

Color $\frac{1}{4}$.

Cut out the square.

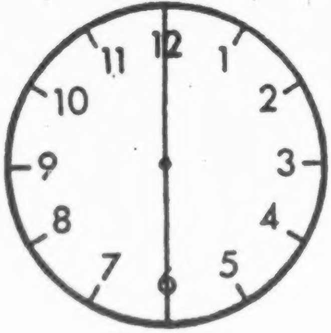
Fold it to make fourths.

Color $\frac{3}{4}$.

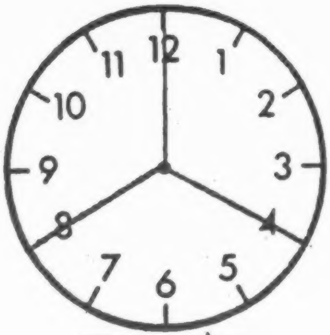
Cut out the rectangle.

Fold it to make thirds.

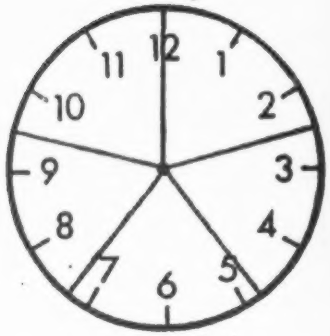
Color $\frac{2}{3}$.


12 1 2 3 4 5 6 7 8 9 10 11

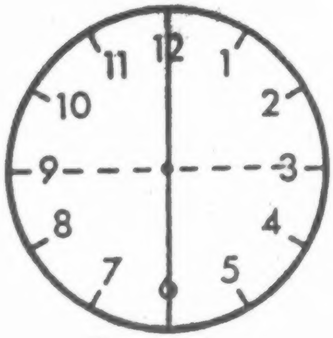
Halves


12 1 2 3 4 5 6 7 8 9 10 11

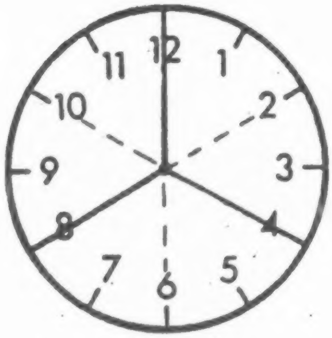
Thirds


12 1 2 3 4 5 6 7 8 9 10 11

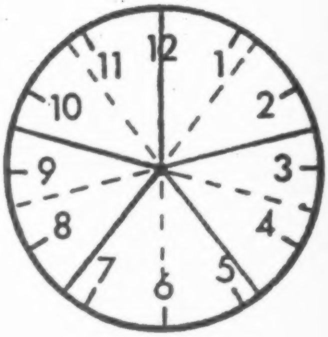
Fifths


12 1 2 3 4 5 6 7 8 9 10 11

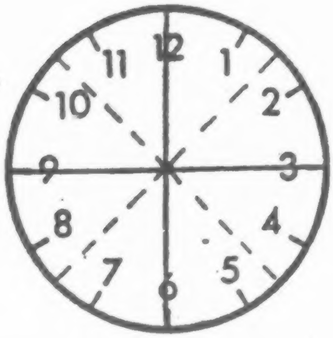
Fourths


12 1 2 3 4 5 6 7 8 9 10 11

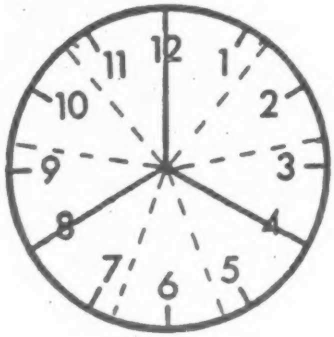
Sixths


12 1 2 3 4 5 6 7 8 9 10 11

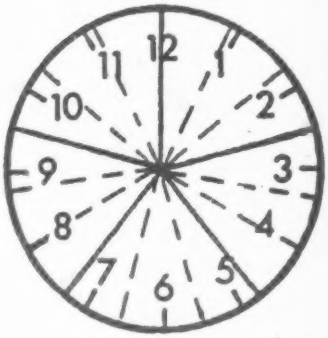
Tenths


12 1 2 3 4 5 6 7 8 9 10 11

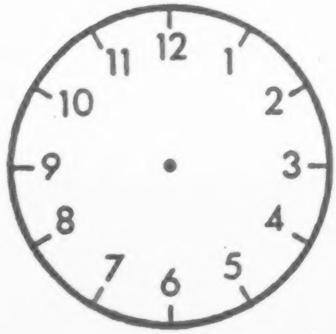
Eighths


12 1 2 3 4 5 6 7 8 9 10 11

Ninths


12 1 2 3 4 5 6 7 8 9 10 11

Fifteenths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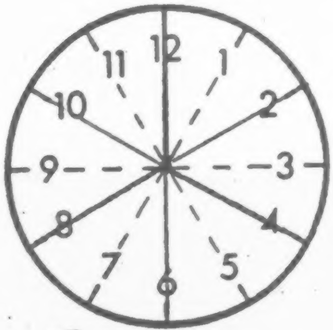
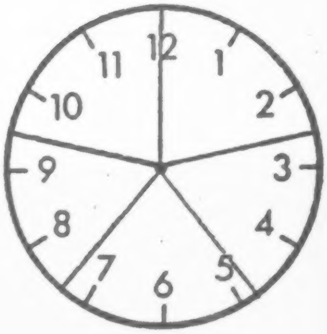
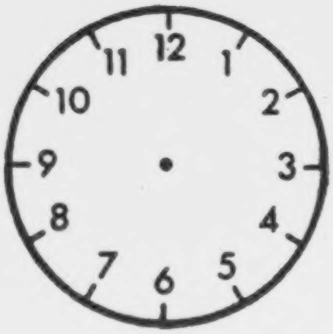
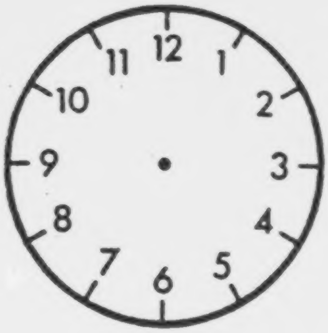

12 1 2 3 4 5 6 7 8 9 10 11

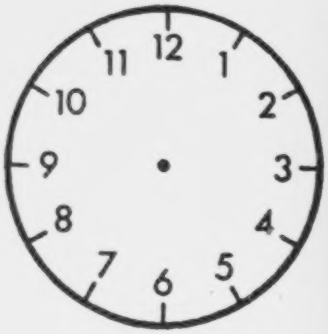
Twelfths

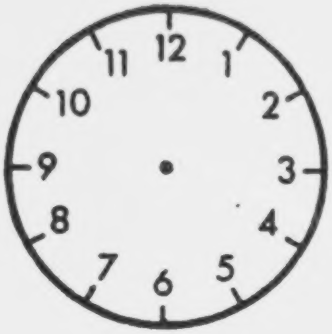

12 1 2 3 4 5 6 7 8 9 10 11

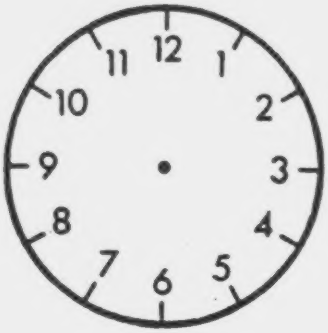
FT-16

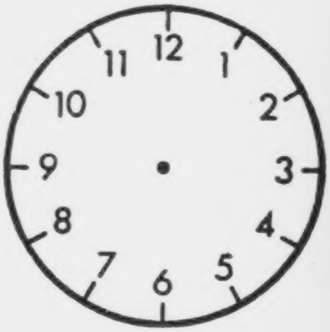


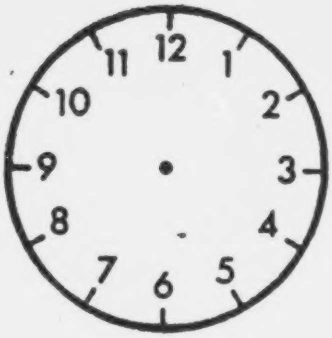


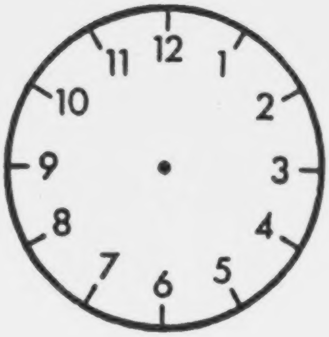


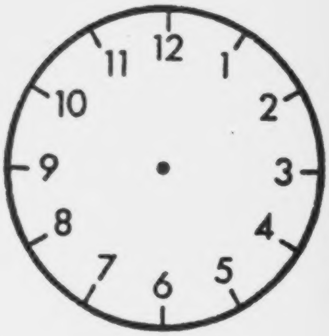


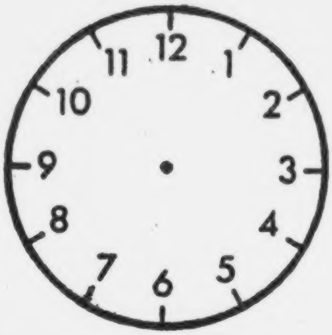


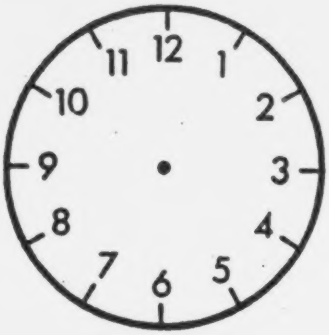


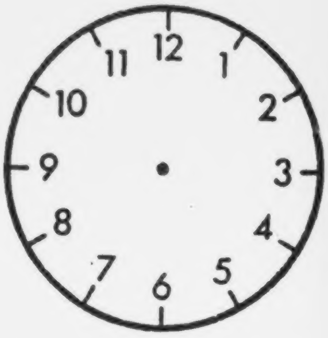












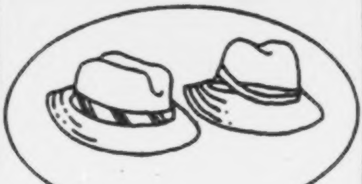
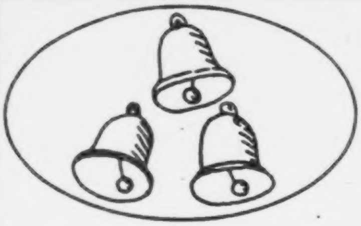
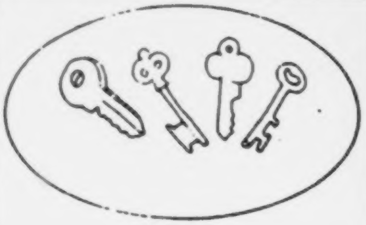
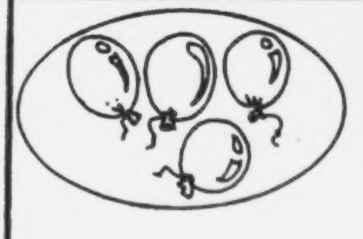
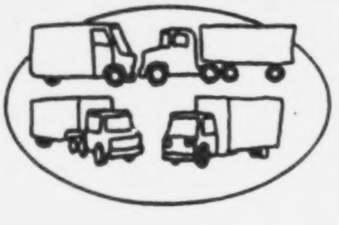
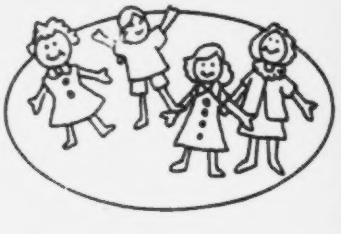
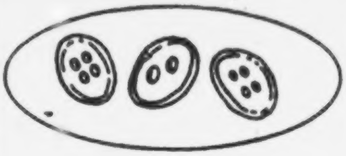
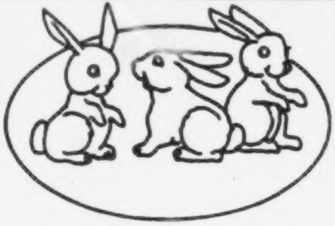
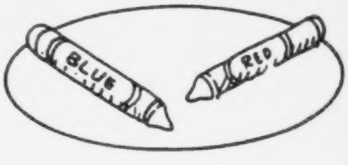
FT-17

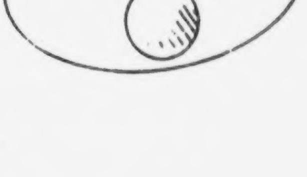
Fractions On A Number Line

44

Part of a Group

Color to show the fraction.

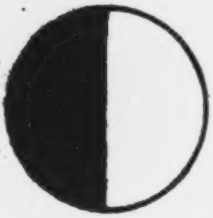
 $\frac{1}{2}$	 $\frac{2}{3}$	 $\frac{1}{4}$
 $\frac{3}{4}$	 $\frac{1}{4}$	 $\frac{2}{4}$
 $\frac{2}{3}$	 $\frac{1}{3}$	 $\frac{1}{2}$



John colored $\frac{1}{3}$ of the balls blue.

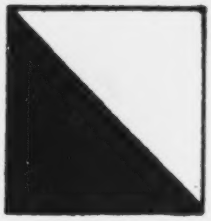
Tom colored $\frac{1}{3}$ of the balls yellow.

What part of the balls are colored? _____

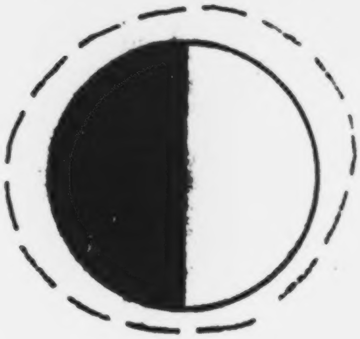


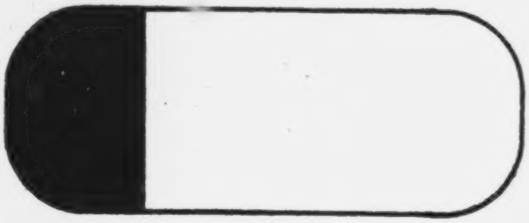
One half

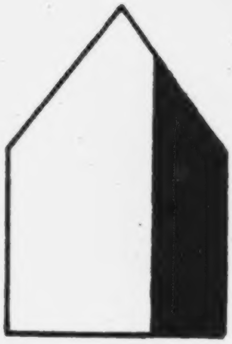
$\frac{1}{2}$

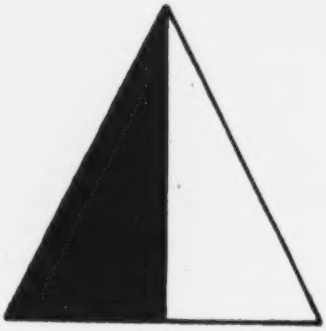


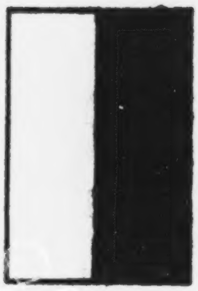
Mark the shapes that show one half.

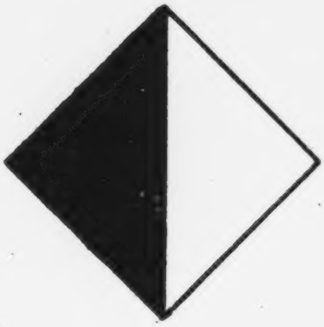


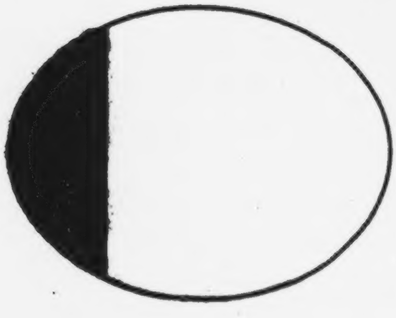


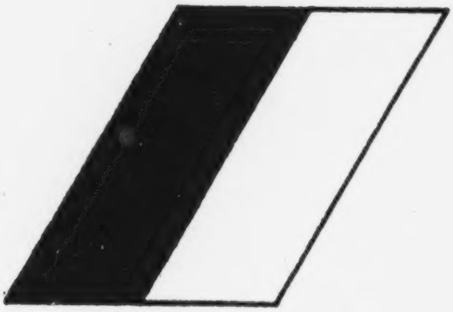












Abstract Identification

$\frac{1}{2}$

$\frac{3}{4}$

$\frac{1}{8} + \frac{2}{8} =$

$\frac{6}{9} - \frac{3}{9} =$

$\frac{3}{5} \times \frac{2}{6} =$

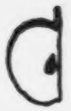
$5 \div \frac{1}{2} =$

Match.

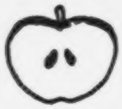
Sal has $\frac{1}{4}$ of an apple.

Bess has $\frac{1}{2}$ of an apple.

Who has more apple?



one fourth



one half

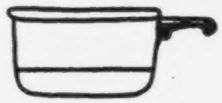
Sal

Bess

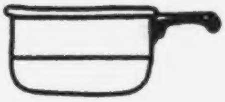
Hal fills $\frac{1}{2}$ of a pot with water.

Gus fills $\frac{1}{3}$ of a pot with water.

Who has more water?



one third



one half

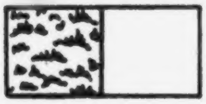
Hal

Gus

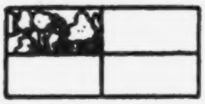
$\frac{1}{2}$ of the park has grass.

$\frac{1}{4}$ of the park has trees.

Is there more grass or trees?



one half



one fourth

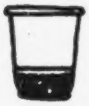
grass

trees

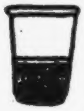
$\frac{1}{3}$ of Mark's glass is full.

$\frac{1}{2}$ of Tom's glass is full.

Who has more in his glass?



one third



one half

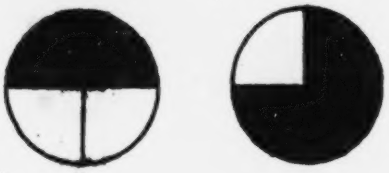
Tom

Mark

Sizing Up Fractions

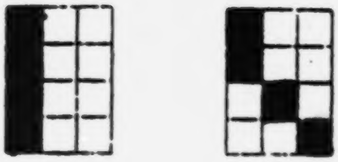
Write a fraction for the shaded part. Place >, <, or = in each ☐.

1.



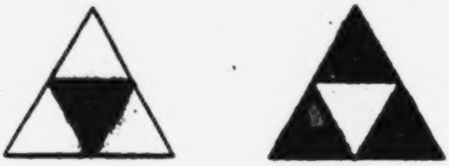
$\frac{2}{4}$ $\boxed{<}$ $\frac{3}{4}$

2.



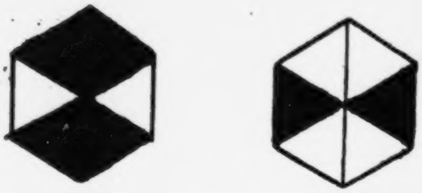
$\frac{4}{12}$ $\boxed{=}$ $\frac{4}{12}$

3.



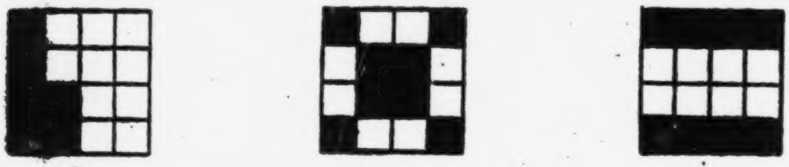
$\frac{1}{4}$ $\boxed{<}$ $\frac{3}{4}$

4.



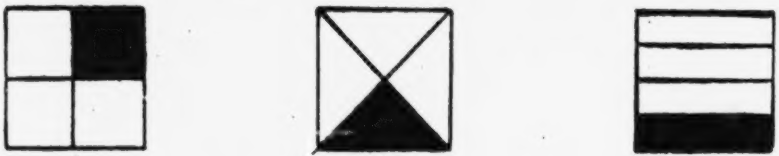
$\frac{4}{6}$ $\boxed{>}$ $\frac{2}{6}$

5.



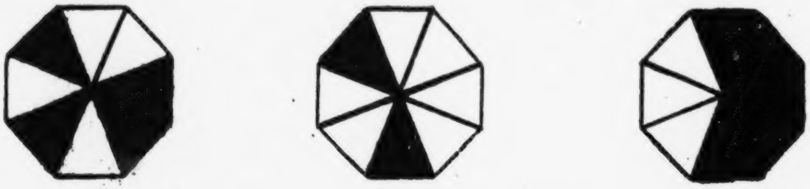
$\frac{6}{16}$ $\boxed{<}$ $\frac{8}{16}$ $\boxed{=}$ $\frac{8}{16}$

6.



$\frac{1}{4}$ $\boxed{=}$ $\frac{1}{4}$ $\boxed{=}$ $\frac{1}{4}$

7.



$\frac{4}{8}$ $\boxed{>}$ $\frac{2}{8}$ $\boxed{<}$ $\frac{5}{8}$

Abstract Comparison

Circle the largest fractional amount.

A. 1/2 , 1/8

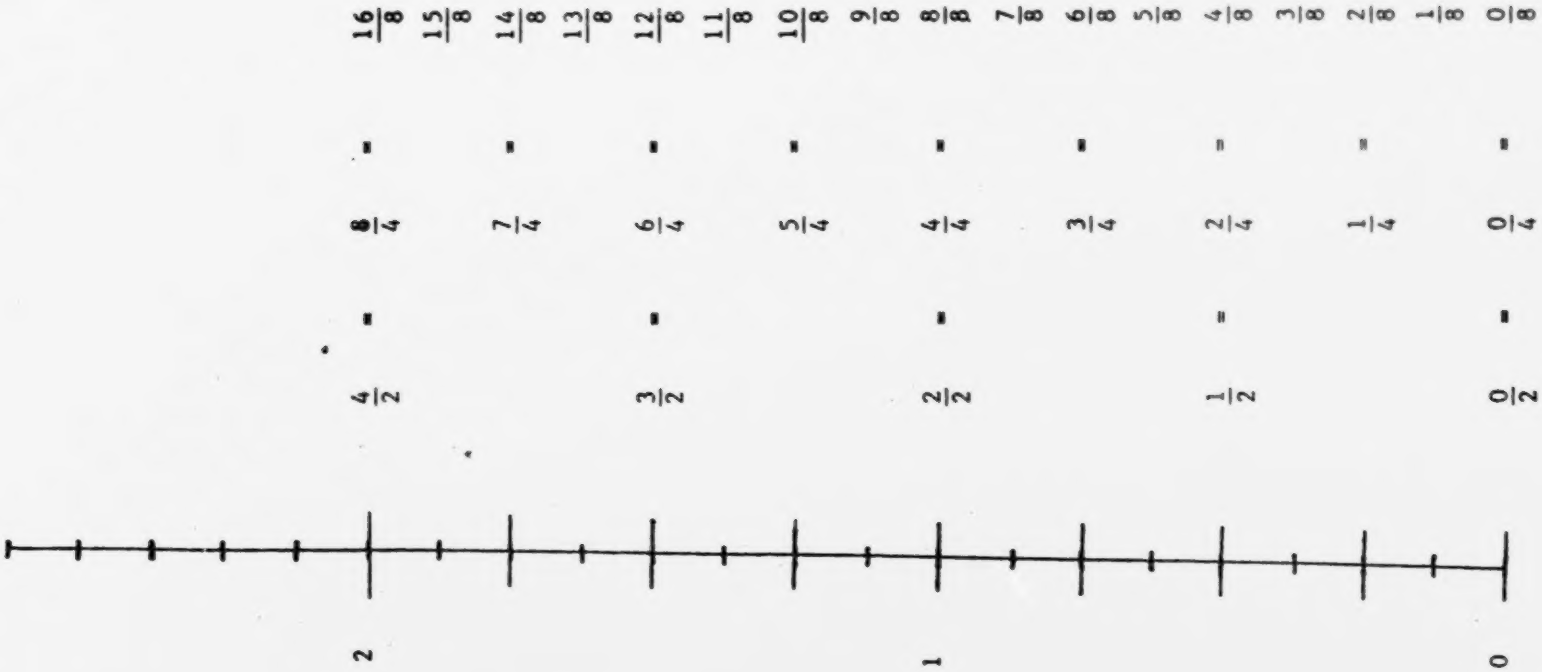
B. 2/4 , 3/4

C. 2/6 , 1/6

D. 1/6 , 1/4

FT-24

Fractions On A Number Line

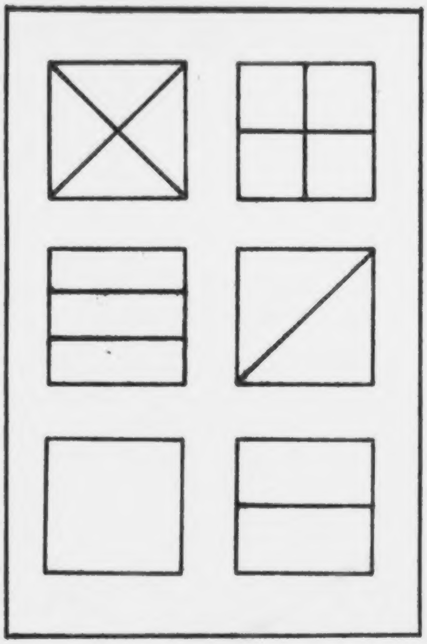

0/8 = 0/4 = 0/2
1/8
2/8 = 1/4
3/8
4/8 = 2/4 = 1/2
5/8
6/8 = 3/4
7/8
8/8 = 4/4 = 2/2
9/8
10/8 = 5/4
11/8
12/8 = 6/4 = 3/2
13/8
14/8 = 7/4
15/8
16/8 = 8/4 = 4/2
0
1
2

Dotpaper FT-25

Centimeter Square Grid

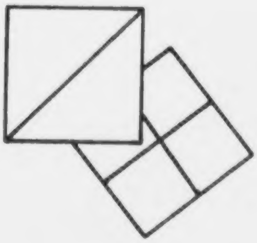
FT-26

FA-1



PATTERN MATCH

USE THE FRACTION BOARD AND SET OF FRACTION PIECES. MATCH THE FRACTION PIECES TO THE PATTERN ON YOUR FRACTION BOARD.



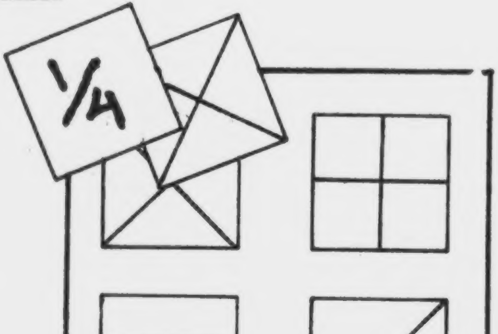
CAN YOU NAME THE FRACTIONAL NUMBER FOR EACH PART?

FRACTION BOARD

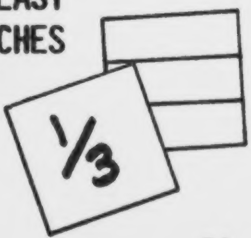
FA-1

FRACTION MATCH

USE YOUR FRACTION BOARD, SET OF FRACTION CARDS AND SET OF FRACTION PIECES. DRAW A CARD AND THEN SELECT THE FRACTION PIECE THAT GOES WITH THE CARD. PLACE THAT PIECE ON THE CORRECT PATTERN ON YOUR FRACTION BOARD.



BE SURE THAT THE LAST FRACTION CARD MATCHES THE LAST FRACTION PIECE.



PATTERN MATCH

OBJECTIVE: CHILDREN WILL CORRECTLY MATCH FRACTION PIECES TO PATTERNS ON FRACTION BOARD.

MATERIALS: FRACTION BOARD (F-1) 9" X 12" TAGBOARD, FRACTION PIECES (DUPLICATES OF PATTERNS ON F-1, CUT INTO CARDS AND MOUNTED ON COLORED PAPER.)

NOTE: PATTERNS ON THE FRACTION BOARDS MAY OCCUR IN DIFFERENT PLACES ON EACH BOARD.

FRACTION MATCH

OBJECTIVE: CHILDREN WILL CORRECTLY MATCH FRACTION PIECES TO FRACTION CARDS AND PATTERN ON FRACTION BOARD.

MATERIALS: FRACTION BOARD (F-1) 9" X 12" TAGBOARD, FRACTION PIECES (DUPLICATES OF PATTERNS ON F-1, CUT INTO CARDS AND MOUNTED ON COLORED PAPER.) FRACTION CARDS (SAME SIZE CARDS AS USED FOR FRACTION PIECES BUT INSTEAD OF PATTERNS OF FRACTIONAL PARTS THEY HAVE THE NUMERAL WRITTEN.) EACH SET OF FRACTION CARDS SHOULD CONSIST OF 2 EACH, 1/2 AND 1/4, 1 EACH, 1 AND 1/3.

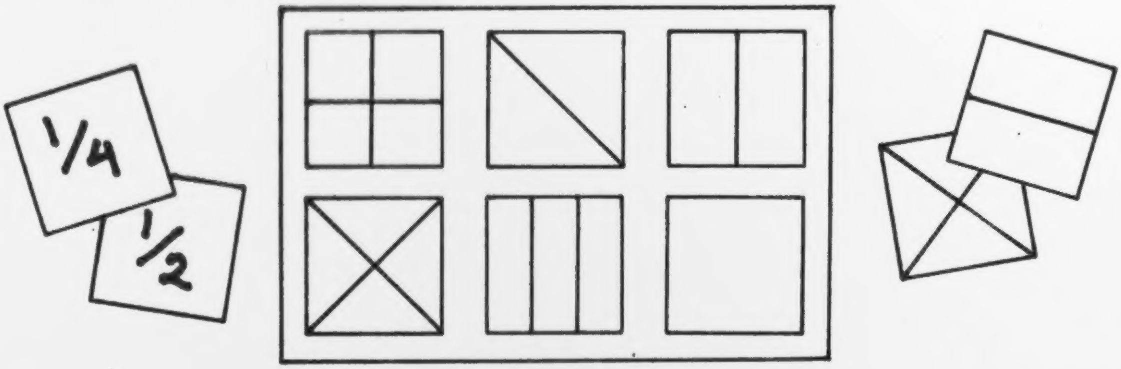
NOTE: PATTERNS ON THE FRACTION BOARDS MAY OCCUR IN DIFFERENT PLACES ON EACH BOARD.

(MEANING)

FRACTION BOARD GAME #1

(2-3 PLAYERS) EACH PLAYER HAS A FRACTION BOARD AND FRACTION PIECES. USE THE FRACTION CARDS, TOO.

SHUFFLE THE FRACTION CARDS AND TURN THEM FACE DOWN. EACH PLAYER, IN TURN, DRAWS A CARD FROM THE PILE AND MATCHES A FRACTION PIECE ON HIS BOARD TO THE CARD. AS THE CARDS ARE USED UP, THE STACK IS RESHUFFLED. THE WINNER IS THE FIRST PERSON TO COVER HIS/HER BOARD.

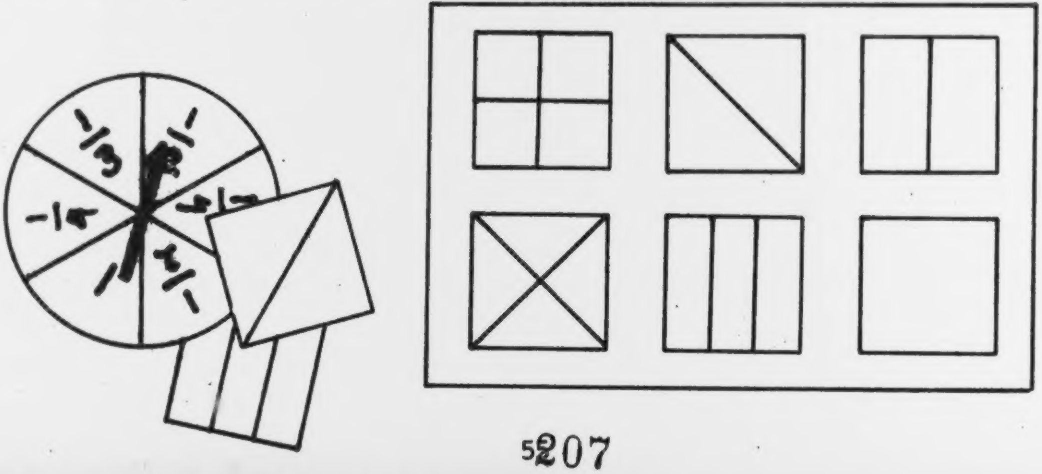


FA-1

FRACTION BOARD GAME #2

YOU WILL NEED A FRACTION BOARD AND FRACTION PIECES FOR EACH PLAYER. EACH PLAYER, IN TURN, SPINS THE FRACTION SPINNER TO DETERMINE WHAT PIECE TO USE AND MATCHES IT WITH THE CORRESPONDING PATTERN ON THE BOARD.

THE WINNER IS THE FIRST PLAYER TO COVER HIS/HER BOARD.



FRACTION BOARD GAME #1

Objective: Children will correctly match FRACTION PIECES to FRACTION CARDS and pattern on FRACTION BOARD.

Materials: FRACTION BOARD (F-1) 9" X 12" tagboard, FRACTION PIECES (duplicates of patterns on F-1, cut into cards and mounted on colored paper.) FRACTION CARDS (same size cards as used for FRACTION PIECES but instead of patterns of fractional parts they have the numeral written) 2 each, 1/2 and 1/4, 1 each, 1 and 1/3.

FRACTION BOARD GAME #2

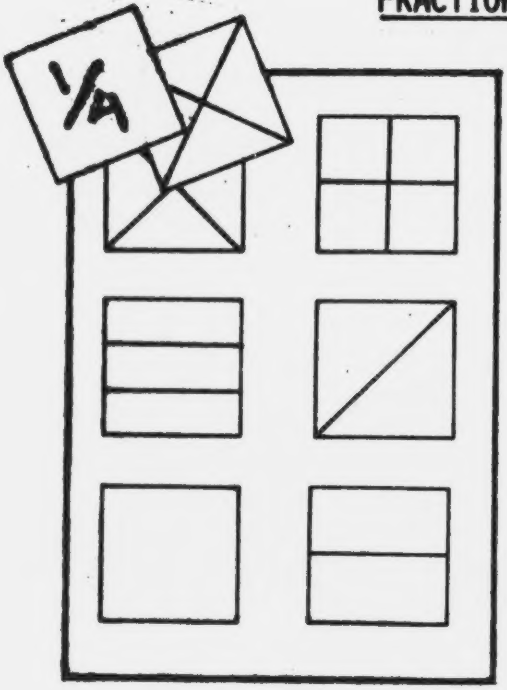
Objective: Children will correctly match FRACTION PIECES to fractions indicated on spinner.

Materials: FRACTION BOARDS (F-1) 9" X 12" tagboard, FRACTION PIECES (duplicates of patterns on F-1, cut into cards and mounted on colored paper.) and fraction spinner.

NOTE: Spinner may be made form tagboard or plastic margerine tub lid. It should be marked to indicate 2 each, 1/2 and 1/4, 1 each, 1 and 1/3. A popsicle stick or tagboard strip may be used for the pointer.

FA-2

FRACTION BOARD GAME #3



(AS MANY PLAYERS AS BOARDS)

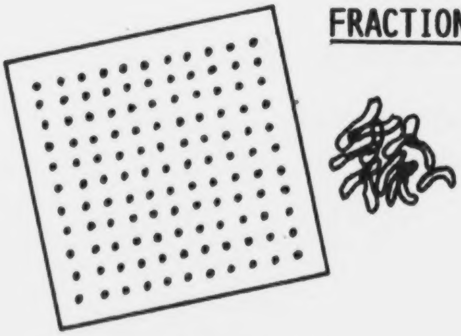
EACH PLAYER STARTS BY COVERING HIS/HER BOARD WITH THE FRACTION PIECES. THEN AS THE FRACTION CARDS ARE DRAWN (OR SPINNER SPUN) THE PLAYERS REMOVE THOSE PIECES FROM THEIR BOARDS.

THE FIRST PLAYER TO GET AN EMPTY BOARD IS THE **WINNER!**

REMEMBER - THE FRACTION ON THE **CARD** REPRESENTS EACH FRACTIONAL PART.

FA-2

FRACTIONS ON A GEOBOARD #1



USING YOUR RUBBER BANDS, DIVIDE THE GEOBOARD INTO:

A. 2 EQUAL PARTS
B. 4 EQUAL PARTS
C. 8 EQUAL PARTS
D. 16 EQUAL PARTS

WRITE THE FRACTION THAT TELLS WHAT EACH PART WOULD REPRESENT IN:

A._____, B._____, C._____, D._____

YOU MAY WISH TO DO THIS ACTIVITY ON THE GEOBOARD ACTIVITY WORKSHEET (F-4) OR THE LAMINATED DOT PAPER. USE PENCIL OR CRAYON TO MARK THE EQUAL PARTS.

FRACTION BOARD GAME #3

OBJECTIVE: CHILDREN WILL CORRECTLY IDENTIFY FRACTIONAL PART REPRESENTED BY EACH PATTERN ON THE FRACTION BOARD (F-1).

MATERIALS: FRACTION BOARDS (F-1) 9" X 12" TAGBOARD AND A SET EACH OF FRACTION PIECES (DUPLICATES OF PATTERNS ON F-1, CUT INTO CARDS AND MOUNTED ON COLORED PAPER.) AND FRACTION CARDS (SAME SIZE CARDS AS USED FOR FRACTION PIECES BUT INSTEAD OF PATTERNS OF FRACTIONAL PARTS THEY HAVE THE NUMERAL WRITTEN.) EACH SET OF FRACTION CARDS SHOULD CONSIST OF 2 EACH, 1/2 AND 1/4, 1 EACH, 2 AND 1/3.

NOTE: PATTERNS ON THE FRACTION BOARDS MAY OCCUR IN DIFFERENT PLACES ON EACH BOARD.

(MEANING)

FRACTIONS ON A GEOBOARD #1

OBJECTIVE: WHEN THE WHOLE AREA OF THE GEOBOARD IS "1" THE CHILDREN WILL DIVIDE THE AREA TO REPRESENT HALVES, FOURTHS AND EIGHTHS.

MATERIALS: GEOBOARDS (F-2, F-3) AND RUBBER BANDS OR USE LAMINATED DOT PAPER WITH GREASE PENCILS OR GEOBOARD ACTIVITY WORKSHEET (F-4).

NOTE: STUDENTS MAY USE GEOBOARDS FOR THIS ACTIVITY AND RECORD WHAT THEY HAVE DONE ON THE WORKSHEET (F-4) FOR TEACHER'S RECORDS.

(MEANING)

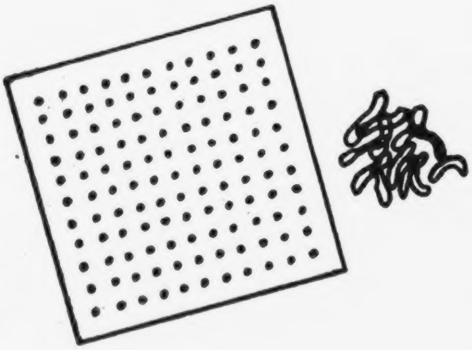
FA-2

FRACTIONS ON A GEOBOARD #2

SHOW 1/2 ON THE GEOBOARD (USING THE WHOLE GEOBOARD AS 1). ON THE OTHER SIDE, SHOW 4/8 (USING THE WHOLE GEOBOARD AS 1). WHAT DO YOU NOTICE ABOUT 1/2 AND 4/8?

SHOW 2/8. ON ANOTHER PART SHOW 1/4. WHAT DO YOU NOTICE ABOUT 2/8 AND 1/4?

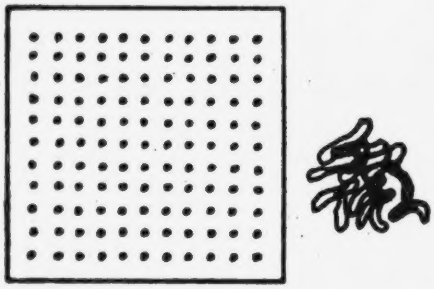
SHOW OTHER FRACTIONAL PARTS USING YOUR GEOBOARD. HOW MANY PARTS CAN YOU MAKE THAT ARE SIMILAR TO THE ONES ABOVE?



FA-2

FRACTIONS ON A GEOBOARD #3

SHOW 1/4 ON THE GEOBOARD (USING THE WHOLE GEOBOARD AS 1). ON ANOTHER PART, SHOW 3/8 (USING THE WHOLE GEOBOARD AS 1). WHICH FRACTIONAL PART IS LARGER?



WHICH FRACTION IS SMALLER 3/16 OR 1/4? PROVE YOUR ANSWER ON THE GEOBOARD.

MAKE OTHER FRACTIONAL PARTS ON THE GEOBOARD AND COMPARE. WHICH IS LARGER?

FRACTIONS ON A GEOBOARD #2

__OBJECTIVE:__ CHILDREN WILL IDENTIFY EQUIVALENT FRACTIONS AS REPRESENTED ON THE GEOBOARD.

__MATERIALS:__ GEOBOARDS (F-2, F-3) AND RUBBER BANDS __OR__ LAMINATED DOT PAPER AND GREASE PENCILS.

NOTE: STUDENTS MAY USE GEOBOARDS FOR THIS ACTIVITY AND RECORD WHAT THEY HAVE DONE ON THE WORKSHEET (F-4) FOR TEACHER'S RECORDS.

(EQUIVALENCE)

FRACTIONS ON A GEOBOARD #3

__OBJECTIVE:__ CHILDREN WILL COMPARE FRACTIONAL PARTS AND IDENTIFY THE PART THAT IS LARGER.

__MATERIAL:__ GEOBOARDS (F-2, F-3) AND RUBBER BANDS __OR__ USE LAMINATED DOT PAPER WITH GREASE PENCILS OR GEOBOARD ACTIVITY WORKSHEET.

NOTE: STUDENTS MAY USE THE GEOBOARD ACTIVITY WORKSHEET TO RECORD THE WORK DONE ON THE GEOBOARD.

(EQUIVALENCE)

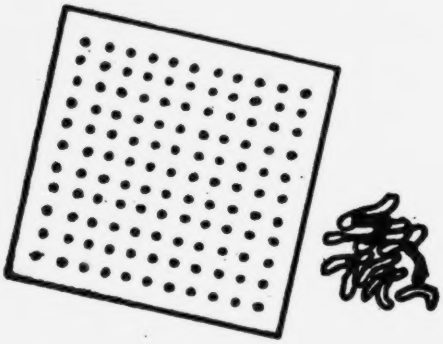
FA-3

FRACTIONS ON A GEOBOARD #4

REPRESENT 1/8 ON THE GEOBOARD. LEAVING THIS ON, REPRESENT 3/8 NEXT TO IT. (YOU HAVE SET UP 1/8 + 3/8.)

HOW MANY EIGHTHS DO YOU HAVE ALL TOGETHER?______

WHAT IS AN EQUIVALENT FRACTION TO THIS?______



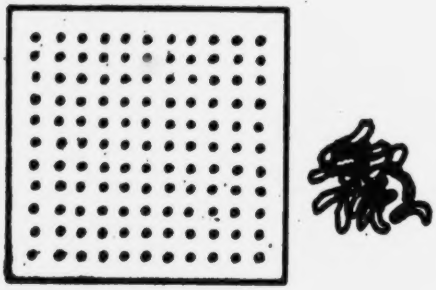
FA-3

FRACTIONS ON A GEOBOARD #5

REPRESENT 3/8 ON THE GEOBOARD. NOW "TAKE-AWAY" 1/4 FROM THE 3/8. WHAT FRACTIONAL PART IS LEFT?______

CAN YOU MAKE UP ANOTHER FRACTION "TAKE-AWAY" PROBLEM?

REPRESENT ______ ON THE GEOBOARD.



NOW, "TAKE-AWAY" _____ FROM YOUR FIRST FRACTION PART.

WHAT FRACTIONAL PART IS LEFT?_____

MAKE UP SOME MORE "TAKE-AWAY" FRACTION PROBLEMS. WRITE THEM ON PAPER IF YOU WISH.

FRACTIONS ON A GEOBOARD #4

OBJECTIVE: CHILDREN WILL REPRESENT THE ADDITION OF 2 PROPER FRACTIONS WITH LIKE DENOMINATORS ON THE GEOBOARD.

MATERIALS: GEOBOARD (F-2, F-3) AND RUBBER BANDS **OR** LAMINATED DOT PAPER (F-5 OR F-6) AND GREASE PENCIL.

GEOBOARD ACTIVITY WORKSHEET MAY BE USED FOR RECORDING WORK.

(ADDITION)

FRACTIONS ON A GEOBOARD #5

OBJECTIVE: CHILDREN WILL REPRESENT THE SUBTRACTION OF PROPER FRACTIONS ON THE GEOBOARD.

MATERIALS: GEOBOARD (F-2, F-3) AND RUBBER BANDS **OR** LAMINATED DOT PAPER (F-5 OR F-6) AND GREASE PENCIL.

GEOBOARD ACTIVITY WORKSHEET MAY BE USED FOR RECORDING WORK.

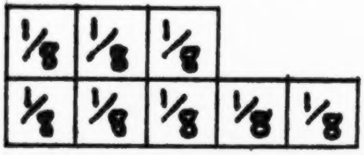
(SUBTRACTION)

FA-3

ORDERING FRACTION STRIPS

YOU WILL NEED A SET OF FRACTION STRIPS. SELECT A PAIR OF FRACTION STRIPS. DECIDE WHICH OF THE FRACTION STRIPS IS LONGER.

WRITE A NUMBER SENTENCE TO DESCRIBE THE RELATIONSHIP BETWEEN THE 2 STRIPS.

FRACTION PAIR	FRACTION STRIPS	SENTENCE
5/8, 3/8		5/8>3/8

SELECT ANOTHER PAIR OF FRACTION STRIPS. WRITE A NUMBER SENTENCE TO DESCRIBE THE 2 STRIPS.

CONTINUE THE PROCESS.

FA-3

ADDITION OF FRACTIONS

YOU WILL NEED A SET OF FRACTION STRIPS (F-10) AND FRACTION CHART A (F-8) TO HELP YOU ADD.
HERE'S HOW: LET'S FIND THE SUM OF 1/2 AND 1/3 --

1. USE YOUR FRACTION STRIPS FOR 1/2 AND 1/3.
2. START WITH THE TOP ROW ON CHART A. CHECK EACH ROW UNTIL YOU FIND ONE THAT CAN BE USED TO RENAME BOTH 1/2 AND 1/3.
3. LAY YOUR FRACTION STRIPS END TO END UNDER THIS ROW.
4. WRITE A NUMBER SENTENCE TO TELL WHAT YOU FIND ON SENTENCE: 1/2 + 1/3 = 3/6 + 2/6 = 5/6

NOW FIND THESE SUMS:
1/4 + 2/3 1/6 + 1/3 3/8 + 1/6 (ETC.)

TRY THESE:
3/4 + 3/8 5/12 + 2/3 5/6 + 1/2

ORDERING FRACTION STRIPS

Objective: Children will compare FRACTION STRIPS to order fractions.

Materials: A set of FRACTION STRIPS should be prepared according to the directions on F-7. Each student should be provided with an evelope containing a complete set of FRACTION STRIPS.

NOTE: Laminating will give FRACTION STRIPS a much longer "life."

(EQUIVALENCE)

ADDITION OF FRACTIONS

Objective: Children will add proper fractions with unlike denominators.

Materials Fraction CHART A (F-8). Envelope containing one set of FRACTION STRIPS (F-7).

NOTE: A demonstration of this activity will be helpful for most students.

(ADDITION)

FA-4

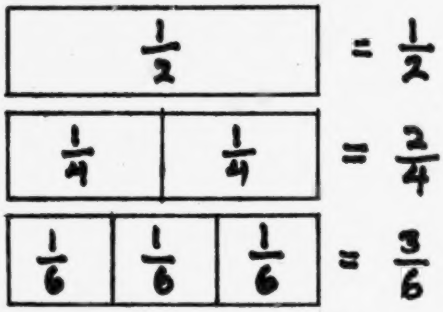
EQUIVALENT FRACTIONS ACTIVITY

YOU WILL NEED A SET OF FRACTION STRIPS. GET OUT THE FRACTION STRIP FOR 1/2. NOW MULTIPLY THE NUMERATOR AND THE DENOMINATOR OF THE FRACTION 1/2 BY 2.

WHAT FRACTION DID YOU GET?__________

GET OUT THE FRACTION STRIP FOR THIS FRACTION.

HOW DOES IT COMPARE TO THE STRIP FOR 1/2?



NOW MULTIPLY THE NUMERATOR AND DENOMINATOR OF 1/2 BY 3.

WHAT DID YOU GET?__________

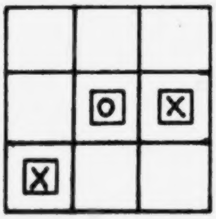
GET OUT THE FRACTION STRIP FOR THIS FRACTION. HOW DOES IT COMPARE TO THE OTHER 2 FRACTION STRIPS?

CAN YOU CARRY OUT THIS SAME ACTIVITY WITH OTHER FRACTION STRIPS?

FA-4

FRACTION TIC-TAC-TOE

(2 PLAYERS) YOU WILL NEED A TIC-TAC-TOE GAME BOARD AND A SET OF X AND O EXERCISE CARDS. ONE PLAYER USES ALL THE X CARDS, WITH THE EXERCISES FACE DOWN. THE OTHER PLAYER USES ALL THE O CARDS THE SAME WAY.



THE X PLAYER TURNS OVER THE TOP CARD, WORKS THE EXERCISE, AND CHECKS THE ANSWER. IF THE ANSWER IS CORRECT, THE PLAYER PUTS THE CARD, WITH THE X FACING UP, ON ANY SPACE OF THE GAME BOARD.

IF THE ANSWER IS INCORRECT, THE PLAYER PUTS THE CARD AT THE BOTTOM OF THE STACK.

THE O PLAYER DOES THE SAME.

IF MORE THAN ONE GAME IS PLAYED, THE X AND O PLAYERS SHOULD TAKE TURNS STARTING EACH GAME.

THE FIRST PLAYER TO GET THREE Xs OR THREE Os IN A ROW HORIZONTALLY, VERTICALLY, OR DIAGONALLY - **WINS.**

EQUIVALENT FRACTIONS ACTIVITY

OBJECTIVE: CHILDREN WILL DISCOVER A RULE FOR FINDING EQUIVALENT FRACTIONS:

$A/B = \frac{A \times C}{B \times C}$ FOR ALL A, B, & C, WHERE B & C ARE NOT 0.

MATERIALS: ENVELOPE CONTAINING ONE SET OF FRACTION STRIPS MADE ACCORDING TO DIRECTIONS ON F-7.

NOTE: ASK CHILDREN, "WHEN YOU MULTIPLY THE NUMERATOR AND THE DENOMINATOR OF A FRACTION BY THE SAME NUMBER, WHAT HAPPENS?" (INDICATE NON-ZERO WHOLE NUMBERS.)

(EQUIVALENCE)

FRACTION TIC-TAC-TOE

OBJECTIVE: STUDENTS WILL PRACTICE ADDITION OF FRACTIONS.

MATERIALS: TIC-TAC-TOE BOARD (LARGE ENOUGH TO ACCOMMODATE 3x5 INDEX CARDS) AND 12 INDEX CARDS, EACH WITH AN ADDITION OF FRACTIONS EXERCISE ON ONE SIDE. MARK AN X ON THE REVERSE SIDE OF 6 CARDS, AND AN O ON THE REVERSE SIDE OF THE OTHER 6 CARDS.

NOTE: VARIATIONS MAY BE PLAYED WITH SUBTRACTION OF FRACTION EXERCISES.

(ADDITION/DIVISION)

FA-4

ADD THE FRACTIONS

(2 PLAYERS) YOU WILL NEED A GAME BOARD, 16 ANSWER CARDS AND AN ANSWER SHEET. (INSTRUCTIONS FOR MAKING GAME BOARD ARE ON THE BACK.)

-PLACE THE ANSWER CARDS FACE DOWN ON THE PLAYING SURFACE. EACH PLAYER DRAWS FIVE CARDS.

-THE FIRST PLAYER FINDS AN ANSWER CARD THAT IS THE CORRECT SUM FOR ONE ADDITION ON THE BOARD AND PLACES THAT CARD IN THE APPROPRIATE SQUARE ON THE GAME BOARD.

-IF THE ANSWER CARD IS NOT IN THE CORRECT SPACE, THE OTHER PLAYER THEN LOSES A TURN.

-THE SECOND PLAYER MUST FIND AN ANSWER CARD IN HIS OR HER HAND THAT FILLS A SQUARE TOUCHING ONE SIDE OF THE FIRST ANSWER CARD ON THE BOARD.

-IF THE SECOND PLAYER DOES NOT HOLD A TOUCHING ANSWER CARD, HE/SHE MUST DRAW A CARD FROM THE STACK AND WAIT UNTIL THE NEXT TURN TO PLAY AGAIN.

-THE PLAYER WHO RUNS OUT OF CARDS IS THE WINNER.

FA-4

FRACTION SLIDE RULE ACTIVITY #1

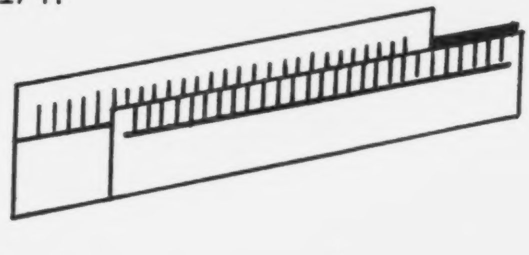
YOU WILL NEED A FRACTION SLIDE RULE (F-10) FOR THIS ACTIVITY.

LET'S USE THE SLIDE RULE TO ADD FRACTIONS.

CAN YOU FIND ANOTHER NAME FOR 2/12 + 3/12?
SLIDE THE 0 ON THE BOTTOM SCALE OVER UNTIL IT'S UNDER THE 2/12 ON THE TOP SCALE. NOW FIND THE 3/12 ON THE BOTTOM SCALE. THE SUM WILL APPEAR ON THE TOP SCALE, RIGHT ABOVE THE 3/12.

CAN YOU FIND THE SUM FOR 2/6 + 1/4?

HOW ABOUT 1/6 + 3/12?

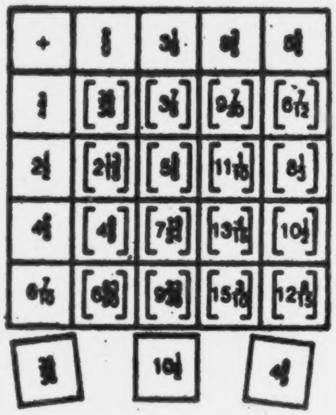


ADD THE FRACTIONS

OBJECTIVE: STUDENTS WILL PRACTICE ADDITION OF FRACTIONS.

MATERIALS: GAME BOARD, 16 ANSWER CARDS, ANSWER SHEET.

MAKE A GAME BOARD, SIMILAR TO THE ONE IN THE ILLUSTRATION, FROM A PIECE OF CARDBOARD APPROXIMATELY 20" SQUARE. YOU CAN MAKE AN ANSWER SHEET ON A PIECE OF PAPER. MAKE ANSWER CARDS TO FIT INTO THE SQUARES ON THE GAME BOARD. MAKE 1 ANSWER CARD FOR EACH OF THE 16 ADDITION EXERCISES ON THE GAME BOARD.



(ADDITION)

FRACTION SLIDE RULE ACTIVITY #1

OBJECTIVE: STUDENTS WILL ADD PROPER FRACTIONS AND VERIFY ANSWERS USING THE FRACTION SLIDE RULE (F-10).

MATERIALS: GLUE THE FRACTION SCALES (F-10) ON CONSTRUCTION PAPER TO SHOW ADDITION AND SUBTRACTION IDEAS.

NOTE: STUDENTS MAY WISH TO WRITE ADDITION SENTENCES OF THEIR OWN.

ALSO: FRACTION SLIDE RULE MAY BE USED TO "CHECK" FOR CORRECT ANSWERS ON WORK SHEET, TIC-TAC-TOE AND OTHER GAMES.

FRACTION SLIDE RULE ACTIVITY #2

YOU WILL NEED THE FRACTION SLIDE RULE (F-10) FOR THIS ACTIVITY.

LET'S SEE IF YOU CAN USE IT TO SUBTRACT FRACTIONS.

TO FIND THE ANSWER TO 9/12 - 5/12, WE LINE UP 9/12 ON THE TOP SCALE WITH 5/12 ON THE BOTTOM SCALE. THE ANSWER APPEARS RIGHT ABOVE THE 0 ON THE BOTTOM SCALE.

NOW: CAN YOU FIND 2/3 - 3/12?

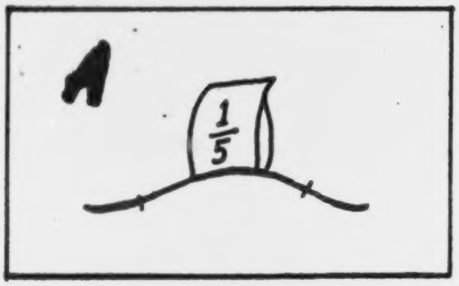
HOW ABOUT 3/4 - 5/12?

MAKE UP SOME SUBTRACTION SENTENCES OF YOUR OWN!

THE HIDDEN "ONE" #1

YOU WILL NEED A RECORD CARD AND ENVELOPES A, B, AND C.

EACH ENVELOPE HAS INSTRUCTIONS FOR YOU TO FOLLOW. WRITE YOUR ANSWER ON THE RECORD CARD.



(17)

FRACTION SLIDE RULE ACTIVITY #2

OBJECTIVE: STUDENTS WILL SUBTRACT PROPER FRACTIONS AND VERIFY ANSWERS USING THE FRACTION SLIDE RULE.

MATERIALS: GLUE THE FRACTION SCALES (F-10) ON CONSTRUCTION PAPER TO SHOW ADDITION AND SUBTRACTION IDEAS.

NOTE: ENCOURAGE STUDENTS TO WRITE APPROPRIATE SUBTRACTION SENTENCES FOR DEVELOPMENT OF SKILLS AND CONCEPTS.

> ASK: "WHAT CAN YOU TELL ME ABOUT ALL OF THE PAIRS THAT ARE LINED UP?"

(SUBTRACTION)

⑱

THE HIDDEN "ONE" #1

OBJECTIVE: STUDENTS WILL MATCH OBJECTS BASED ON THE RELATIONSHIP OF UNIT FRACTION TO THE WHOLE.

MATERIALS: HIDDEN "ONE" ENVELOPES A, B, AND C (F-12) PREAPRED ACCORDING TO INSTRUCTIONS.
RECORD CARDS (F-11) SHOULD ALSO BE PROVIDED FOR EACH STUDENT.

NOTE: THE FOCUS OF THIS ACTIVITY IS ON THE RELATION BETWEEN THE UNIT FRACTION AND THE UNIT OF REFERENCE. THE STUDENT IS TOLD: "HERE IS A PIECE OF WIRE THAT IS ONE-FOURTH AS LONG AS ONE OF THE PIECES OF STRING IN THIS ENVELOPE. FIND THAT PIECE OF STRING."

(MEANING)

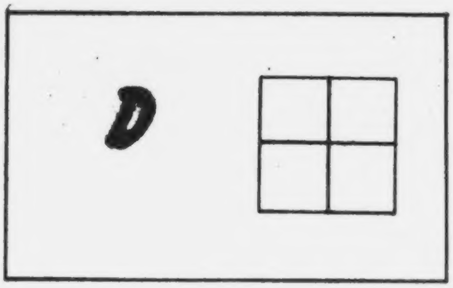
FA-5

THE HIDDEN "ONE" #2

ENVELOPES D, E, AND F MUST BE USED FOR THIS ACTIVITY. INSIDE THE EVELOPE ARE PIECES OF SQUARED PAPER.

YOUR JOB IS TO FOLLOW THE DIRECTIONS GIVEN ON THE ENVELOPE TO FIND THE HIDDEN "ONE."

RECORD YOUR ANSWER ON THE RECORD CARD IN THE SPACE PROVIDED.

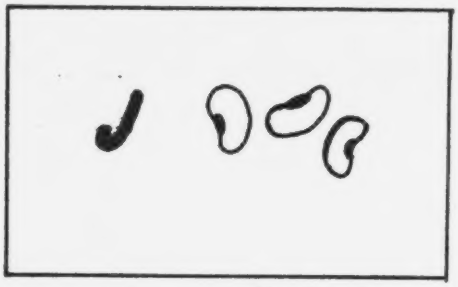


FA-5

THE HIDDEN "ONE" #3

ENVELOPES I AND J ARE NEEDED FOR THIS ACTIVITY. CAN YOU FIND THE HIDDEN "ONE" WHEN GROUPS OF OBJECTS ARE INVOLVED?

FOLLOW THE INSTRUCTIONS ON EACH ENVELOPE AND RECORD YOUR ANSWER ON THE RECORD CARD.



THE HIDDEN "ONE" #2

OBJECTIVE: STUDENTS WILL MATCH SQUARED PAPER BASED ON THE RELATIONSHIP OF UNIT FRACTION TO THE UNIT OF REFERENCE.

MATERIALS: HIDDEN "ONE" EVELOPES D, E, AND F, PREPARED ACCORDING TO INSTRUCTIONS (F-13). MATERIAL SHEET F-14 IS NEEDED TO COMPLETE THE ENVELOPES. STUDENTS MAY USE THE RECORD CARD (F-9) FOR ALL THREE HIDDEN "ONE" ACTIVITIES.

(MEANING)

THE HIDDEN "ONE" #3

OBJECTIVE: STUDENTS WILL MATCH GROUPS OF OBJECTS BASED ON THE RELATIONSHIP OF THE UNIT FRACTION TO THE UNIT OF REFERENCE.

MATERIALS: HIDDEN "ONE" ENVELOPES I AND J, PREPARED ACCORDING TO INSTRUCTIONS (F-15) AND RECORD CARDS FOR EACH STUDENT (F-11). THE SAME RECORD CARD IS USED FOR HIDDEN "ONE" ACTIVITIES #1, 2, AND 3.

NOTE: THE HIDDEN ONE ACTIVITIES ARE NOT NECESSARILY TO BE DONE IN ANY GIVEN SEQUENCE. IT IS ESSENTIAL TO THE SUCCESS OF THE EXPERIENCES THAT EACH STUDENT BECOME INVOLVED IN THE ACTIVITIES.

(MEANING)

FA-6

NAMING FRACTIONS WITH RODS

YOU WILL NEED A COLLECTION OF CUISENAIRE RODS TO ANSWER THE FOLLOWING QUESTIONS:

- IF THE RED ROD IS 1, WHAT IS THE WHITE ROD?
- IF THE YELLOW ROD IS 1, WHAT IS THE WHITE ROD?
- IF THE BROWN ROD IS 1, WHAT IS THE RED ROD?
- IF THE BLACK ROD IS 1, WHAT ROD IS 1/2?
- IF THE DARK GREEN ROD IS 2, WHAT ROD IS 1?
- IF THE DARK GREEN ROD IS 2, WHAT IS THE RED ROD?
- IF THE ORANGE ROD IS 1, WHAT IS THE BLUE ROD?

YOU MAY WISH TO WRITE THE ANSWER ON PAPER.

FA-6

NAMING FRACTIONS WITH "HAPPY FACE PIECES"

IF THE "Y" STRIP OF HAPPY FACE PIECES IS 1, WHAT IS:

IF "Y" IS 1, WHAT IS "W"?
IF "Y" IS 1, WHAT IS "P"?
IF "Y" IS 1, WHAT IS "Y" + "G"?
IF "Y" IS 1, WHAT IS "N"?
IF "Y" IS 1, WHAT IS "D" + "G"?
IF "Y" IS 1, WHAT IS "O" + "Y"?

YOU MAY WISH TO WRITE THE ANSWER ON PAPER.

NAMING FRACTIONS WITH RODS

Objective: Students will identify and name fractional relationships between Cuisenaire rods.

Materials: Set of Cuisenaire Rods.

(meaning)

NAMING FRACTIONS WITH "HAPPY FACE PIECES"

Objective: Students will identify and name fractional relationships between HAPPY FACE strips.

Materials: HAPPY FACE STRIPS - labeled to coordinate with the symbols used for each Cuisenaire rod (F-16).

HAPPY FACE PIECES - labeled and cut in strips of 1 to 10 faces in each strip. Each strip should be labeled:

1 = W
2 = R
3 = G
4 = P
5 = Y
6 = D
7 = K
8 = N
9 = E
10 = O

(meaning)

FA-6

ADDING FRACTIONS WITH RODS

LET'S USE THE CUISENAIRE RODS TO ADD FRACTIONS.

REMEMBER - WHEN ADDING LIKE FRACTIONS (THOSE WHOSE DENOMINATORS ARE THE SAME NUMBER), WE MUST FIND THE ROD THAT CAN BE DIVIDED INTO THE SAME NUMBER OF PARTS AS THE DENOMINATOR --

SO - WHEN ADDING 2/7 + 3/7
THE BLACK ROD "K" = 1
THE RED ROD "R" = 2/7
THE LIGHT GREEN ROD "G" = 3/7

SINCE THE BLACK ROD "K" IS 1, THEN THE TRAIN FORMED BY PUTTING R + G IS 5/7 UNITS LONG.

CAN YOU ADD OTHER FRACTIONS WITH THE RODS?

FA-6

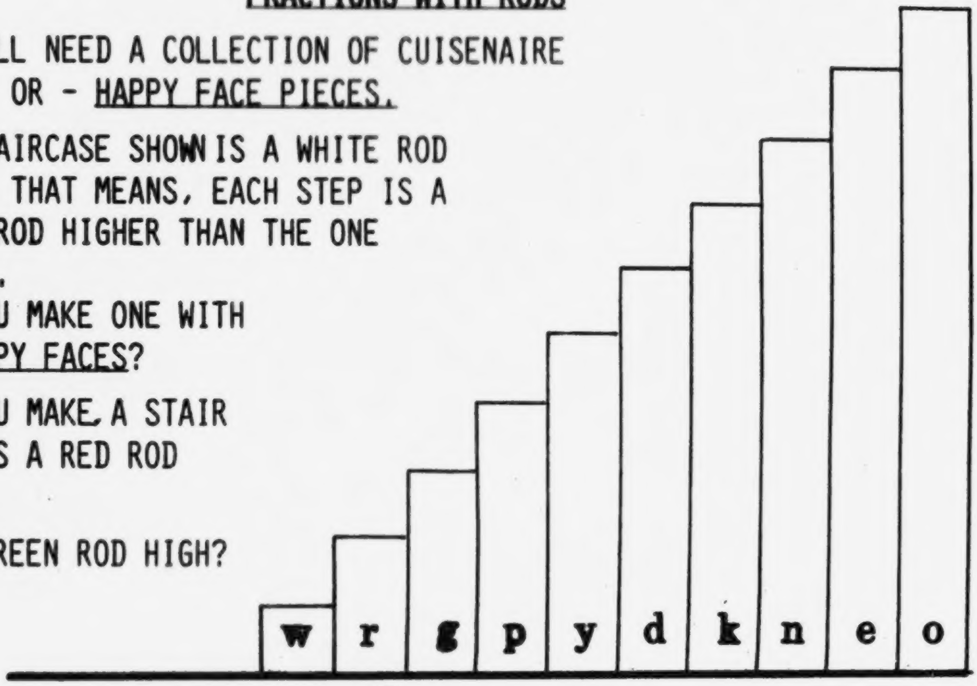
FRACTIONS WITH RODS

YOU WILL NEED A COLLECTION OF CUISENAIRE RODS - OR - HAPPY FACE PIECES.

THE STAIRCASE SHOWN IS A WHITE ROD HIGH. THAT MEANS, EACH STEP IS A WHITE ROD HIGHER THAN THE ONE BEFORE.
CAN YOU MAKE ONE WITH HAPPY FACES?

CAN YOU MAKE A STAIR THAT IS A RED ROD HIGH?

...A GREEN ROD HIGH?



ADDING FRACTIONS WITH RODS

OBJECTIVES: STUDENTS WILL SHOW CONCRETELY THE ADDITION OF 2 PROPER FRACTIONS WITH LIKE DENOMINATORS.

MATERIALS: SET OF CUISENAIRE RODS.

(ADDITION)

FRACTIONS WITH RODS

OBJECTIVE: STUDENTS WILL MANIPULATE RODS OF HAPPY FACE STRIPS AND DISCOVER RELATIONSHIPS AMONG THEM.

MATERIALS: SET OF CUISENAIRE RODS - OR - SET OF HAPPY FACE PIECES CUT AND LABELED ACCORDING TO THE CUISENAIRE SYMBOLS (F-16).

(RODS)

ADDING UNLIKE FRACTIONS WITH RODS

WHEN ADDING UNLIKE FRACTIONS, LIKE 1/2 + 1/3, FIRST TAKE A ROD SO THAT YOU CAN FIND 1/2 AND 1/3 OF IT. IN THIS CASE, THE FIRST ROD THAT WORKS IS THE DARK GREEN ROD. (THE RED ROD IS 1/3 OF IT AND THE LIGHT GREEN ROD IS 1/2 OF IT). FORM A TRAIN WITH THE RED AND THE LIGHT GREEN RODS. SINCE THE DARK GREEN REPRESENTS 1, THE TRAIN OF LIGHT GREEN + RED IS 5/6.

NOW:	ADD:	1/2 + 1/4	WHAT ROD TRAIN IS 1?
			WHAT ROD IS 1/2?
			WHAT ROD IS 1/4?
			FRACTIONAL ANSWER?
	ADD:	1/4 + 1/3	WHAT ROD TRAIN IS 1?
			WHAT ROD IS 1/4?
			WHAT ROD IS 1/3?
			FRACTIONAL ANSWER?

SUBTRACTING FRACTIONS WITH RODS

WHEN SUBTRACTING LIKE FRACTIONS, YOU MUST APPLY THE SAME IDEAS AS IN THE ADDITION OF LIKE FRACTIONS.
THIS TIME YOU "UNDO" THE PROCESS.

A. SUBTRACT 3/4 - 1/4	WHAT ROD TRAIN IS 1?
	WHAT ROD IS 1/4?
	WHAT ROD IS 3/4?
	FRACTIONAL ANSWER?
B. SUBTRACT 3/5 - 1/5	WHAT ROD TRAIN IS 1?
	WHAT ROD IS 3/5?
	WHAT ROD IS 1/5?
	FRACTIONAL ANSWER?

FA-7

SUBTRACTING UNLIKE FRACTIONS WITH RODS

A. SUBTRACT: 3/4 - 1/3
WHAT ROD TRAIN IS 1?
WHAT ROD IS 3/4?
WHAT ROD IS 1/4?
FRACTIONAL ANSWER?

B. SUBTRACT: 5/7 - 2/3
WHAT ROD TRAIN IS 1?
WHAT ROD IS 1/7?
WHAT ROD IS 5/7?
WHAT ROD IS 1/3?
WHAT ROD IS 2/3?
FRACTIONAL ANSWER?

CAN YOU MAKE UP OTHER SUBTRACTION SENTENCES TO SOLVE WITH THE RODS?

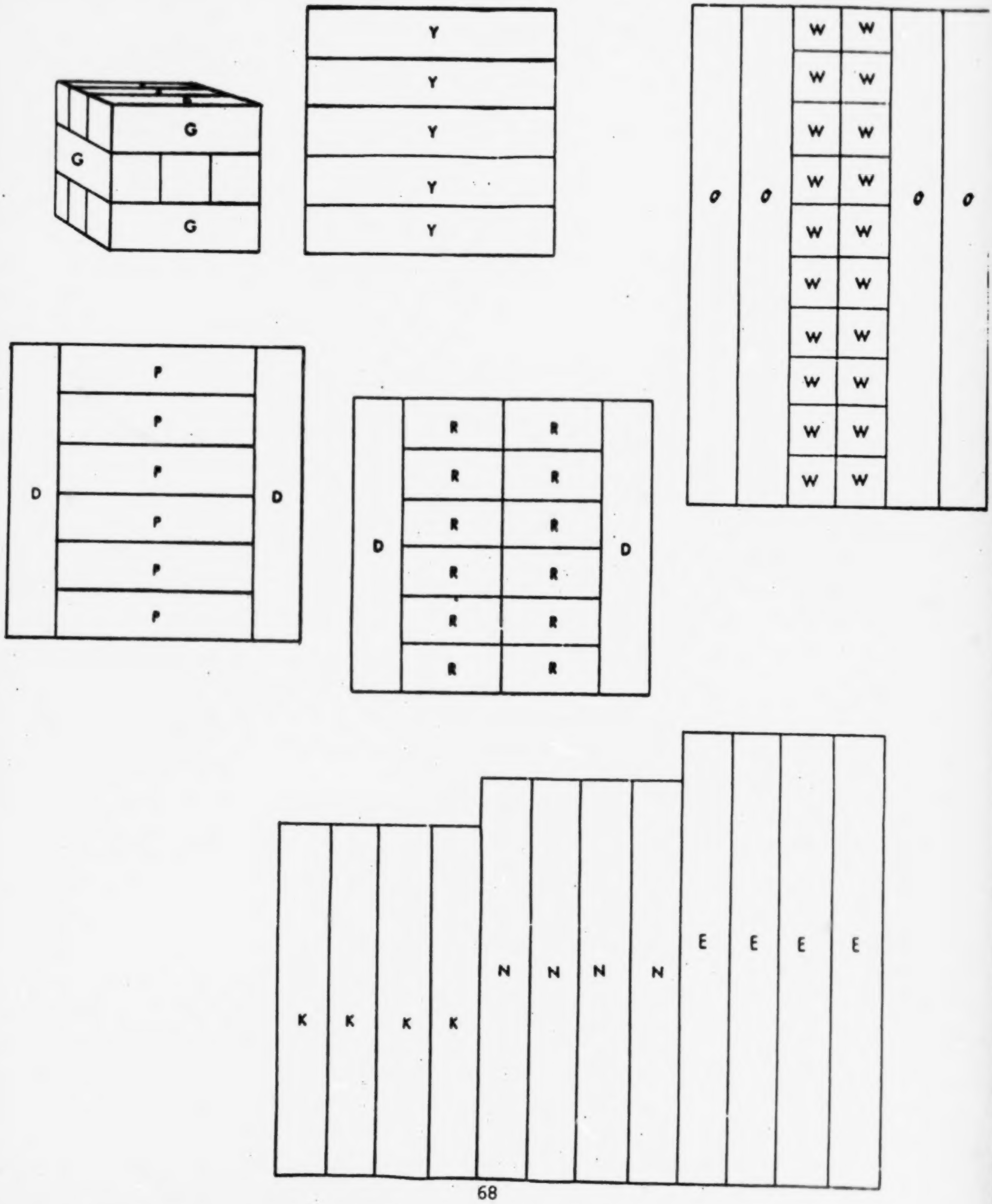
FA-7

MULTIPLYING TWO FRACTIONS WITH RODS

1/5 X 1/3 ASKS THE QUESTION: "WHAT IS ONE-FIFTH OF ONE-THIRD?" TO FIND THE ANSWER YOU CAN USE RODS. TRY THIS: FIND A ROD THAT CAN BE DIVIDED INTO FIFTHS AND THIRDS. DID YOU DECIDE ON THE ORANGE ROD PLUS THE YELLOW ROD? MAKE THE TRAIN O + Y. THE QUESTION IS: "WHAT ROD REPRESENTS 1/5 OF THIS YELLOW ROD?" ANSWER: A WHITE ROD. THE WHITE ROD IS THEN THE ROD THAT REPRESENTS 1/5 OF 1/3. THE REPRESENTATION FOR ONE WAS THE ORANGE PLUS THE YELLOW ROD. ONE WHITE ROD IS 1/15 OF THE TRAIN.

MULTIPLY: 2/3 X 1/4
WHAT ROD TRAIN IS 1?
WHAT ROD IS 1/4?
WHAT ROD IS 2/3 OF THE 1/4 ROD?
FRACTIONAL ANSWER?

MULTIPLY: 1/2 X 1/3
WHAT ROD TRAIN IS 1?
WHAT ROD IS 1/3?
WHAT ROD IS 1/2 OF THE 1/3 ROD?
FRACTIONAL ANSWER?


G
G
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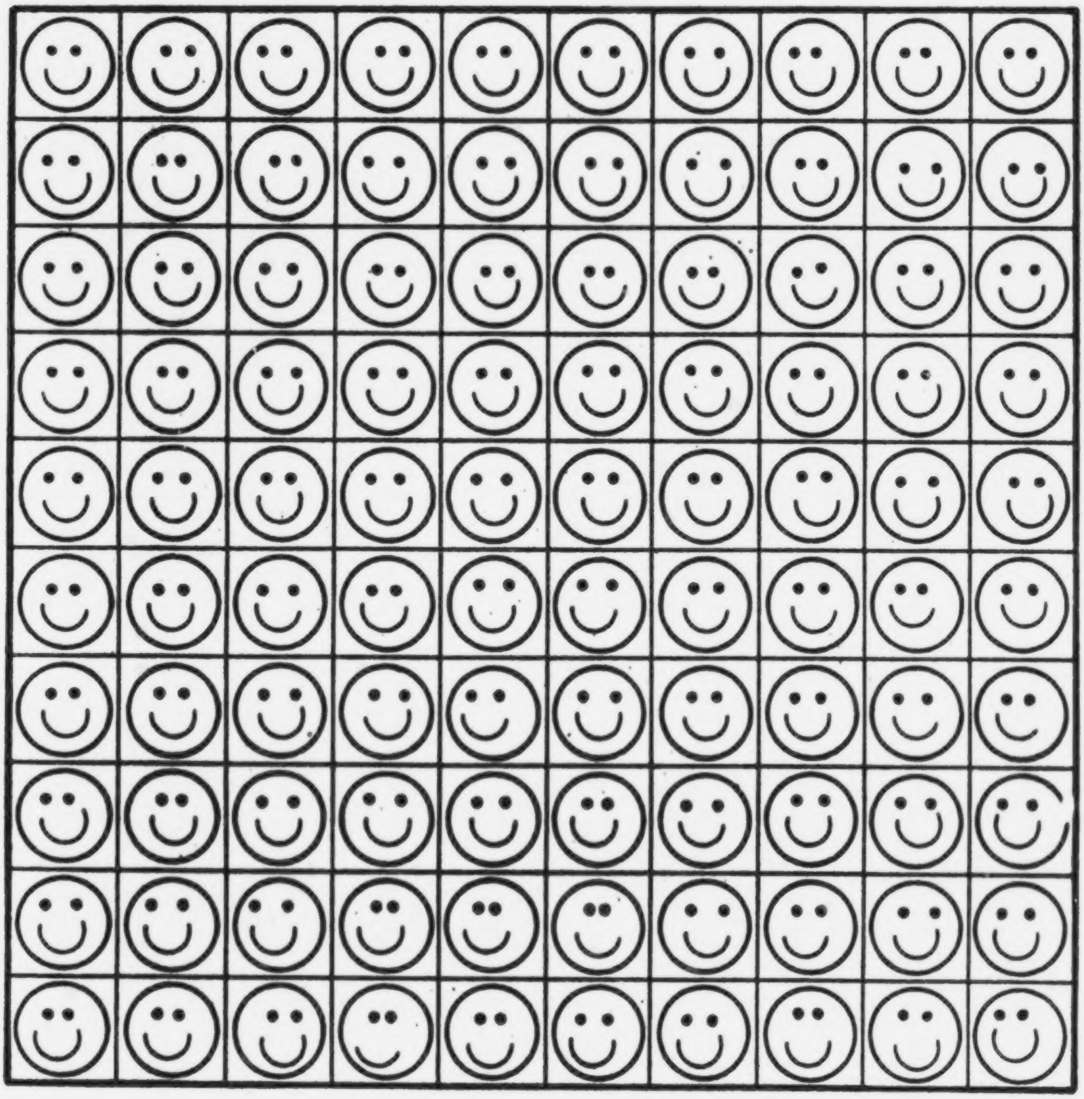
R R
R R
D
D
K K K K
N N N N
E E E E

Pattern for checking the 72 rods in each student's set.

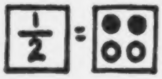
Happy Face Pieces

Reproduce as needed on heavy paper. Cut out hundreds, ten-strips, and singles.

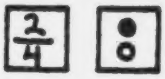


Play continues until someone uses all of his cards. The player with the most pairs is the winner. Note: Players may use a turn to swap a card they have already played for one in their hand that has no other match. Example:

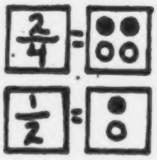
Already played:



Draws:



Swap cards to pair:



Fractions – Skill 1 – Learning Activities

FRACTIONAL PARTS PUZZLES

Using only one color, make several poster board circles, squares, rectangles, and hexagons. Divide one of each shape into halves, thirds, fourths, fifths, sixths and eighths wherever possible. Initially, you might store the shapes separately. Later when students are very familiar with these pieces, mix them all together.

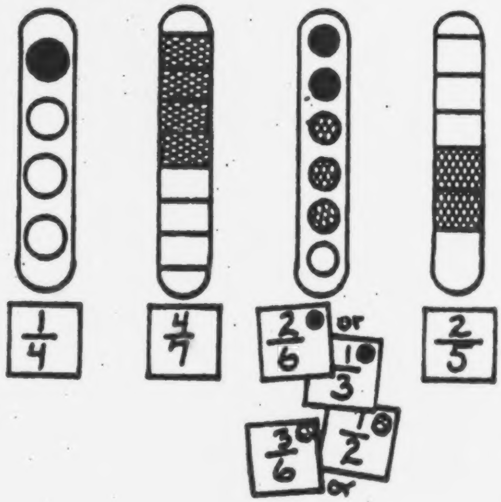
Students work in pairs and try to put together all the whole units. Then they place on each part the correct fractional name.

For very young children, the unit pieces can be color coded making it easier to put the puzzles together.

FRACTION STICKS

To help students grasp the concept of fractions, tongue depressors are used to represent various fractions. Use felt pen to color the sticks and draw the circles. You may wish to print all the possible answers on the back of each stick for self-correction. Write the fractions on cards.

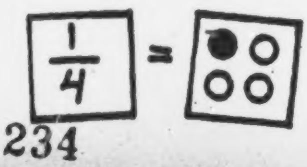
The teacher or another student may hold up a card with a fraction written on it. The student locates the stick that represents that fraction.



FRACTION GO FISH

Make two sets of fraction cards—picture the fractions on one set and write the fraction numbers on the other. See illustration below. We used these fractions: 1/2, 1/3, 1/4, 1/5, 3/4, 2/3, 2/5, 3/6, 3/8, 4/8, 1/8, 1/6, 2/4, 3/5, 5/8, 4/5, 2/6, 2/8, 6/8, 7/8, 4/6, 5/6. The object of the game is to match equivalent pairs. It can be played by two to four players.

Mix cards and deal five to each player. The game is played like "Fish" with the first player asking for a card he needs to make a pair. If he doesn't get one, he must fish for it. If a match can be made, the equivalent pair is laid on the table for all to see. Players draw from the "pond" to replace cards laid down.



Number of players: 2–4

Materials: Pattern block hexagons, trapezoids, blue parallelograms, triangles.
One die or a 1–6 spinner.

Rules:

1) Choose one player to be captain.

2) Give each player three hexagons.

3) Each player, in turn, rolls the die (or spins). The number rolled is the number of triangles that the player must pay to the captain. Each player must have the fewest number of blocks possible a' all times.

Example: A player rolls four, trades one hexagon for six triangles, pays the captain four triangles, and trades the remaining two triangles for one blue parallelogram.

4) The winner is the first player to get rid of all of his blocks. Winner must *go out* exactly. To win, player must have exact amount rolled on die.

8) Beth is cutting a board for a shelf. The board is $39\frac{11}{16}$" long. The shelf will be $35\frac{1}{2}$" long. How much should Beth cut off the board? _____ inches

9) Mimi needs $6\frac{3}{8}$ yards of ribbon for a craft project. She has $3\frac{3}{4}$ yards of ribbon at home How much more ribbon does she need? _____ yards

10) Jessica is $61\frac{7}{16}$" tall. William is $69\frac{1}{4}$" tall. How much taller is William than Jessica? _____ inches

11) The Chess Club raised some money at a pancake breakfast. They spent $\frac{3}{5}$ of the money to go to a chess tournament, and $\frac{1}{10}$ of the money to buy new equipment. What part of the money did they spend? _____ of the money

12) Mitchell is mailing two packages. One package weighs $9\frac{5}{8}$ pounds. The other package weighs $\frac{1}{2}$ pound. How much do the packages weigh together? _____ pounds

13) Terry had a piece of material that was 4 yards long. He used $3\frac{5}{8}$ yards of it to make some pillows. How much material was left over? _____ yards

14) The school bookstore sold $3\frac{1}{5}$ boxes of pencils last month. This month they sold $5\frac{1}{3}$ boxes. How many pencils were sold during the two months? _____ boxes

15) Mr. Cheng works $8\frac{3}{4}$ hours every day. So far he has worked $4\frac{2}{3}$ hours today. How many more hours does he have to work? _____ hours

16) A share of stock in The Math Company costs $12\frac{1}{8}$ dollars. A share of stock in The Reading Company costs $8\frac{7}{8}$ dollars. How much less is a share of Reading Company stock? _____ dollars

17) Yesterday, $\frac{1}{6}$ of our class was home sick with the flu. Also, half the class spent the day at the library. What part of the class was gone from the room? _____ of the class

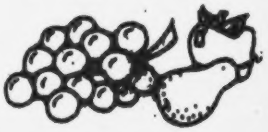
18) Carlos is baking cookies. He needs $2\frac{3}{4}$ cups of flour for the chocolate chip cookies and $3\frac{1}{2}$ cups of flour for the peanut butter cookies. How much flour does he need altogether? _____ cups

Sign on a Fruit Stand

Solve each problem. Find the answer in the Code Box. Write the corresponding letter in the blank(s) above the problem number. The first one is done for you.

Oranges are $\frac{\ }{16}\ \frac{\ }{14}\ \frac{E}{1}\ \frac{\ }{5}\ \frac{\ }{10}\quad \frac{\ }{12}\ \frac{\ }{15}\ \frac{\ }{2}\ \frac{\ }{11}\ \frac{\ }{7}\ \frac{\ }{4}\ \frac{\ }{9}\ \frac{\ }{14}\ \frac{\ }{11}\ \frac{\ }{3}$

but $\frac{\ }{18}\ \frac{\ }{17}\ \frac{\ }{8}\ \frac{\ }{13}\ \frac{E}{1}\ \frac{\ }{16}$ **have** $\frac{\ }{18}\ \frac{\ }{8}\ \frac{\ }{17}\ \frac{\ }{6}\ \frac{\ }{2}\ \frac{\ }{13}$.



Code Box

$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{7}{10}$	$1\frac{1}{4}$	$1\frac{4}{5}$	$2\frac{1}{8}$	$2\frac{5}{8}$	$2\frac{7}{10}$	$3\frac{1}{4}$
L	B	S	P	R	I	C	E	T	A	S

$3\frac{3}{10}$	$4\frac{1}{12}$	$4\frac{3}{16}$	$5\frac{1}{5}$	$5\frac{7}{8}$	$6\frac{1}{4}$	$7\frac{1}{2}$	$7\frac{13}{16}$	$8\frac{8}{15}$	$10\frac{1}{8}$	$12\frac{3}{4}$	$16\frac{5}{8}$
O	H	P	E	N	A	A	Y	E	C	W	D

1) Mrs. Morales needs $1\frac{1}{4}$ yards of material to make a skirt, and $\frac{7}{8}$ yard to make a blouse. How much material should she buy? $\underline{2\frac{1}{8}}$ yards

2) Jeff painted the fence around his yard. When he started he had about $3\frac{1}{2}$ cans of paint. Now he has about $\frac{4}{5}$ can left. How much paint did he use? About ______ cans.

3) Lisa grew $2\frac{3}{8}$" last year. Her sister grew $1\frac{3}{4}$". How much more did Lisa grow than her sister? ________ inches

4) Todd rides his bike $\frac{9}{10}$ mile to school. How far does he ride each day going to school and back home again? ________ miles

5) Yesterday, a share of stock in The Math Company cost $14\frac{3}{8}$ dollars. Today the price went up $2\frac{1}{4}$ dollars per share. What is the new price? ________ dollars

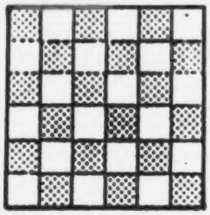
6) My car gas tank holds $16\frac{1}{2}$ gallons. It took $11\frac{3}{10}$ gallons to fill it today. How much gas was in the tank before I filled it? ________ gallons

7) Judie worked $3\frac{2}{3}$ hours on Saturday. She worked $3\frac{5}{6}$ hours on Sunday. How many hours did Judie work in all? ________ hours

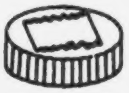
Greater Than, Less Than Game

Materials: pencil, paper, ruler, 6 red counters, 6 black counters, masking tape
Number of players: two

1. Copy this game board. Your counters should fit into the small squares



2. Put a piece of masking tape on each counter.



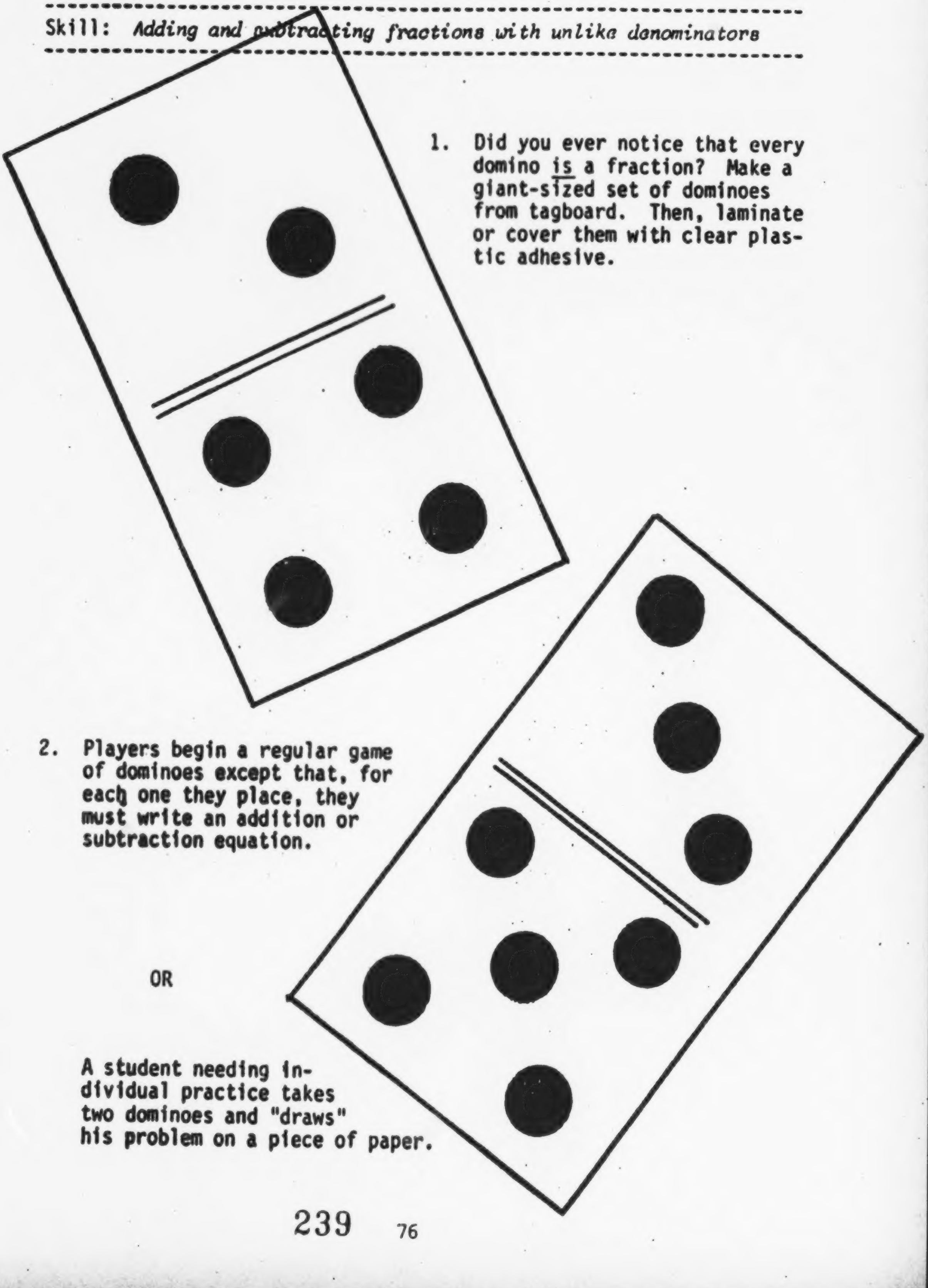
3. Write one of these fractions on the tape on each red counter. Then write one on each black counter.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{5}{6}$

4. Play this game like checkers, except:
   a. you can jump a counter only if your fraction is greater than or equal to the fraction on the other counter.
   b. when you get a king, turn the counter over. Kings can jump any counter. Kings can be jumped only by other kings.
5. The last player who is able to move a counter is the winner.

DOMIN-KNOWS!

Skill: *Adding and subtracting fractions with unlike denominators*



1. Did you ever notice that every domino is a fraction? Make a giant-sized set of dominoes from tagboard. Then, laminate or cover them with clear plastic adhesive.

2. Players begin a regular game of dominoes except that, for each one they place, they must write an addition or subtraction equation.

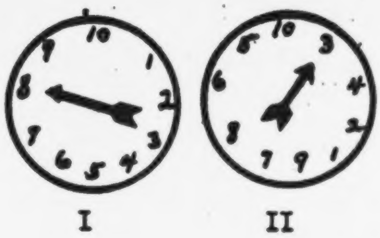
OR

A student needing individual practice takes two dominoes and "draws" his problem on a piece of paper.

Fraction Roulette*

Fraction roulette may be played either as a team game or as a table game for two or more players. Two spinners will be needed as shown, with arrows pointing to numerators and feather-ends pointing to denominators of fractions:

Spinner I shows the fraction $\frac{8}{3}$



Spinner II show the fraction $\frac{3}{7}$

One student spins on number I and a second student spins on number II (if in teams, student no. 1 will be on Team A, student no. 2 on Team B). The game can proceed in several ways from here:

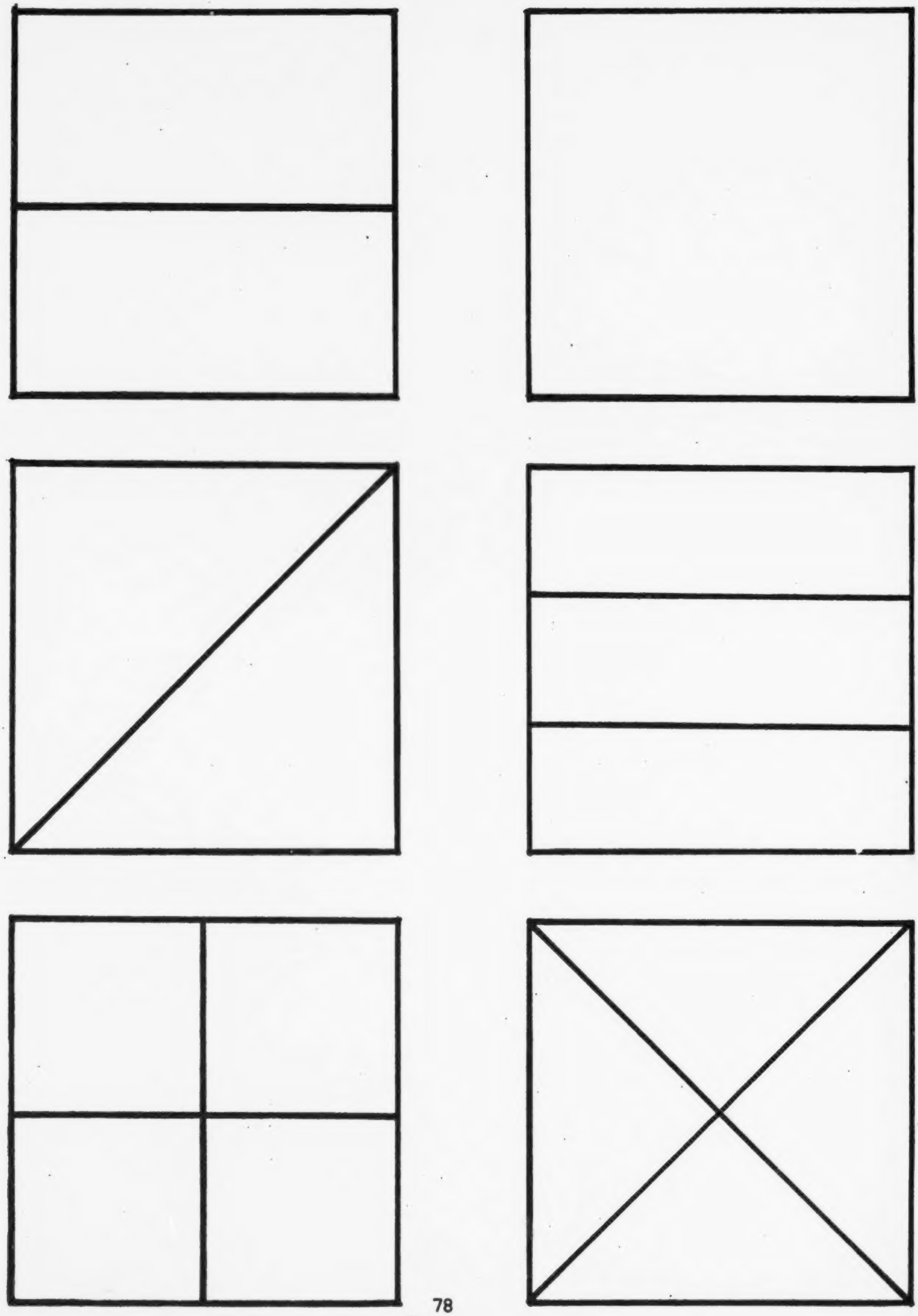
One team option: The first student to find the common denominator (or sum, difference, product, quotient, etc.) of the two fractions wins a point for his/her team.

Table game option: Student number 2 must work the problem (find the greatest fraction, the sum, or whatever the objective for the lesson covers). then the two students spin again and the first student must work the problem. The spinners are then passed to the next two students.

The following topics may be taught using these fraction spinners:

Identifying proper and improper fractions
Finding common denominators
Comparing fractions
Performing basic operations using fractions.

F-1



Making a Geo-Board

Material: (for 1 geo-board)

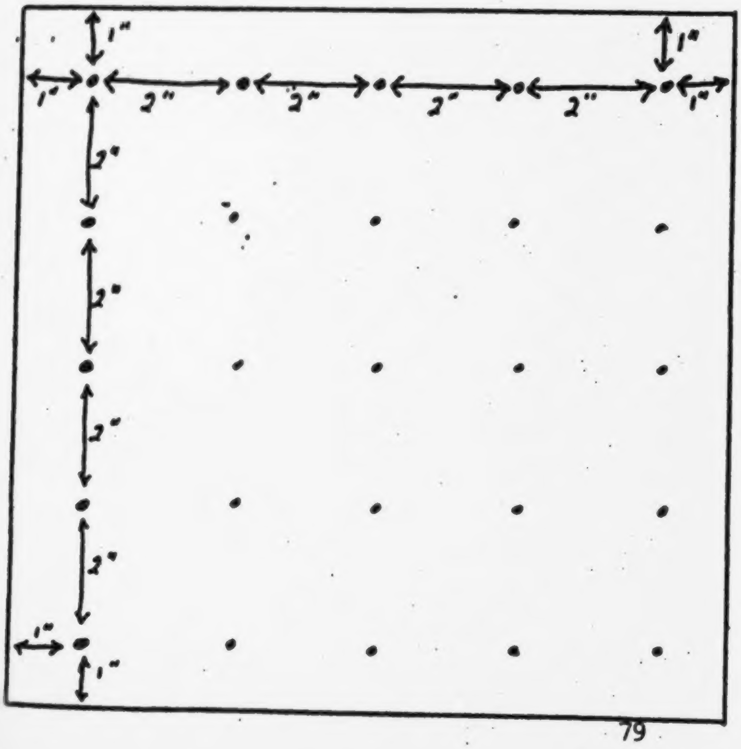
10" x 10" x 1/2" plywood

25 brass linoleum nails (5/8" long)

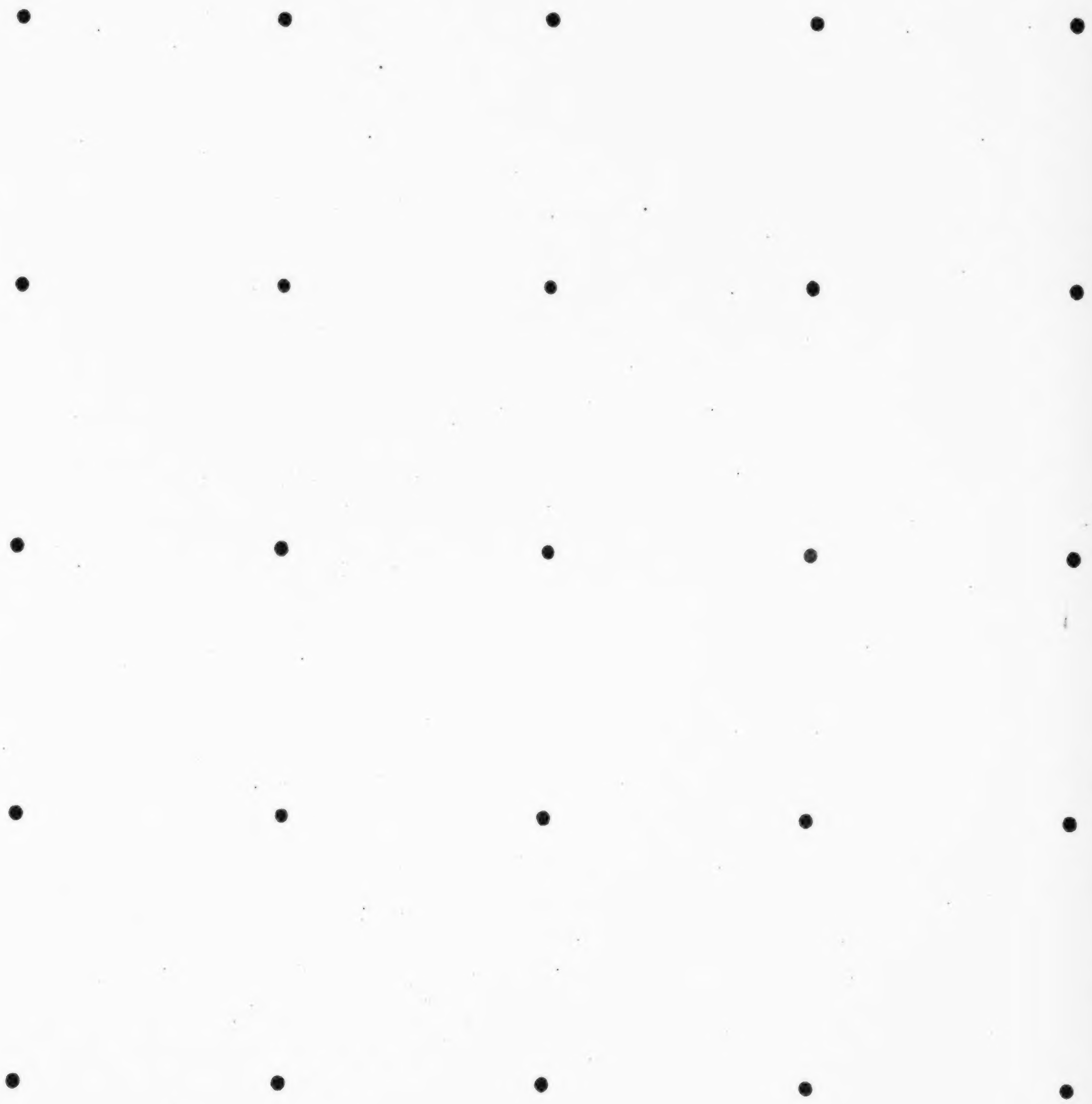
Sandpaper

Paint/varnish (optional)

1. Sand the plywood and paint it if desired.
2. Place the corner nails so that they are 1" from the corner on two sides.
3. The remaining nails are placed two inches from one another, vertically and horizontally.

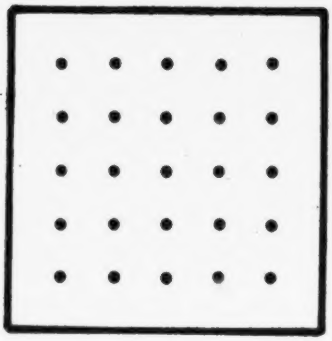


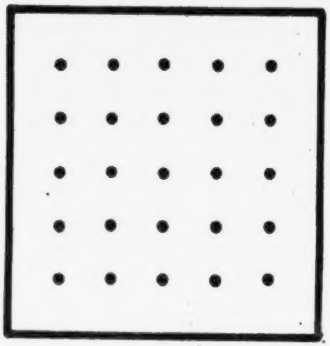
Template For Making A Geoboard

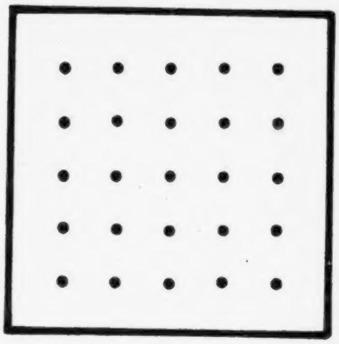


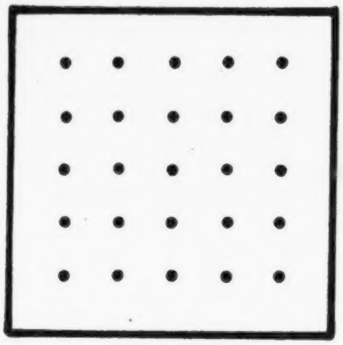
r-4

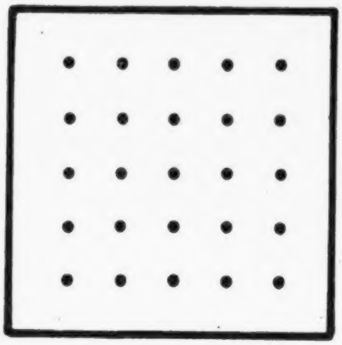
Geoboard Activity Worksheet

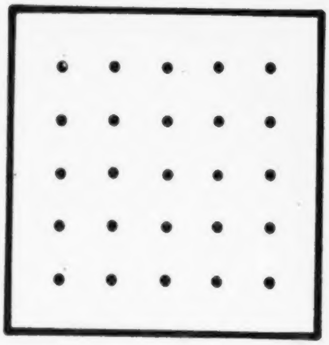


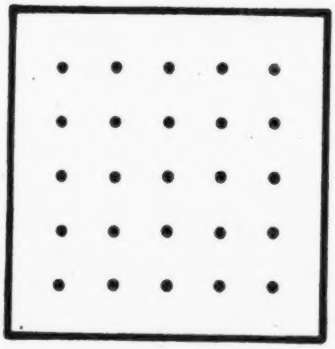


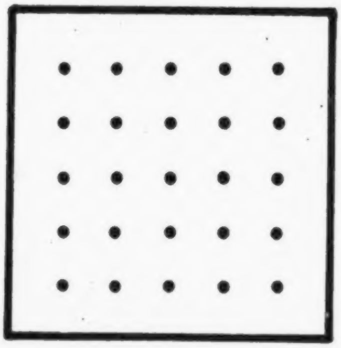


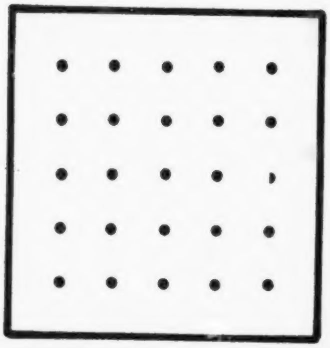


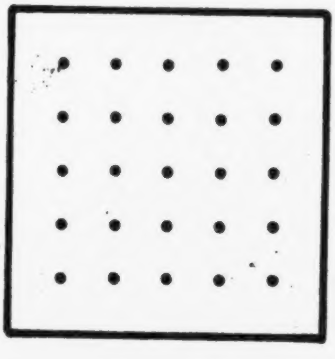


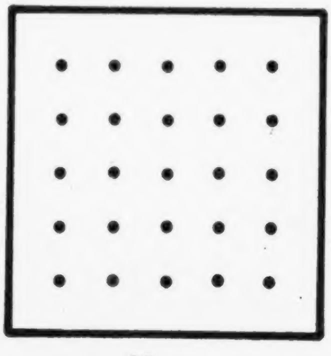


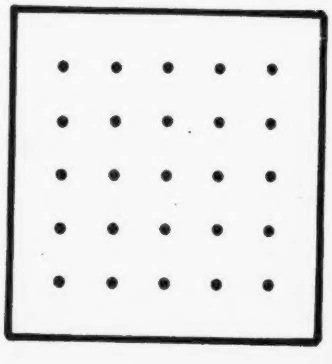








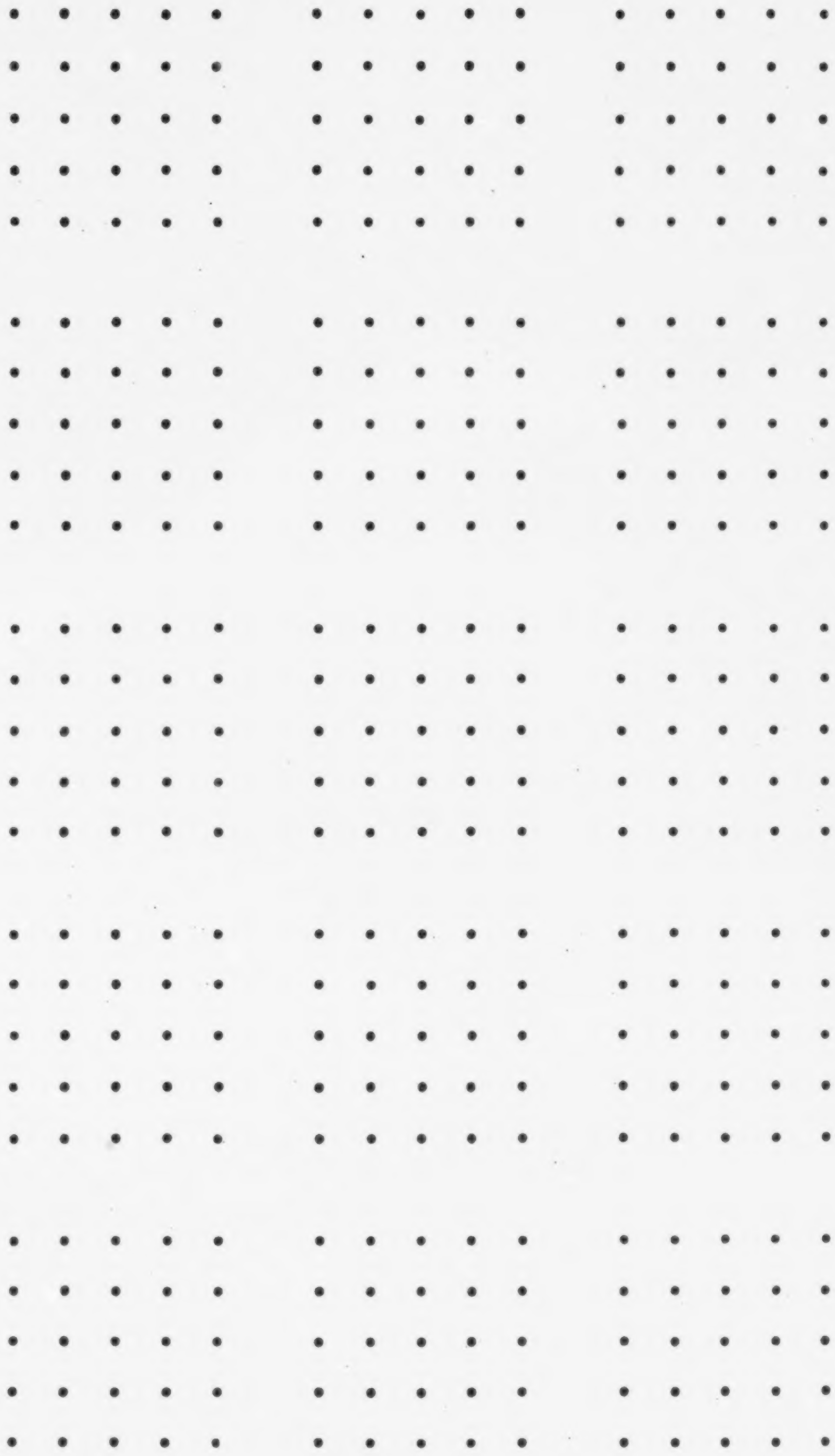




F-5

Dotpaper

F-6



FRACTION STRIPS

Instruction for making FRACTION STRIPS:

Paste 2 of FRACTION CHART A and 2 of FRACTION CHART B (F-8, F-9) on construction paper. Then cut out pieces for each of these fractions:

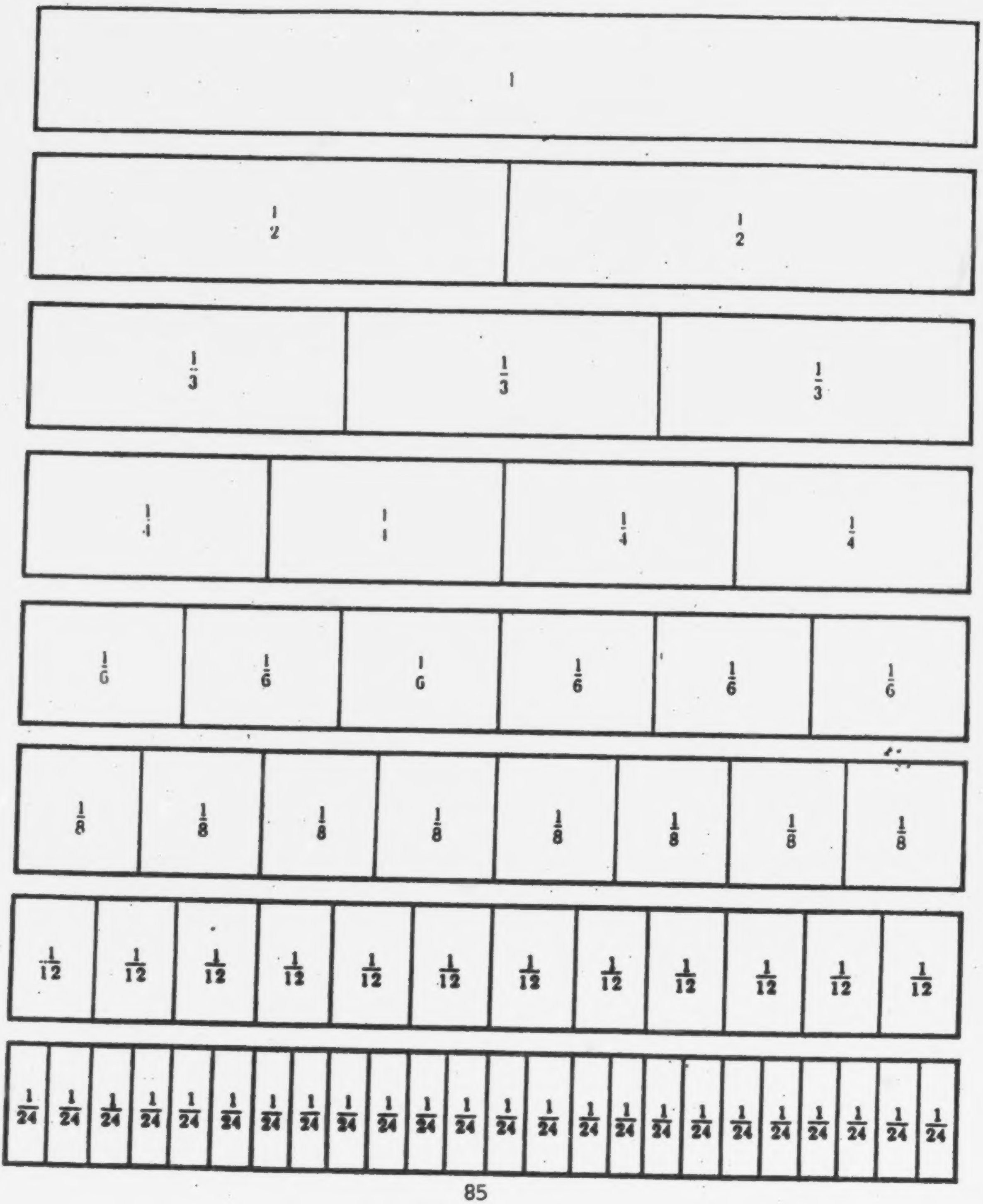
1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/10, 1/12,
3/8, 5/8, 2/3, 3/4, 2/5, 3/5, 4/5,
5/6, 7/8, 3/10, 7/10, 9/10, 5/12, 7/12, 11/12

Label each STRIP on the back with the fraction.

Keep FRACTION STRIPS in an envelope labeled

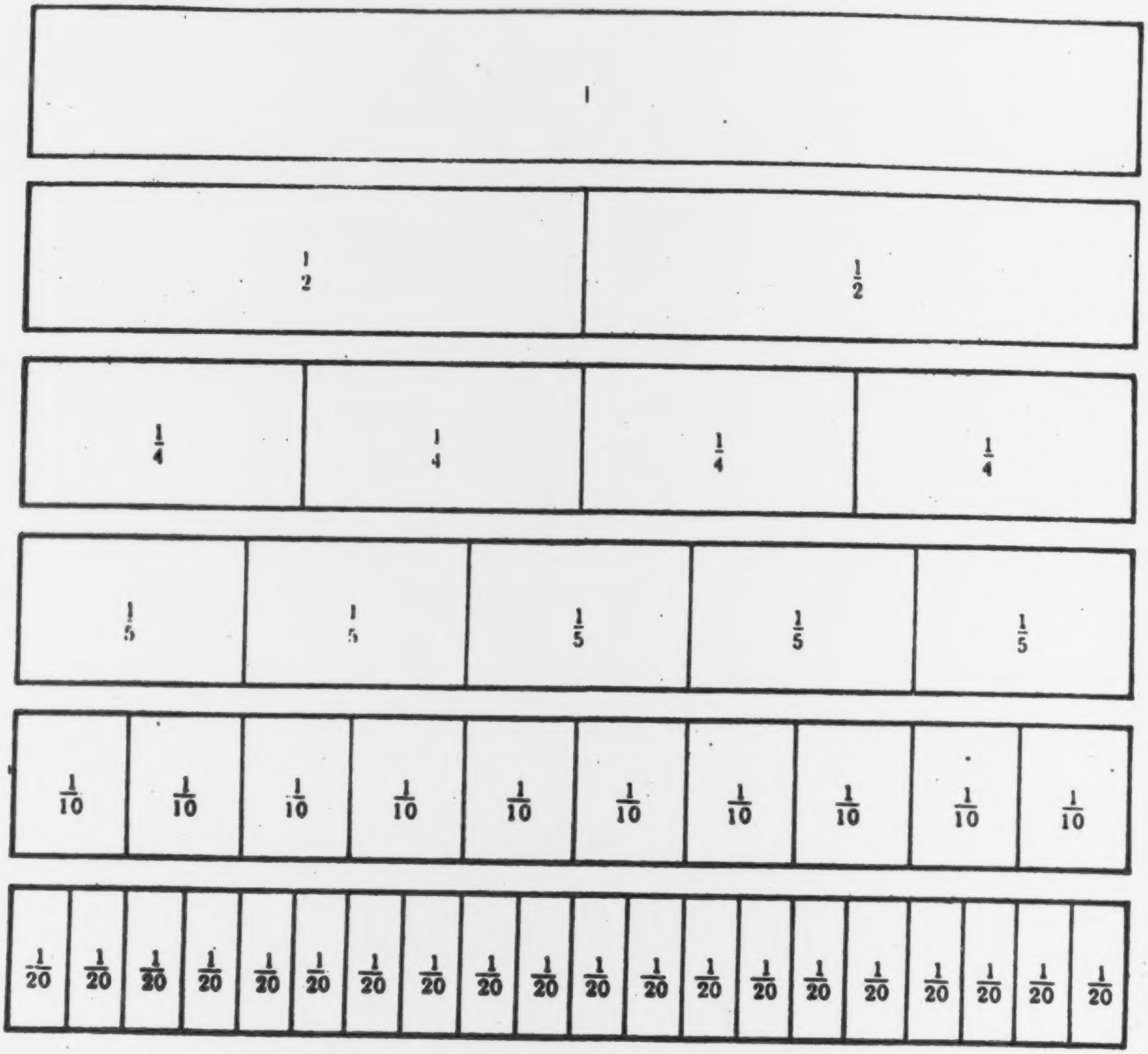
FRACTION STRIPS

Fraction Chart A


1
1/2 1/2
1/3 1/3 1/3
1/4 1/4 1/4 1/4
1/6 1/6 1/6 1/6 1/6 1/6
1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8
1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12
1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24 1/24

F-9

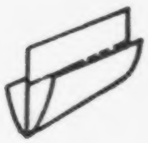
Fraction Chart B

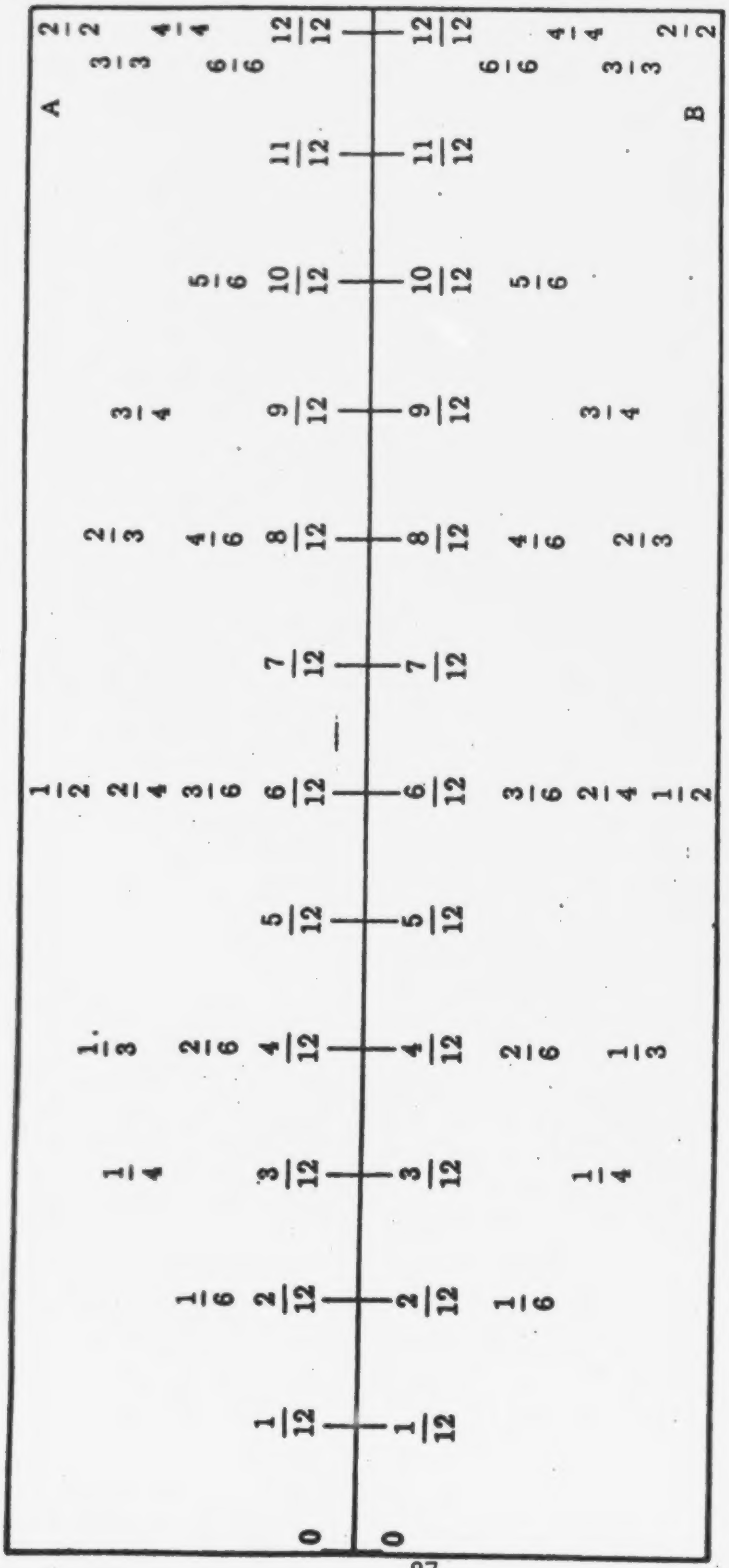

1
$\frac{1}{2}$ $\frac{1}{2}$
$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$
$\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$
$\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$ $\frac{1}{20}$

F-10

FRACTION SLIDE RULE AND DECIMAL SLIDE RULE

Glue the scales on construction paper to show addition and subtraction ideas.




A
B
0
1/12
2/12
3/12
4/12
5/12
6/12
7/12
8/12
9/12
10/12
11/12
12/12
1/6
2/6
3/6
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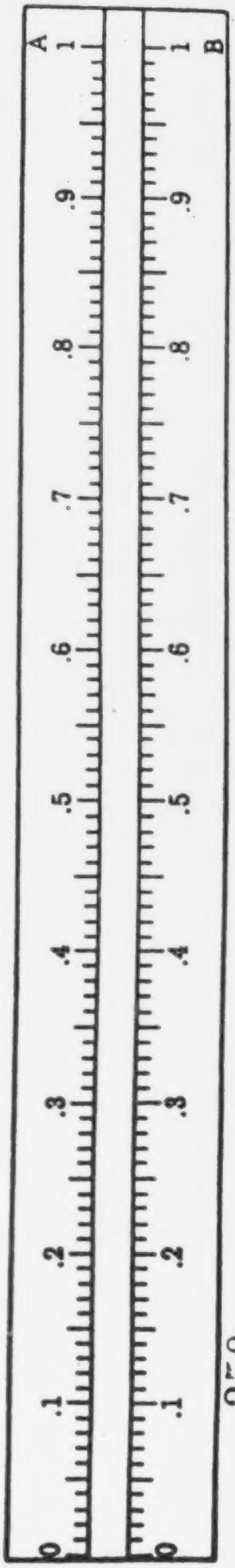
1/2
2/2


A
B
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1

F-11

RECORD CARD

Experience I

Name ______________________

1

Envelope A ______
Envelope B ______
Envelope C ______

2

Envelope D ______
Envelope E ______
Envelope F ______

3

______
______

Envelope I ______
Envelope J ______

RECORD CARD

Experience I

Name ______________________

1

Envelope A ______
Envelope B ______
Envelope C ______

2

Envelope D ______
Envelope E ______
Envelope F ______

3

______
______

Envelope I ______
Envelope J ______

RECORD CARD

Experience I

Name ______________________

1

Envelope A ______
Envelope B ______
Envelope C ______

2

Envelope D ______
Envelope E ______
Envelope F ______

3

______
______

Envelope I ______
Envelope J ______

A

This wire is $\frac{1}{5}$ as long as one of the four pieces of wire in this envelope.

Find that piece of wire.

Write its name on your record card.

B

This toothpick is $\frac{1}{3}$ as long as one of the four pieces of ribbon in this envelope.

Find that piece of ribbon.

Write its name on your record card.

C

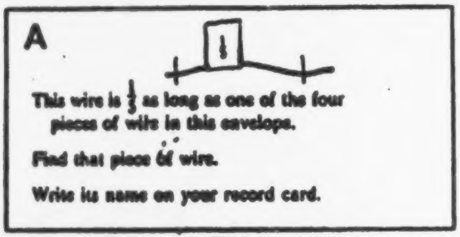
This piece of straw is $\frac{1}{4}$ as long as one of the four pieces of string in this envelope.

Find that piece of string.

Write its name on your record card.

Station L. Prepare Envelopes A, B, and C by gluing to them the labels reproduced from the Teaching Package.

The wire attached to Envelope A is 1½ inches long and is labeled ⅕ with masking tape, as shown in figure 2. There are four pieces of wire inside the envelope. They have lengths of 5 inches, 6 inches, 7½ inches, and 9 inches, and they are labeled A-1, A-2, A-3, and A-4, respectively.



This wire is $\frac{1}{5}$ as long as one of the four pieces of wire in this envelope.

Find that piece of wire.

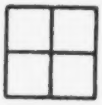
Write its name on your record card.

Fig. 2

The toothpick stapled to Envelope B is 2⅓ inches long and is labeled ⅓ with masking tape. Inside are four pieces of ribbon of lengths approximating 5, 6⅔, 9, and 10 inches and labeled B-1, B-2, B-3, and B-4, respectively.

The piece of straw glued to Envelope C is 3½ inches long and is labeled ¼. The pieces of string inside should be lengths approximating 12, 14, 16, and 17 inches and should be labeled C-1, C-2, C-3, and C-4, respectively.

D

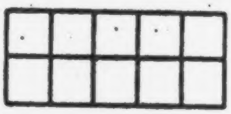


This piece of paper is $\frac{1}{4}$ as large as one of the five pieces of paper in this envelope.

Find that piece of paper.

Write its name on your record card.

E

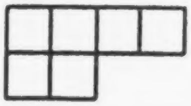


This piece of paper is $\frac{1}{3}$ as large as one of the five pieces of paper in this envelope.

Find that piece of paper.

Write its name on your record card.

F

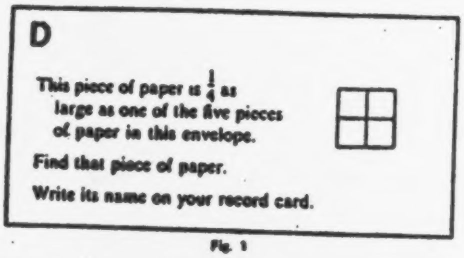


This piece of paper is $\frac{1}{4}$ as large as one of the four pieces of paper in this envelope.

Find that piece of paper.

Write its name on your record card.

Station 2. Envelopes D, E, and F must be prepared for this station. The labels for the envelopes and the pieces of squared paper which go inside the envelopes may be reproduced from the Teaching Package. Figures 3 and 4 show the materials given in the Teaching Package for Envelope D.



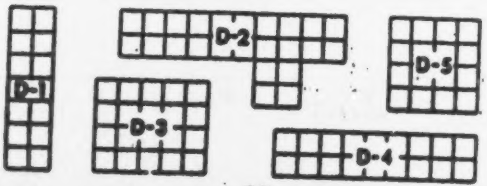
D

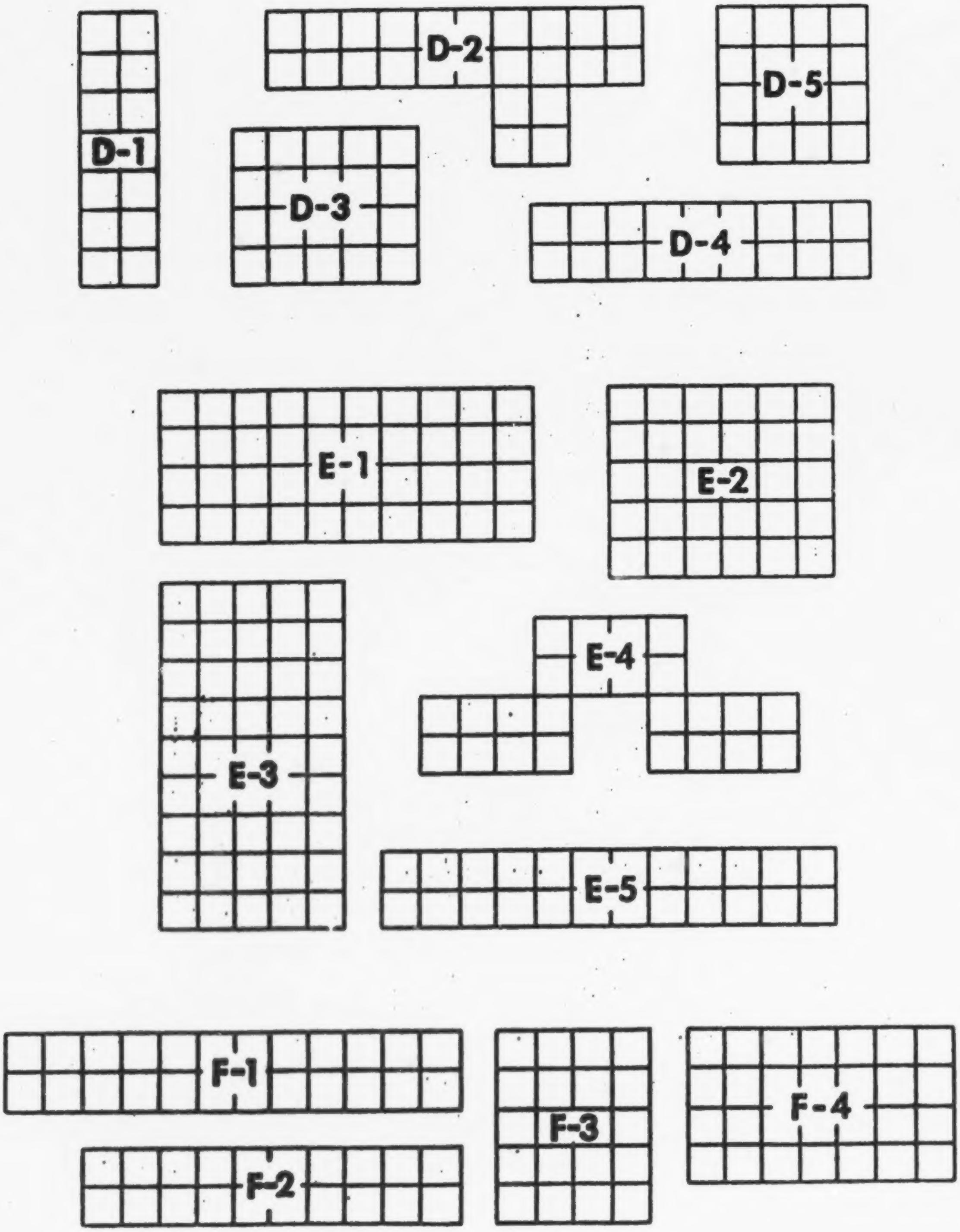
This piece of paper is $\frac{1}{4}$ as large as one of the five pieces of paper in this envelope.

Find that piece of paper.

Write its name on your record card.

Fig. 1




D-1
D-2
D-5
D-3
D-4
E-1
E-2
E-3
E-4
E-5
F-1
F-2
F-3
F-4

F-15

I

This group has $\frac{1}{4}$ as many lima beans as one of the groups in this envelope.

Find that group.

Write its name on your record card.

J

This group has $\frac{1}{3}$ as many lima beans as one of the groups in this envelope.

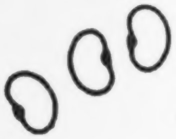
Find that group.

Write its name on your record card.

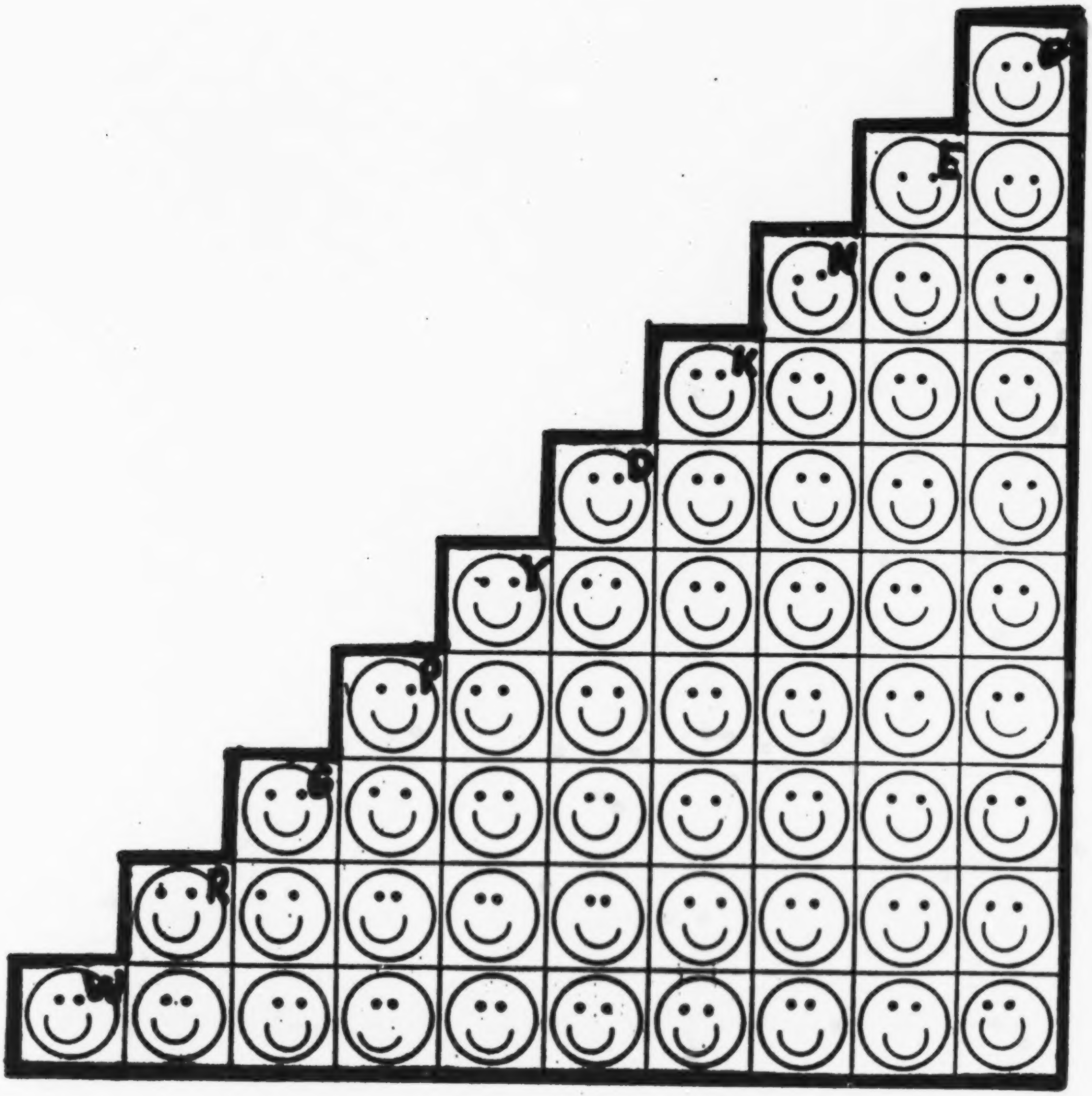
At this station are Envelopes I and J with labeled index cards inside. The labels for the envelopes may be reproduced from the Teaching Package.

Envelope I has two lima beans taped or glued securely to the label. Inside are cards labeled I–1, I–2, I–3, and I–4. Attached to the cards are 6, 7, 8, and 9 lima beans, respectively.

Envelope J has three lima beans on the label. The cards are labeled J–1, J–2, J–3, and J–4 and have 7, 10, 12, and 15 lima beans, respectively.

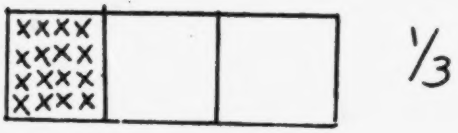


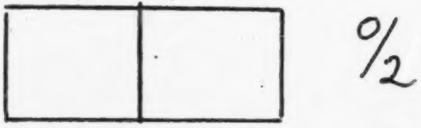
F-16



FRACTION BARS

The fraction bars consist of a set of 64 colored bars divided into 2,3,4,6, and 12 parts. The denominators of the fractions are represented by the number of parts to a bar, and the numerator by the number of shaded parts.





Each bar is six inches long. The color coding is as follows:

halves ------ green

thirds ------ yellow

fourths ----- blue

sixths ------ red

twelves -----orange

There are 32 different fractions bars in five colors (a complete set has two of each bar).

The halves consist of: 0/2, 1/2, 2/2

The thirds consist of: 0/3, 1/3, 2/3, 3/3

The fourths consist of: 0/4, 1/4, 2/4, 3/4, 4/4

and so on for the sixths and the twelves.

One conviction that led to the development of fraction bars was: "The need for an adequate readiness period for fraction concepts before the use of fraction symbols and rules. Without sufficient readiness the students must rely solely on the manipulation of symbols."

From Fraction Bars Teacher's Guide by Albert B. Bennett, Jr. and Patricia S. Davidson. Copyright 1973 by Scott Resources, Inc. Used by Permission of publisher.

Centimeter Square Grid F-18

FRACTION CIRCLES/SQUARES

F-19a

CIRCLES: Whole - White 1/4- Yellow
1/2 - Orange 1/6- Green
1/3 - Blue 1/8- Red

There are undoubtedly hundreds of games and variations using fraction circles and squares. However, this afternoon we will only have time to learn a couple, but I am sure you can see other variations to the rules to adapt the game to your classroom.

FRACTION RUMMY

MATERIALS: FRACTION SQUARES OR CIRCLES
Non-player randomly deals all pieces, except white whole, to 3 - 4 players.

Play continues for 2 rounds and whoever makes the most wholes or # pairs of equivalents wins. Decide first on one or the other, not both whole and equivalents.

In case of a tie, greatest sum wins. After deal, each player forms as many ones as possible.(or equivalents)

Then each player passes one piece of their choice to the player on their left. Players once again try to form one's or equivalences. Then a final pass is made and one's or equivalence again attempted.

Winner has most one's (or equivalents).

ROLL TO MAKE ONE

Using fraction circles and one octahedral die or spinner.

A Roll of	Means
1	draw 1 whole - score 1 pt.
2	draw 1/2 (Orange)
3	draw 1/3 (Blue)
4	draw 1/4 (Yellow)
5	draw one piece from anyone
6	draw 1/6 (Green)
7	Player on right takes one of yours
8	draw 1/8 (Red)

Object: To Form Wholes exactly

a. When point is scored, return pieces to "bank"

b. If no piece of variety rolled is available, choose from bank the equivalent (if possible) or lose your turn.

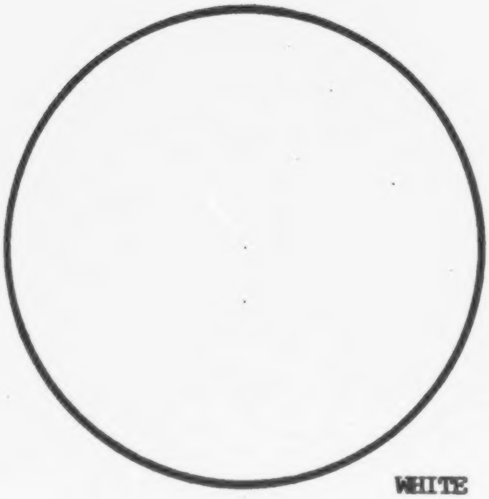
c. You may build more than one set at a time and interchange at will.

WINNER: Greatest score in a given time period or first to reach a pre-set score.

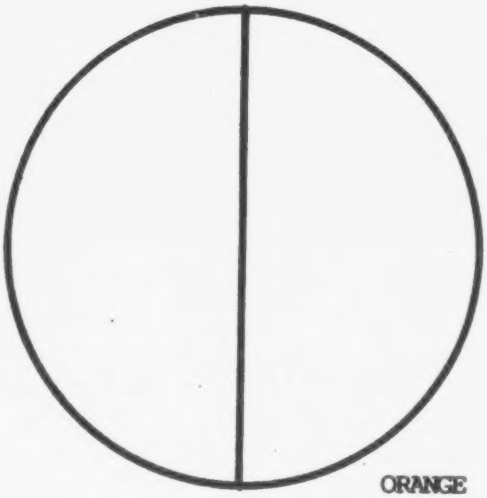
BEST PLAYED WITH 3-5 Players.

VARIATIONS: Using 2 or 3 sets, begin with 3 ones and subtract or differ rules oppositely, i.e. instead of drawing give etc.

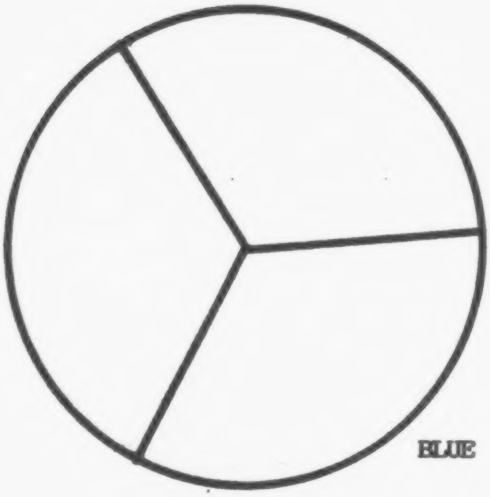
F-19b



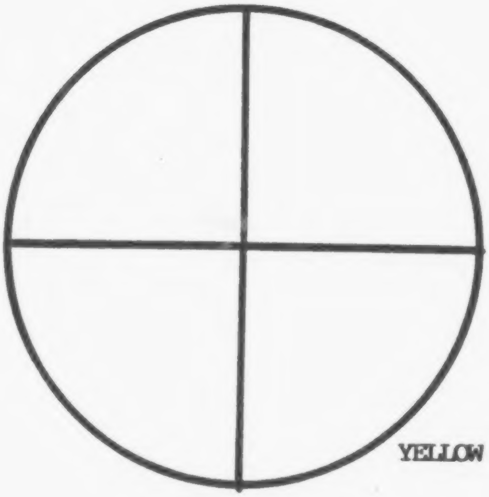
WHITE



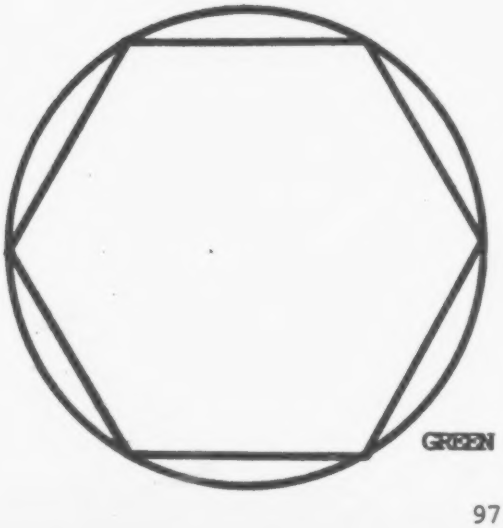
ORANGE



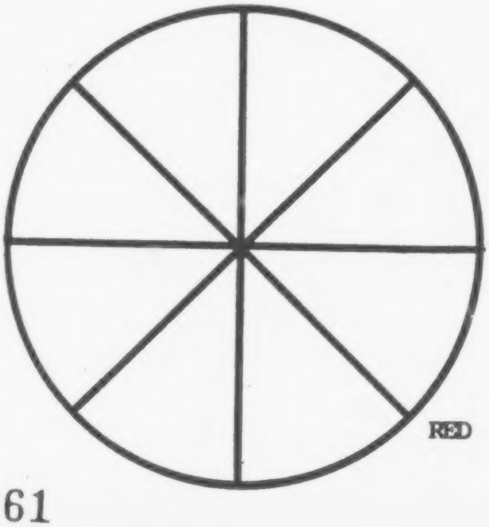
BLUE



YELLOW



GREEN

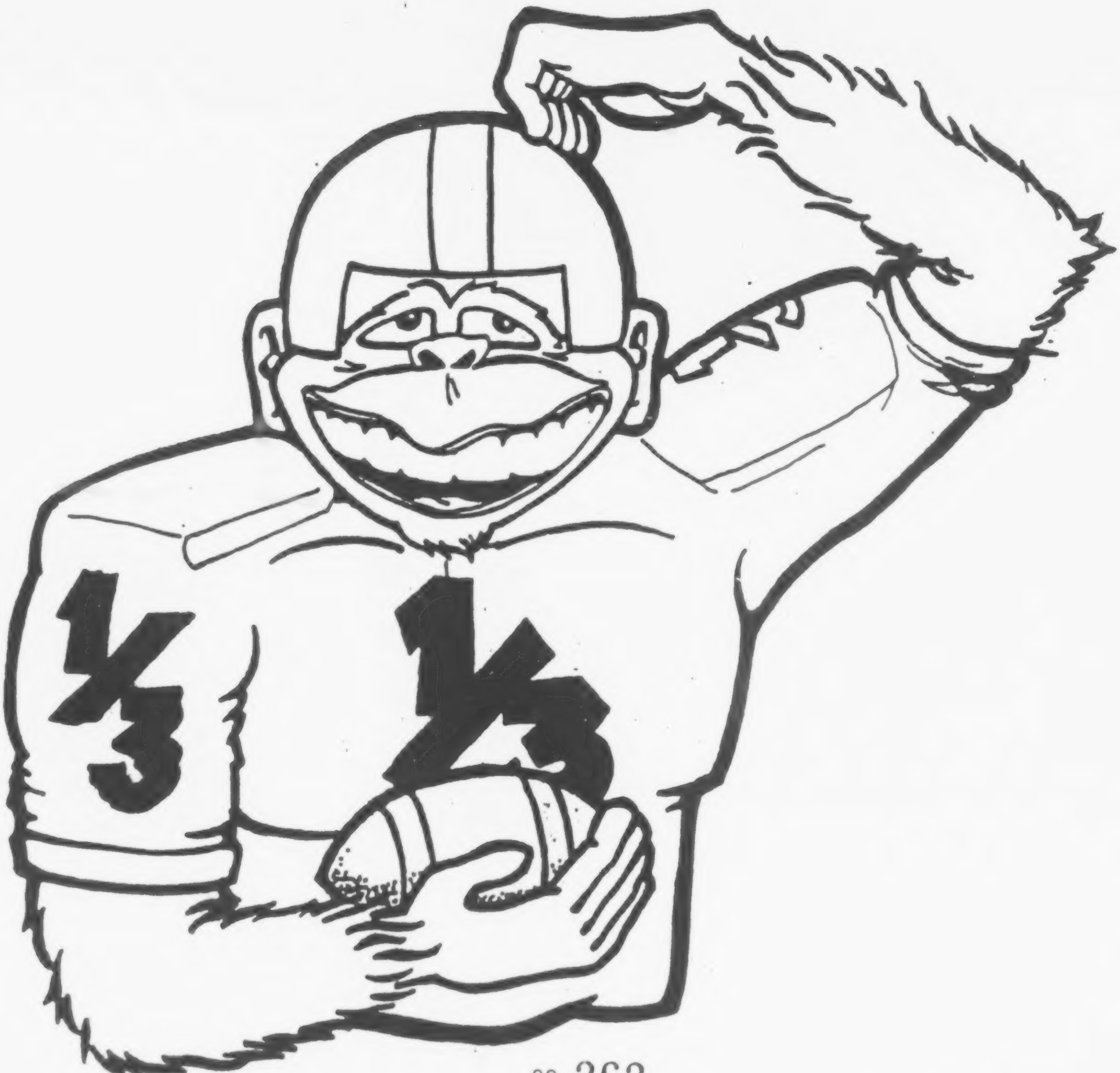


RED

F-20a

NUMERATOR OR DENOMINATOR?

EQUIVALENT FRACTIONS



F-20b

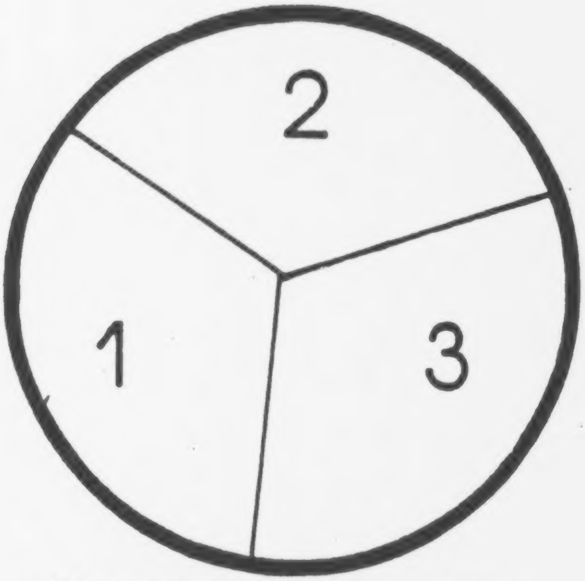
Numerator or Denominator?

Equivalent Fractions

Players: 2 to 4
Materials: 1 game marker per player

How to Play

1. Place markers on START.
2. In turn, spin and move forward. Name the missing numerator or denominator on that space.
3. Partner checks answer card.
4. If incorrect, move back to where you were.
5. First player to reach END and answer problem Z correctly wins.



(Place circle opposite gameboard in folder. Add spinner)

Answer Card
(Numerator or Denominator?)

A. 2
B. 15
C. 10
D. 1
E. 4
F. 10
G. 2
H. 1
I. 9
J. 6
K. 4
L. 15
M. 2
N. 8
O. 1
P. 8
Q. 3
R. 2
S. 20
T. 4
U. 1
V. 3
W 3
X. 7
Y. 4
Z. 30

(Place answer card in envelope on folder.)

Numerator or Denominator

Equivalent Fractions

F-20c

H. $\frac{\square}{3} = \frac{2}{6}$

G. $\frac{1}{2} = \frac{\square}{4}$

F. $\frac{1}{5} = \frac{2}{\square}$

E. $\frac{3}{\square} = \frac{12}{16}$

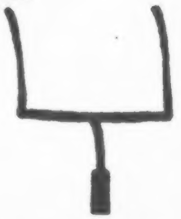
D. $\frac{\square}{4} = \frac{3}{12}$

C. $\frac{2}{3} = \frac{\square}{15}$

B. $\frac{1}{3} = \frac{5}{\square}$

A. $\frac{1}{\square} = \frac{7}{14}$

START



I. $\frac{3}{4} = \frac{\square}{12}$

J. $\frac{1}{2} = \frac{\square}{12}$

K. $\frac{2}{3} = \frac{\square}{6}$

L. $\frac{1}{5} = \frac{3}{\square}$

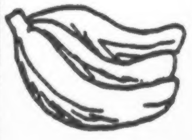
M. $\frac{1}{\square} = \frac{3}{6}$

N. $\frac{1}{4} = \frac{2}{\square}$

O. $\frac{\square}{3} = \frac{4}{12}$

P. $\frac{1}{2} = \frac{4}{\square}$

Q. $\frac{\square}{4} = \frac{6}{8}$



END

Z. $\frac{5}{6} = \frac{25}{\square}$

Y. $\frac{\square}{5} = \frac{16}{20}$

X. $\frac{3}{\square} = \frac{9}{21}$

W. $\frac{2}{\square} = \frac{8}{12}$

V. $\frac{1}{3} = \frac{\square}{9}$

U. $\frac{\square}{2} = \frac{5}{10}$

T. $\frac{1}{\square} = \frac{4}{16}$

S. $\frac{1}{5} = \frac{4}{\square}$

R. $\frac{\square}{3} = \frac{6}{9}$

F-21a

SPIN THE FRACTION

SIMPLIFYING FRACTIONS

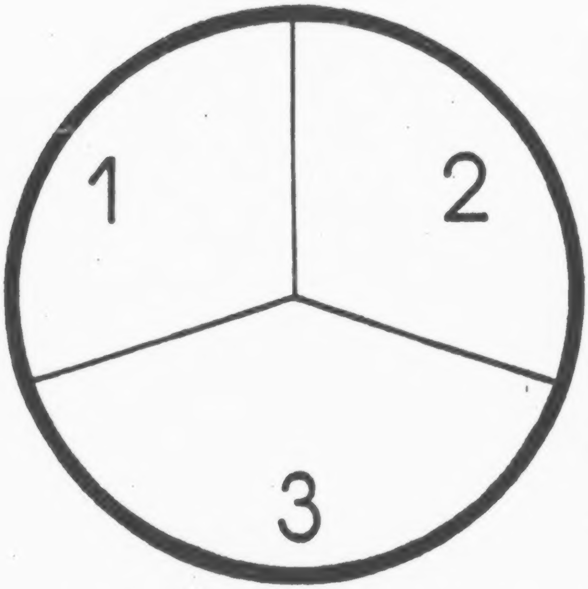
F-21b

Spin the Fraction
Simplifying Fractions

Players: 2 to 4
Materials: 30 game tokens for each player

How to Play

1. In turn, spin a fraction on the gameboard and give it in lowest terms.
2. Partner checks answer card.
3. If correct, spin the 1-2-3 spinner to see how many tokens to remove from your pile of 30. If incorrect, do not spin.
4. First player to use up all 30 tokens wins.



(Place circle opposite gameboard in folder. Add spinner)

Answer Card
(Spin the Fraction)

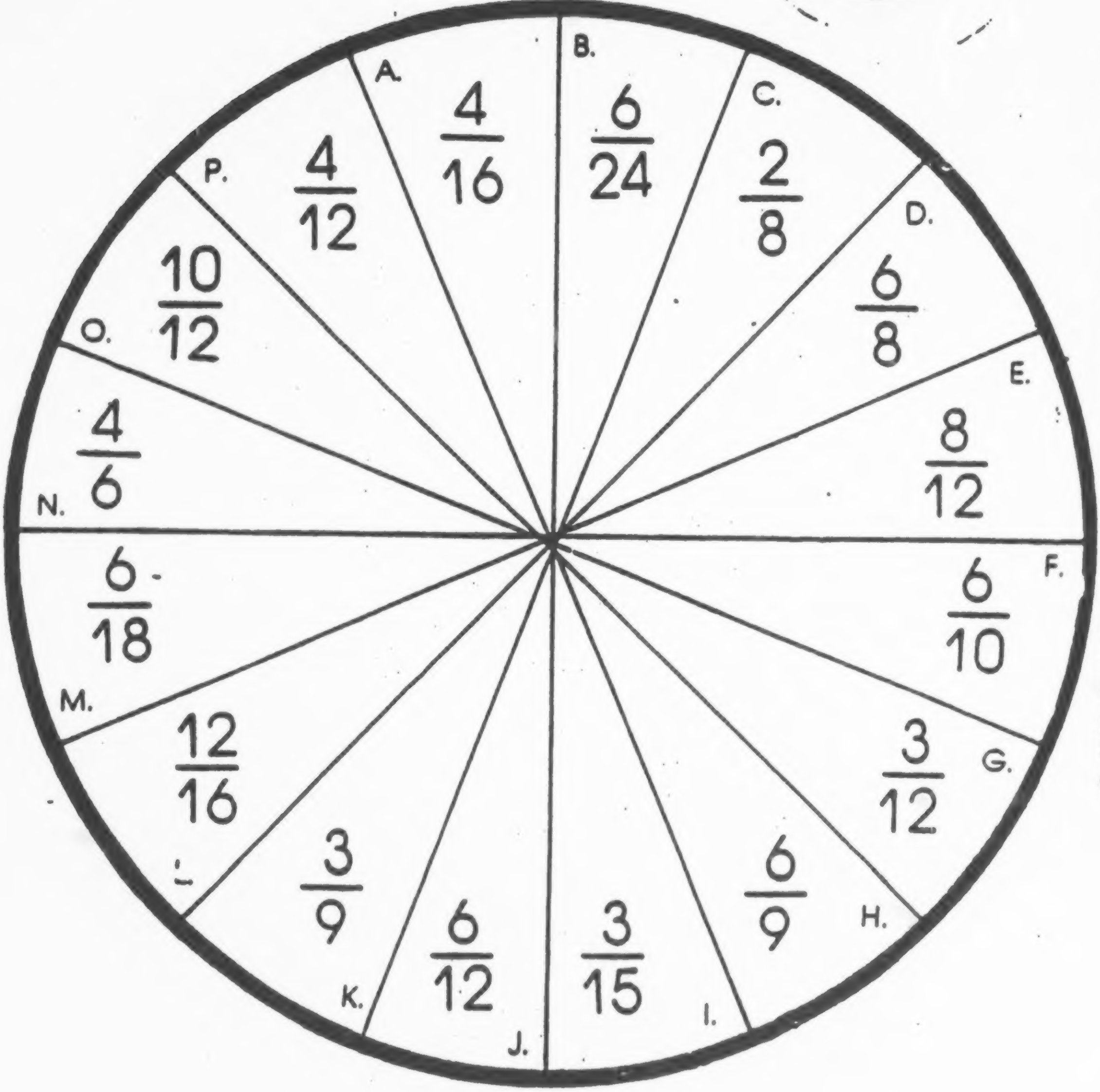
A. $\frac{1}{4}$
B. $\frac{1}{4}$
C. $\frac{1}{4}$
D. $\frac{3}{4}$
E. $\frac{2}{3}$
F. $\frac{3}{5}$
G. $\frac{1}{4}$
H. $\frac{2}{3}$
I. $\frac{1}{5}$
J. $\frac{1}{2}$
K. $\frac{1}{3}$
L. $\frac{3}{4}$
M. $\frac{1}{3}$
N. $\frac{2}{3}$
O. $\frac{5}{6}$
P. $\frac{1}{3}$

(Place answer card in envelope on folder.)

Spin the Fraction

Simplifying Fractions

F-21c


A.
$\frac{4}{16}$
B.
$\frac{6}{24}$
C.
$\frac{2}{8}$
D.
$\frac{6}{8}$
E.
$\frac{8}{12}$
F.
$\frac{6}{10}$
G.
$\frac{3}{12}$
H.
$\frac{6}{9}$
I.
$\frac{3}{15}$
J.
$\frac{6}{12}$
K.
$\frac{3}{9}$
L.
$\frac{12}{16}$
M.
$\frac{6}{18}$
N.
$\frac{4}{6}$
O.
$\frac{10}{12}$
P.
$\frac{4}{12}$

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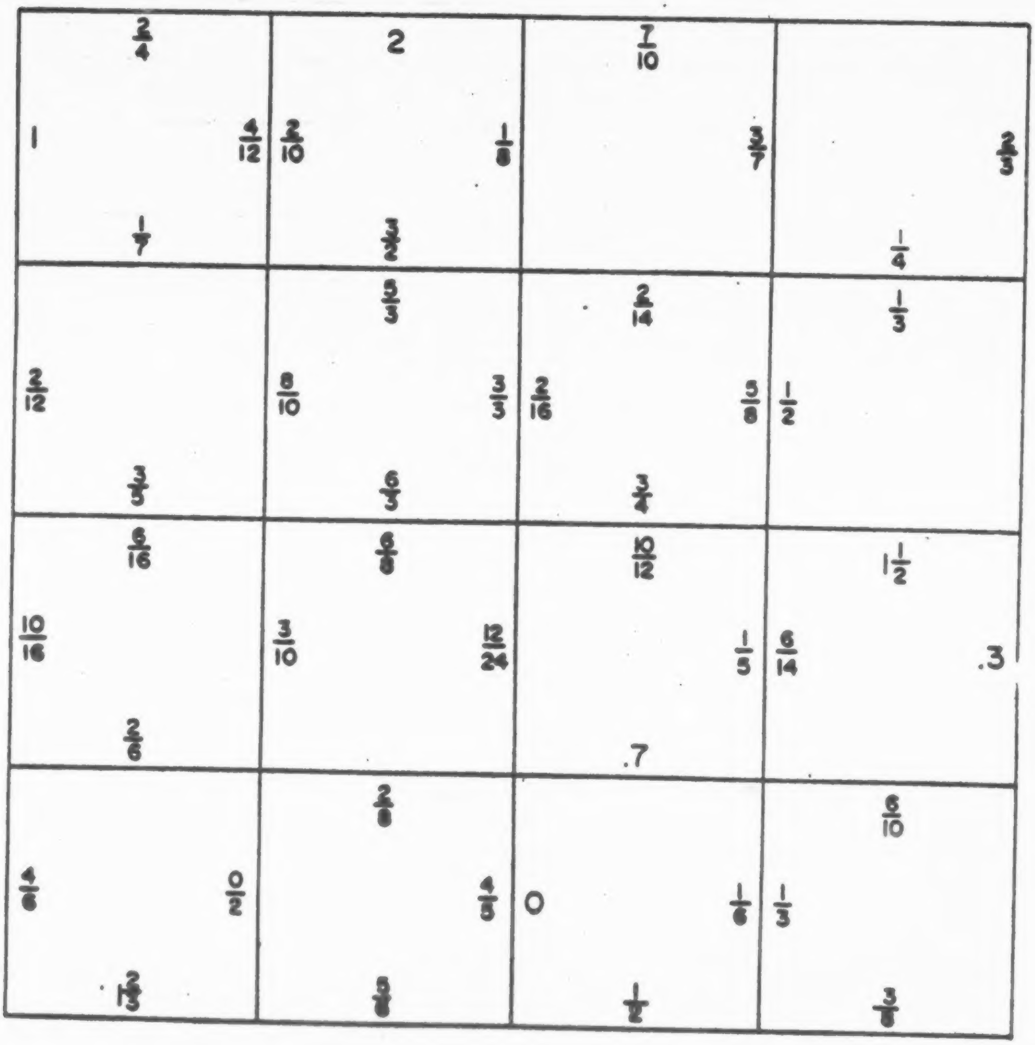
(Add spinner.)

F-22a

Directions: Cut out the squares. Fit them together so that the edges that touch name the same number.


$\frac{2}{4}$
1
$\frac{4}{12}$
$\frac{1}{7}$
2
$\frac{2}{10}$
$\frac{1}{8}$
$\frac{3}{2}$
$\frac{7}{10}$
$\frac{3}{7}$
$\frac{2}{3}$
$\frac{1}{4}$
$\frac{2}{12}$
$\frac{3}{5}$
$\frac{5}{3}$
$\frac{8}{10}$
$\frac{3}{3}$
$\frac{6}{3}$
$\frac{2}{14}$
$\frac{2}{16}$
$\frac{5}{8}$
$\frac{3}{4}$
$\frac{1}{3}$
$\frac{1}{2}$
$\frac{6}{16}$
$\frac{10}{16}$
$\frac{2}{6}$
$\frac{6}{8}$
$\frac{3}{10}$
$\frac{12}{24}$
$\frac{10}{12}$
$\frac{1}{5}$
.7
$1\frac{1}{2}$
$\frac{6}{14}$
.3
$\frac{4}{6}$
$\frac{0}{2}$
$1\frac{2}{3}$
$\frac{2}{8}$
$\frac{4}{5}$
$\frac{5}{8}$
0
$\frac{1}{6}$
$\frac{1}{2}$
$\frac{6}{10}$
$\frac{1}{3}$
$\frac{3}{8}$

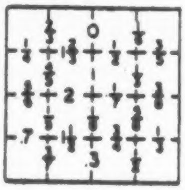
F-22b

Objective: Experience with the concept of equals.

Directions:

1. Provide each student with a copy of the activity sheet and a pair of scissors.
2. If necessary, give the hint that when the squares are fit together correctly, a 4 by 4 square is once again found.

Comments: The fewer directions required the better. Students need experience in figuring things out for themselves.



FRACTION STRIPS

OBJECTIVES

1. To explore a method for identifying equivalent fractions
2. To use the method for addition and subtraction of fractions with unlike denominators

GROUP SIZE

Individual, small group

MATERIALS

Two sets of multiplication strips 1–9 (see Figure 223)

1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
			⋮							
9	9	18	27	36	45	54	63	72	81	90

Figure 223
Multiplication Strips

DESCRIPTION

To show fractions equivalent to $\frac{3}{5}$, place the 3-strip above the 5-strip (see Figure 224).

3	3	6	9	12	15	18	21	24	27	30
5	5	10	15	20	25	30	35	40	45	50

Figure 224
Fractions Equivalent to $\frac{3}{5}$

This shows that fractions equivalent to $\frac{3}{5}$ are $\frac{6}{10}$, $\frac{9}{15}$, $\frac{12}{20}$ and so on.

Addition and subtraction of fractions can be accomplished easily using these strips. To add $\frac{3}{5}$ and $\frac{1}{2}$, place the $\frac{3}{5}$ fractions strips above the $\frac{1}{2}$ fractions strips. Look for identical denominators (see Figure 225).

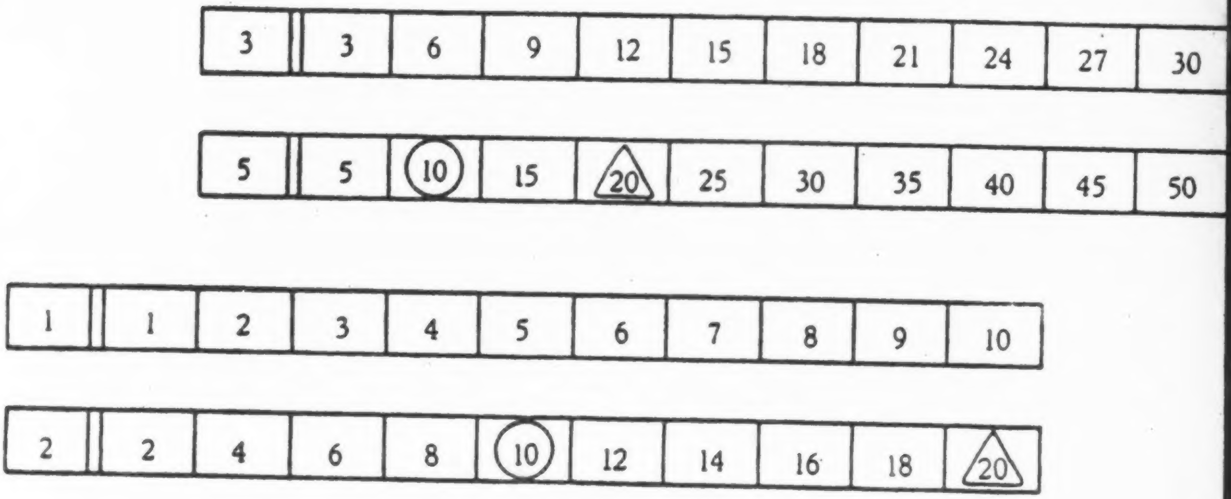

3 | 3 6 9 12 15 18 21 24 27 30
5 | 5 10 15 20 25 30 35 40 45 50
1 | 1 2 3 4 5 6 7 8 9 10
2 | 2 4 6 8 10 12 14 16 18 20

Figure 225
$\frac{3}{5}$ and $\frac{1}{2}$ Fractions Strips

$\frac{3}{5} + \frac{1}{2}$ is equivalent to $\frac{6}{10} + \frac{5}{10} = \frac{11}{10}$ or $\frac{12}{20} + \frac{10}{20} = \frac{22}{20}$

Write an improper fraction for each mixed number or whole number.

1. $\underset{\times}{\overset{+}{3\frac{1}{4}}} = \frac{\boxed{13}}{4}$

First, 4 × 3 = 12
Then 12 + 1 = 13

2. $\underset{\times}{\overset{+}{3\frac{2}{5}}} = \frac{\boxed{17}}{5}$

3. $5 = \frac{\boxed{}}{2}$

4. $7 = \frac{\boxed{}}{4}$

5. $2\frac{3}{8} = \frac{\boxed{}}{8}$

6. $4\frac{2}{3} = \frac{\boxed{}}{3}$

7. $1\frac{1}{4} = \frac{\boxed{}}{4}$

8. $5\frac{1}{6} = \frac{\boxed{}}{6}$

9. $7\frac{2}{3} = \frac{\boxed{}}{3}$

10. $6\frac{1}{4} = \frac{\boxed{}}{4}$

11. $2\frac{5}{6} = \frac{\boxed{}}{6}$

What do you find in a marching band and a flock of turkeys?

Write an improper fraction for each mixed number. Then find your answer below. Write the letter for that answer. One answer is used twice.

12. $2\frac{5}{8}$ ____ U

13. $1\frac{7}{9}$ ____ C

14. $3\frac{2}{3}$ ____ T

15. $6\frac{5}{6}$ ____ M

16. $4\frac{2}{5}$ ____ K

17. $2\frac{3}{4}$ ____ D

18. $1\frac{8}{9}$ ____ S

19. $5\frac{1}{10}$ ____ I

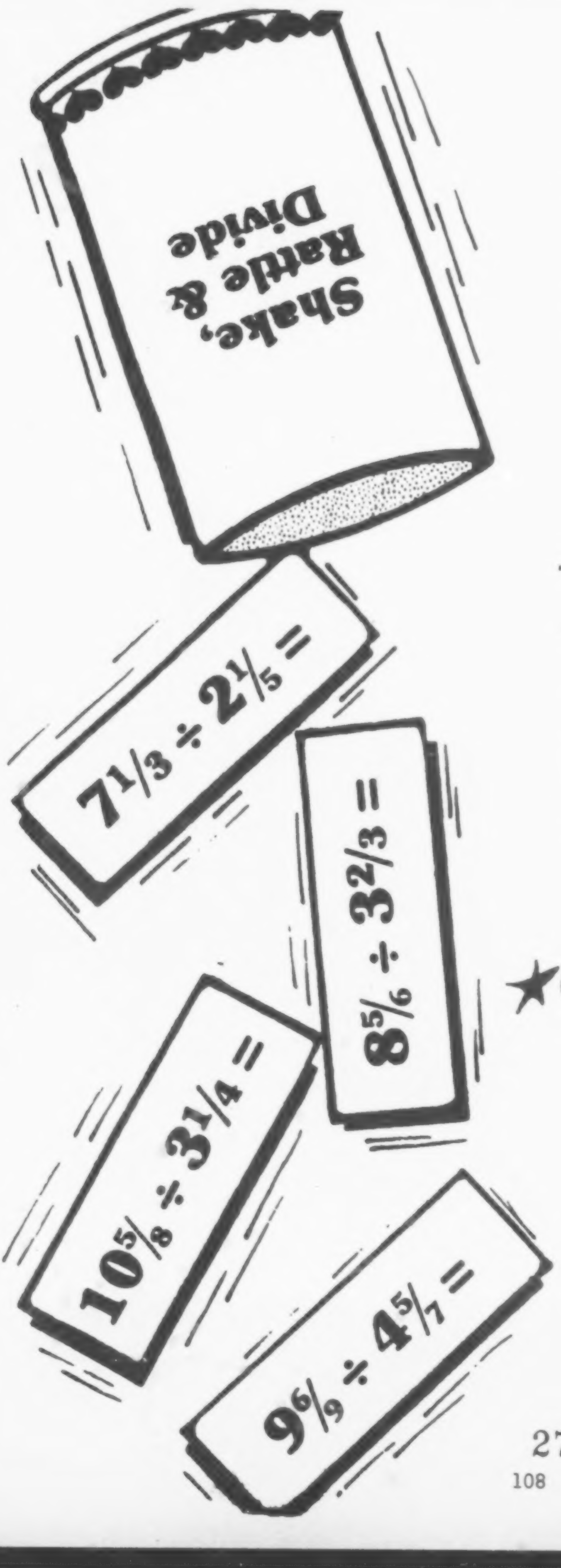
20. $5\frac{4}{7}$ ____ R

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

$\frac{11}{4}$ $\frac{39}{7}$ $\frac{21}{8}$ $\frac{41}{6}$ $\frac{17}{9}$ $\frac{11}{3}$ $\frac{51}{10}$ $\frac{16}{9}$ $\frac{22}{5}$ $\frac{17}{9}$

Centimeter Square Grid

F-25


Shake, Rattle & Divide
$7\frac{1}{3} \div 2\frac{1}{5} =$
$8\frac{5}{6} \div 3\frac{2}{3} =$
$10\frac{5}{8} \div 3\frac{1}{4} =$
$9\frac{6}{9} \div 4\frac{5}{7} =$

Shake, Rattle And Divide

Skill #327
Mixed and Mixed

Objective:
The student will compute a quotient.

Directions: See #2 below

Place three or four students in a circle. In alternating turns, each student shakes out a card and computes an answer for the division problem. For each correct quotient, he scores seven points. The first student to reach twenty-one wins.

Follow-up Activities:

1. Present the students with a problem and have them compute the quotient on individual answer boards.
2. ★ Have the students form two lines and provide each team with a can of division problems. In relay style, each team competes to answer its can of problems first.

DAFFY DIVISION

OBJECTIVES

1. To introduce a method for dividing fractions
2. To identify the least common denominator for two fractions
3. To practice renaming fractions using the least common denominator

GROUP SIZE

Individual, small group, whole class

MATERIALS

Paper and pencil

DESCRIPTION

The following procedure is an alternative method for dividing fractions.

Example: $\frac{3}{5} \div \frac{1}{5}$ means *how many one-fifths are there in three-fifths?* There are $3 \div 1$, or 3.

Example: $\frac{9}{15} \div \frac{3}{5}$ means *how many three-fifths are in nine-fifteenths?* To make the problem simpler, find the common denominator for $\frac{9}{15}$ and $\frac{3}{5}$. The common denominator is 15. Therefore, the problem may be rewritten as $\frac{9}{15} \div \frac{9}{15}$. Now the question is, how many nine-fifteenths are in nine-fifteenths? There are $9 \div 9$, or 1 (see numerators).

Example: $\frac{3}{4} \div \frac{1}{6}$ means *how many one-sixths are there in three-fourths?* Simplify the problem by renaming $\frac{3}{4}$ and $\frac{1}{6}$ using their least common denominator— $\frac{9}{12} \div \frac{2}{12}$. The question is now, how many two-twelfths are in nine-twelfths? There are $9 \div 2$, or $\frac{9}{2}$.

The students may check the solutions to these problems by using the traditional method for dividing fractions.

Note: Using this technique, mixed numbers should be changed into improper fractions.

FM-1b

materials to the objectives of their day-by-day lessons. Teachers usually do not have enough time to plan the use of these materials. Teaching materials must include instructions for the use of manipulatives along with the development of the lessons.

Many teachers are concerned about the amount of time needed to organize and to manage materials within the classroom. Teachers need to be encouraged to begin slowly. Simple materials should be used at first. If sticks tied in bundles of ten are easier to manage and explain than base-ten blocks when teaching place value, teachers should begin with these. Teachers can use an overhead projector with all students for a beginning activity. Later when the topic is presented, the teacher may try a small-group approach or individualized instruction using manipulatives. By developing the concept first with the entire class, the teacher can control questions and obtain valuable feedback from the children. By progressing gradually from simple to more complex activities teachers gradually become comfortable using manipulatives.

Some teachers do not use manipulatives because they think the children will feel the materials are below their level. Instead of large blocks a teacher can use cards of sports figures to demonstrate a multiplication array to older children.

Manipulatives are often left in the closet because of the amount of time needed for their use and care. It may take only a few minutes to *tell* students how to rename in addition, but it may take a half hour for children to work with base-ten blocks the first time they are used for this topic. The internalization and retention of the concept, however, compensates for the extra time it takes to use manipulatives. The long-term clarity and understanding achieved makes their use well worth consideration. ✚

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from:
"Current Issues in
Mathematics,"

Houghton Mifflin

FM-2a

MANIPULATIVE AIDS FOR EXPERIENCES WITH FRACTIONS AND DECIMALS

1. ABACUS - individual and demonstration

2. ADDING MACHINE TAPE
   May be used to make strips of tickets, or for folding of paper strips as an aid to addition of unlike fractions.

3. BEADS
   May be strung on string, wire or frame. Can be made into individual students "tens counter."

4. BUTTER CARTON AND STICKS OF WOOD TO REPRESENT STICKS OF BUTTER
   This aid provides a real-life situation for dealing with fractional parts.

5. CALENDAR
   Provides a real-life number line. It is also helpful for solution of ratio problems. (as "two days out of three. . .")

6. CARDBOARD
   May be used to make geometric regions, and shapes, fractional parts and strips and interlocking circles.

7. COLORED RODS
   Different types of colored rods are available to aid in work with counting, set operations, place value, fractions, measurement, etc.
   Cuisenaire Rods
   Arithma stix
   Minnebars
   Napier's Rods

8. COUNTERS
   Sets of miscellaneous objects to demonstrate and identify a fraction with a "part" of a unit that is a physical model. Materials may include:
   lima beans and plastic bags
   acorns
   tongue depressors
   play money
   washers
   checkers
   popsicle sticks & rubber bands
   bottle caps
   applicator sticks
   strips of colored paper
   plastic squares or disks
   cubes

9. CUP, marked in fractions and metric system
   A "resl-life" situation for the use of fractions and tens.

10. EGG CARTONS
    May be cut apart into various size parts. Each group may be painted a different color and then used to illustrate fractional parts and equivalent fraction.

11. FLANNEL BOARD AND FELT PIECES AND YARN
    Felt sets of objects
    Felt pieces to show fractional parts
    Felt pieces to show fraction board
    Felt symbols and numerals

12. FRACTION BLOCKS
Cabinet with four shelves 4" depth and 24" length.
1 block 24" length for 1 unit
2 blocks 12" length for 1/2's
3 blocks 8" length for 1/3's
4 blocks 6" length for 1/4's

13. FRACTION BOARD
Excellent for exploring equivalent fractions. May be made from cardboard strips placed in a pocket chart. The top pocket should contain 1 strip for 1 unit. The next pocket should contain 2 strips for 1/2's, the next 3 strips for 1/3's and so forth through 1/12's.

Strips may be made from felt (or construction paper backed with sandpaper) for use on the flannel board.

14. FRACTIONEER
A device to aid the teacher and students in constructing interlocking paper circles. Each circle should be a different color and marked to show different fractional parts. The circles are slit along one of the lines (radii of the circle) and by interlocking them, they can be revolved and thirds matched against sixths, halves against sixths and so on. Three and more circles may be interlocked.

15. GRID PAPER
This paper is useful for coloring, cutting and folding to represent fractions.

16. GEOBOARDS
Students learn to assign fractional values to shapes, determine fractional values of the smallest square when other squares are called one, determine fractional values for lengths, widths, and areas.

17. CUBES
Students develop lists of equivalent fractions.

18. HUNDREDS BOARD
One hundred cup hooks or small nails are arranged in ten rows of ten hooks on a plywood board (approximately 3 feet by 3 feet). Numerals are written on one side of key tags and multiplication factors may be written on the other side.
Hundreds board may be used with key tage, red on one side and blue on the other to show percent.

19. HUNDRED SQUARES
Decimal values may be assigned in various ways.

20. NUMBER LINE
May be used for demonstration or individual activities
Decimal number line
Fraction number line
vertical number line
Flexible number line
Plastic number line with no numerals, so that pupils may write on it with a grease pencil for varied use.

21. ODOMETER or SUPERMARKET ADD-A-MATIC
The odometer is an indicator on an automobile which shows miles traveled and may be made by students. The Supermarket add-a-matic is a small counter which is an excellent aid in problems involving tenths.

22. PAPER - for folding and cutting
Students develop lists of equivalent fractions, add fractions with unlike denominators and subtract fractions with unlike denominators.

23. PEGBOARD and colored pegs, with strings and rubber bands.
May be used to assign fractional values to various shapes, lengths, widths, and areas.

24. PLACE FINDER
A valuable aid in the study of decimal fractions. The base may be made of wood, cardboard, or any aubstantial material and has place values named in words. A decimal point is shown in ones place. The base remains stationary and the top is notched to fit easily into the base, but can be moved easily. Numerals are written on the upper part to show the decimal fraction.

25. PLAY MONEY - coins to dollars
Useful in work with decimal fractions.

26. POCKET CHART
A most helpful aid in the classroom. Children may use it to construct a Place Value Chart, a Fraction Board, a Decimal Board and a variety of other uses.

27. POP BEADS
Children can assemble beads into sets having 10 beads of the same color. These sets can be combined into closed chains of 100, alternating colors of groups of 10. The chains can be used to assign fractional, decimal or percent values.

28. POSTAGE STAMPS OR CHRISTMAS SEALS
May be used to represent decimal fractions and wholes.

29. RULER-metric
May be used to demonstrate decimal fractions.

30. STRING
Especially helpful in demonstrating half, fourth, etc.

31. TAPE MEASURE - metric
May be used to demonstrate decimal fractions.

SOME WORDS WE NEED TO KNOW!

FM-3a

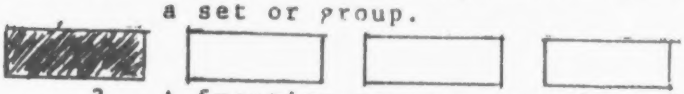
FRACTION, a quantity less than a unit or whole number or the expression of a number relationship. The word fraction is also used with reference to the symbol which expresses the fraction numeral (ex. ½, ¼, etc.).

Fractions have different meanings in different situations and are defined in statements expressing the ideas that may be represented by fractions:

1. A fraction may represent one or more of the equal parts of a whole unit.

¼ of 1 strip

2. A fraction may represent one or more of the equal parts of a set or group.



¼ of a group of 4 strips

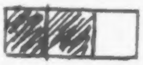
3. A fraction may express the relationship that exists between two or more groups (ex. 3 out of 4 children have books. Expressed with fractions: 3/4 of the children have books.).

4. A fraction may express quotients that cannot be expressed by whole numbers. ex. 2/3 means 2 ÷ 3 or 2 divided by 3.

TERMS, the numerator and denominator are called the terms of the fraction.

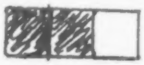
2 --numerator
3 --denominator

NUMERATOR, the numeral in the fraction written above the horizontal bar which indicates how many parts are being considered.



$\frac{2}{3}$ --shaded parts

DENOMINATOR, the numeral in the fraction written below the horizontal bar indicating the number of equal parts into which a whole has been divided.



$\frac{2}{3}$ --number of equal parts

CLASSES OF FRACTIONS: The basic meanings of fractions remain the same even though the forms of fractions vary widely. Some of the various classes of fraction numerals are summarized below:

Classes of Numerals	*Distinguishing Features*	*Examples*
Unit fractions	Numerator = 1	$\frac{1}{2}, \frac{1}{4}$
Nonunit fractions	Numerator ≠ 1	$\frac{3}{4}, \frac{2}{5}$
Proper fractions	Numerator < denominator	$\frac{1}{6}, \frac{3}{4}$
Improper fractions	Numerator > denominator	$\frac{5}{3}, \frac{8}{5}$
Mixed numerals	Whole-number numeral + fraction	$1\frac{1}{2}, 4\frac{1}{4}$
Whole-number numerals	Denominator = 1	3 or $\frac{3}{1}$
Like fractions	Same denominators	$\frac{1}{4}, \frac{3}{4}$
Unlike fractions	Different denominators	$\frac{3}{4}, \frac{3}{5}$

INVERT, to turn a fraction upside down. ex. $\frac{a}{b}$ inverted is $\frac{b}{a}$

RECIPROCALS, if the product of two numbers is 1, the numbers are said to be reciprocals of each other. Therefore, the reciprocal of a number is 1 divided by that number.

RENAMING, rewriting a fraction or mixed numeral in an equivalent form having a specified denominator. ex. $\frac{5}{8} = \frac{?}{24} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$

REDUCE, to write a fraction in lowest terms.
ex. $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

SIMPLIFY, to divide the numerator and denominator by the same nonzero number. When both the denominator and numerator are divided by their greatest common factor, the resulting fraction is in lowest terms.

LOWEST TERMS, a fraction having numerator and denominator which cannot be divided by a factor greater than 1.

COMMON DENOMINATORS, fractions having like denominators.
ex. $\frac{4}{6}$, $\frac{3}{6}$ are said to have common denominators.

FACTOR, a number which when multiplied by another number results in a product. 4 is a factor of 12 since 4 x 3 = 12

MULTIPLE, the product of two or more numbers. 12 is a multiple of 4 since it is the product of 4 x 3.

LEAST COMMON MULTIPLE (L.C.M.) the whole number that is the smallest nonzero multiple that two numbers have in common.

LEAST COMMON DENOMINATOR (L,C.D.) the least common multiple of the denominators of two or more fractions is said to be the least common denominator.

GREATEST COMMON FACTOR (G.C.F.) The largest factor that two numbers have in common.

FM-4

Addresses for Math Materials

Activity Resources Company, Inc.
Post Office Box 4875
Hayward, California 94540

American Guidance Service, Inc.
Publishers' Building
Circle Pines, Minnesota 55014
(Key Math Test)

Creative Publications
5005 West 110th Street
Oak Lawn, Illinois 60453
800-624-0822

Creative Teaching Associates
Post Office Box 7714
Fresno, California 93727

Cuisenaire Co. of America, Inc.
12 Church Street, Box D
New Rochelle, New York 10802

Dale Seymour Publications
Post Office Box 10888
Palo Alto, California 94303

Deltax Industries, Inc.
Attention Dept - 42AB
Post Office Box 442
Marinette, Wisconsin 54143

Dick Blick
Box 1267
Galesburg, Illinois 61401

Gamco Industries
Math-Master
Box 1911
Big Spring, Texas 79720

Ideal School Supply Co.
11000 S. Lavergne Avenue
Oak Lawn, Illinois 60453

L. J. Associates
Post Office Box 2388
Littleton, Colorado 80120

Midwest Publications
Post Office Box 448 Dept. 80
Pacific Grove, California 93950

National Council of Teachers
of Mathematics
1906 Association Drive
Reston, Virginia 22091

Scott Resources, Inc.
Box 2121
Ft. Collins, Colorado 80522

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Special Characteristics of Rational Numbers. Feb., 10-12.

Subtraction of Fractions With Borrowing. Jan., 35.

Teaching Rational Number Division: A Special Problem. Feb., 20-22.

Teaching Rational Numbers -- Primary Grades. Feb., 36-39.

Teaching Rational Numbers -- Intermediate Grades. Feb., 40-42.

Teaching Rational Numbers -- Junior High School. Feb., 43-46.

Ten Often Ignored Applications of Rational-Number Concepts. Feb., 48-50.

FM-6a

SKILLS FOR MATH EDUCATION

FRACTIONS AND MIXED NUMBERS

1. Identify a shape that is divided into half from a group of divided shapes.
2. Identify a shape that is divided into fourths from a given group of divided shapes.
3. Identify a shape that is divided into thirds from a given group of divided shapes.
4. Identify a shape that is divided into fifths from a given group of divided shapes.
5. Add two proper fractions with like denominators without renaming the sum.
6. Subtract two proper fractions with like denominators without renaming.
7. Identify equivalent fractions.
8. Change improper fractions to mixed numerals.
9. Simplify proper fractions.
10. Add two proper fractions with like denominators, renaming the sum.
11. Subtract two proper fractions with like denominators, renaming the difference.
12. Identify fractional amounts of tenths and hundredths.
13. Add mixed numerals with like and unlike denominators.
14. Multiply a whole number and a proper fraction.
15. Add and subtract proper fractions with unlike denominators, regroupi as necessary.
16. Write in lowest terms when given any fraction or mixed number.
17. Use $>$, $<$, or $=$ correctly with proper fractions.
18. Add, subtract, multiply, and divide any given fractions and/or mixed numbers.

DECIMALS

19. Add decimals through tenths without regrouping.
20. Add decimals through hundredths with and without regrouping.
21. Subtract decimals through hundredths with and without regrouping.

22. Multiply whole numbers by tenths.
23. Divide a decimal number through tenths by a whole number.
24. Add, subtract, multiply, and divide decimals with up to three digits.
25. Read decimals through tenths.
26. Read decimals through hundredths.
27. Identify place value -- tenths to thousandths.

PROCEDURAL ERRORS IN ARITHMETIC

Dr. Bobby Moore

PROCEDURAL ERRORS IN ARITHMETIC

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PROCEDURAL ERRORS IN ARITHMETIC

Overview and Rationale

When teachers of arithmetic in grades three through six are asked why their students have difficulty with various algorithms in the basic operations, many respond that the chief culprit is that students do not know their basic facts. This attitude usually results in excessive drill and practice on the basic facts. It is true that in some cases students do not know their basic facts, and this would certainly inhibit their progress in computation. However, there are other pertinent reasons for students having difficulty in their overall mathematics achievement.

This workshop is designed to explore another area that, according to much research, causes the majority of students having difficulty to "get wrong answers." Throughout this workshop, emphasis will be placed on the analyzation of student procedures, primarily their written wcrk. The workshop is designed to help classroom teachers become more sensitive to students' difficulties in learning mathematics and to give teachers the professional skills they need when working with these students. The participants will be engaged in a careful analysis of systematic errors in computation, errors familiar to most classroom teachers, and therefore quickly becomes aware of the subtlety and importance of such an analysis. The ability to identify the probable conceptual or procedural base for errors will be explicitly promoted, as well as the ability to decide what directions are appropriate for follow-up instruction.

Finally, this workshop should result in altering participating teachers' perspectives, and may even lead teachers to modify some of their long-ingrained teaching practices.

Behavioral Objectives of Workshop Participants

At the conclusion of the workshop, each participant will be able to:

1. describe the sequence of arithmetic from beginning work in numbers to the basic operation of addition.
2. properly sequence the teaching of the four basic operations of addition, subtraction, multiplication, and division of whole numbers.
3. discuss and give examples of errors students make in elementary school arithmetic.
4. devise a diagnostic test of addition, subtraction, multiplication, and division of whole numbers.
5. determine procedural errors made by students on a diagnostic test of addition, subtraction, multiplication, and division of whole numbers.
6. demonstrate the ability to recognize error patterns in students' work.

Workshop Leader Behaviors

1. Introduction: Give overview and rationale.
2. Discuss the role of sequence in mathematics.
3. Sequencing activity in the four operations.
4. Constructing diagnostic tests for the four operations.
5. Discuss the concept of procedural errors and error patterns. Show examples and encourage participants to get involved.
6. Discuss the kinds of errors students make in the four basic operations.
7. Analysis of student work to identify procedural errors and determine if a pattern is present.
8. Allow participants to demonstrate their ability to recognize error patterns.

Preparing for the Workshop

Prepare the following materials for the workshop. (Before you begin preparing materials, read the section on Workshop Content and Activities.)

Transparencies:	A-2 and E-1
Handouts:	A-1, B-1, C-1, D-1, F-1, G-1, G-2, G-3, G-4, EPA-1, EPS-1, EPM-1, and EPD-1.

Content and Activities of the Workshop

A. Introduction: Overview and Rationale (Use A-1 and A-2).

B. The Role of Sequence in Mathematics (Use B-1).

1. Discuss early theories on How Children Learn Math.
   a. The brain is a muscle theory . . . work it.
   b. Stimulus-Response Theory (Edward Thorndike). Many drills were established to give students a stimulus and they were to give a response. This is still done today with flash card drills.
   c. Meaning Theory (William Brownell). Children must be able to understand what they are doing in order for their learning to be permanent.
2. Discuss recent theories on How Children Learn.
   a. Developmental Theory (Jean Piaget). Stages of development through which all children go determine what types of learning will take place and how the learning may best take place.

---Sensori-motor State	(0-2 yrs.)
---Preoperational Stage	(2-7 yrs.)
---Concrete Operational Stage	(7-11 yrs.)
---Formal Operational Stage	(11+ yrs.)

Most children in elementary school are in the age level of 7-11 years and are in the concrete operational stage. This tells us that learning takes place best through the use of concrete objects. This needs to be stressed now and throughout the entire workshop.

b. Structured Learning Theory (Jerome Bruner and Robert Gagné).

They studied the structure of learning and broke learning information down into a series of very small steps with the overall skill or area known as a hierarchy. Teachers should be familiar with the steps in each concept of topic in order to teach their children the proper steps. Every succeeding step in the structure depends upon a child learning the preceding steps. The AIM Program in our schools is based on this concept.

3. Results of research indicate how mathematics should be taught.

   a. Concrete or manipulative mode.
   b. Semi-concrete or pictorial mode.
   c. Abstract or symbol mode.

These three levels and the progression or steps they imply are very important and must be stressed. This indicates to us that we do not begin with concrete objects, progress to the pictorial stage and finally wind up using the symbols. Many teachers are guilty of using the symbols first and then wondering why the students do not really understand what they are attempting to teach.

Use the flow chart (B-1) to review the sequence of getting to addition of basic addition facts. Go over each step in detail.

1.) Sets, set membership - Children need to be able to pick out items that are alike and place them into a group or set. (Note: Sets are usually not discussed in current elementary textbooks, but the idea of grouping is still very important in mathematics.) In picking out items that are alike, children normally will learn to pick out objects that are the same color or objects that are the same size and place them into groups. Classifying or grouping becomes more sophisticated as one moves up the grade level ladder.

2.) One-to-one correspondence - After learning to pick out items that are alike and place them into groups, children must learn to match the items in the set with lines or other objects. This first idea of one-to-one correspondence is very basic but it leads to the more sophisticated level mentioned above.

3.) Equivalent sets - After the child is able to match objects through one-to-one correspondence, the child advances to being able to determine if sets are equal (e.g., have the same number of objects).

4.) Conservation of number - After the child is able to tell whether sets are equivalent or not, then the child should be tested to see if conservation of number is present. Conservation of number deals with whether or not the child can understand that numbers have permanence. The way that this can be tested is to have two groups of objects with the same number of objects and arranged in exactly the same number (e.g., align the groups one above or beside the other). If the child can conserve number, even when the objects in one of the groups have been dispersed differently. If the child says there are more objects in the group that has been scattered, it can be said that the child is not conserving number. He, therefore, needs more work on this concept. When a child can conserve number, the dispersion will not fool him and he is ready to progress to the next step.

5.) Cardinal number of a set - In this step, the child is able to count the number of objects in a set using one-to-one correspondence.

6.) Set union - After learning to count the number of objects in a set, the next step is to learn how to join sets and count the number of objects in the union of the two sets.

7.) Meaning of addition of cardinal numbers - In this step the child is introduced to what he was actually doing in set union, but now he is using the symbols and not just the objects. The objects are counted for reinforcement but are actually counted less and less.

8.) Basic ideas concerning place value - Before the basic addition facts are introduced using concrete objects, the child must be taught some basic concepts about place value. The basic ideas about place value include such ideas that only one numeral can be placed in each digit and each digit (or position) is grouped in multiples of ten.

9.) Basic addition facts - Using concrete objects the child is introduced to the basic facts in one of several ways. The following sequence is suggested: First, easy facts are introduced with the more difficult ones to come later. Second, doubles and near doubles are introduced until all facts are introduced. Third, introduce easy facts and then increase facts by one until all basic facts are covered.

C. Sequencing Activity (Use C - 1).

Go through each of the four basic operations by first having the student number the problems from easiest to most difficult. The second page in each series gives the correct order of the sequence and the final page in the series tells in words what the difference among the problems is. This should prove to spark some interesting discussion among the participants as you go through each operation.

D. Constructing Diagnostic Tests for the Four Operations (Use D - 1).

Using the criteria for designing a good diagnostic test, the participants will now design a test for each operation. To design the test for each operation the participant is asked to go by the description of each step for problems in the sequence. Three problems will be made up for each step of the sequence. The first problem should reflect the simplest problem for that particular step. The second problem should be of medium difficulty and the third problem should be the most difficult representing that particular step of difficulty.

The participants should look carefully at their problems as they are making them up to ensure that all of the basic facts are represented equally and used approximately the same number of times throughout the test. They also need to make sure that each sample problem follows the stated step. An example would be: If the step says "two digits plus two digits without regrouping," the sample problems would be something like those below.

$$\begin{array}{r} 2\ 3 \\ +\ 1\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 3\ 4 \\ +\ 2\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 5\ 7 \\ +\ 4\ 2 \\ \hline \end{array}$$

E. Defining Procedural Errors and Error Patterns (Use E - 1).

Using the transparency, a discussion should be held with the participants about the four types of errors that students make in their computation of problems. The first error type is "basic facts" errors. The student just does not know the basic facts. The second error type is "random" errors where the student puts an answer down but there is no rational reason how he got that answer. The third error type is "using the wrong operation". The fourth error type is the "procedural error or error pattern". This last type error is one that re-occurs each time the child faces a problem of the same type where the error first occurred. With this type error, one can predict exactly what mistake the student will make when working the same type problem. The teacher should be able to quickly recognize the procedure that is faulty and then concentrate instruction in that particular problem area. MORE DRILL OR PRACTICE ON THE BASIC FACTS IS NOT ALWAYS WHAT IS NEEDED. An example of this type error could be illustrated in the work of the problems below.

$$\begin{array}{r} 2\ 5 \\ +\ 3\ 8 \\ \hline 513 \end{array} \qquad \begin{array}{r} 3\ 8 \\ +\ 2\ 7 \\ \hline 515 \end{array}$$

In this situation, the child does not "regroup" at all. Instead, he simply write the whole sum down and moves to the tens and adds them. If the teacher detects this, she can work specifically on this particular problem area.

F. Student Errors in the Four Operation (Use F - 1).

Go over each of the various errors noted for each of the four basic operations and discuss each one. This discussion should help teachers see that we have listed several problem types that they have also seen from time to time with the work of their own students.

G. Analysis of Student Work (Use G - 1, G - 2, G - 3, and G - 4).

Using the first page of each set of papers as the "KEY", the participants can quickly grade the set sample papers. They can then go through the problems missed and identify any procedural errors that might be present. A discussion should be held after all the participants have had a chance to analyze the problems missed. Each participant should be able to easily determine the error pattern in each problem that is missed.

H. Participants Demonstrate Their Ability to Analyze Students' Work and Recognize Procedural Errors (Use EPA - 1, EPS - 1, EPM - 1, EPD - 1).

Each participant should analyze the problems in the exercises and then work the last problems just like the student worked the first four problems (e.g., make the same mistake). Of course, these last problems should be worked incorrectly, using the error pattern demonstrated by the student. When the participants are able to successfully complete this exercise, they should be able to analyze which, if any, procedural errors are present.

Suggestions for Conducting This Workshop in Your Local School District ("How To")

The basic problem in setting up this workshop is to make sure enough time is allowed to thoroughly cover the topic being discussed.

A. The overview and rationale, the role of sequence in mathematics, and the sequencing activity could all be presented in a one-hour session.

B. A discussion of designing diagnostic tests and then making out a diagnostic test for one of the four basic operations could be presented in a one-hour session. If more operations are covered, additional time would be needed.

C. A discussion of errors students make in arithmetic problems and then analyzing the types of procedural errors students make in one of the four basic operations could be presented in a one-hour session. If more operations are covered, additional time would be needed. (After thoroughly covering one operation, inferences could be made to the other operations.)

D. Recognizing and understanding procedural errors made in problems students work could again be accomplished in a one-hour session provided not more than two of the four basic operations were covered at one time.

E. Ultimately, remedial activities could be discussed that would assist students who have been identified as making one or more patterns of procedural errors. This could be presented in a one-hour session provided not more than two of the basic operations were covered.

F. Additional suggestions for follow-up assignments to use in staff development training sessions are:

   1. Identify two or three students experiencing extreme difficulty

with one of the four basic operations. Carry these students individually through the steps of difficulty for that operation to determine the particular step at which to begin to develop necessary prerequisite skills for future success.

2. Administer diagnostic test(s) constructed in session to one or more groups of students. Analyze for procedural errors. Write up findings.

3. Require students to verbalize their procedure of working problems they worked. This could be done either orally (at the chalkboard to the class or individually to the teacher) or written. This would be mostly applicable for upper grades such as grades three through six.

4. Have upper grade students develop problems from written descriptions for lower grade students or peers to work.

5. Have students write why they missed the problems on a test as they are allowed to correct the ones they missed. This allows students to analyze their own work and directs teachers to lack of understanding of procedures when students do not know why they missed a certain problem.

6. Upon application of follow-up activities, share findings with fellow teachers in next session. This has been found to be most helpful. The "give and take" of ideas and problems encountered when trying to apply material and concepts learned is most beneficial.

G. Suggestions from workshop participants for additional follow-up assignments or activities may be added to the spaces that follow.

1.

2.

3.

4.

5.

RESOURCES

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PROCEDURAL ERRORS IN ARITHMETIC

(MACTE MATH PROJECT)

Overview and Rationale

When teachers of arithmetic in grades three through six are asked why their students have difficulty with various algorithms in the basic operations, many respond that the chief culprit is that students do not know their basic facts. This attitude usually results in excessive drill and practice on the basic facts. It is true that in some cases students do not know their basic facts, and this certainly inhibits their progress in computation. However, there are other pertinent reasons for students having difficulty in their overall mathematics achievement.

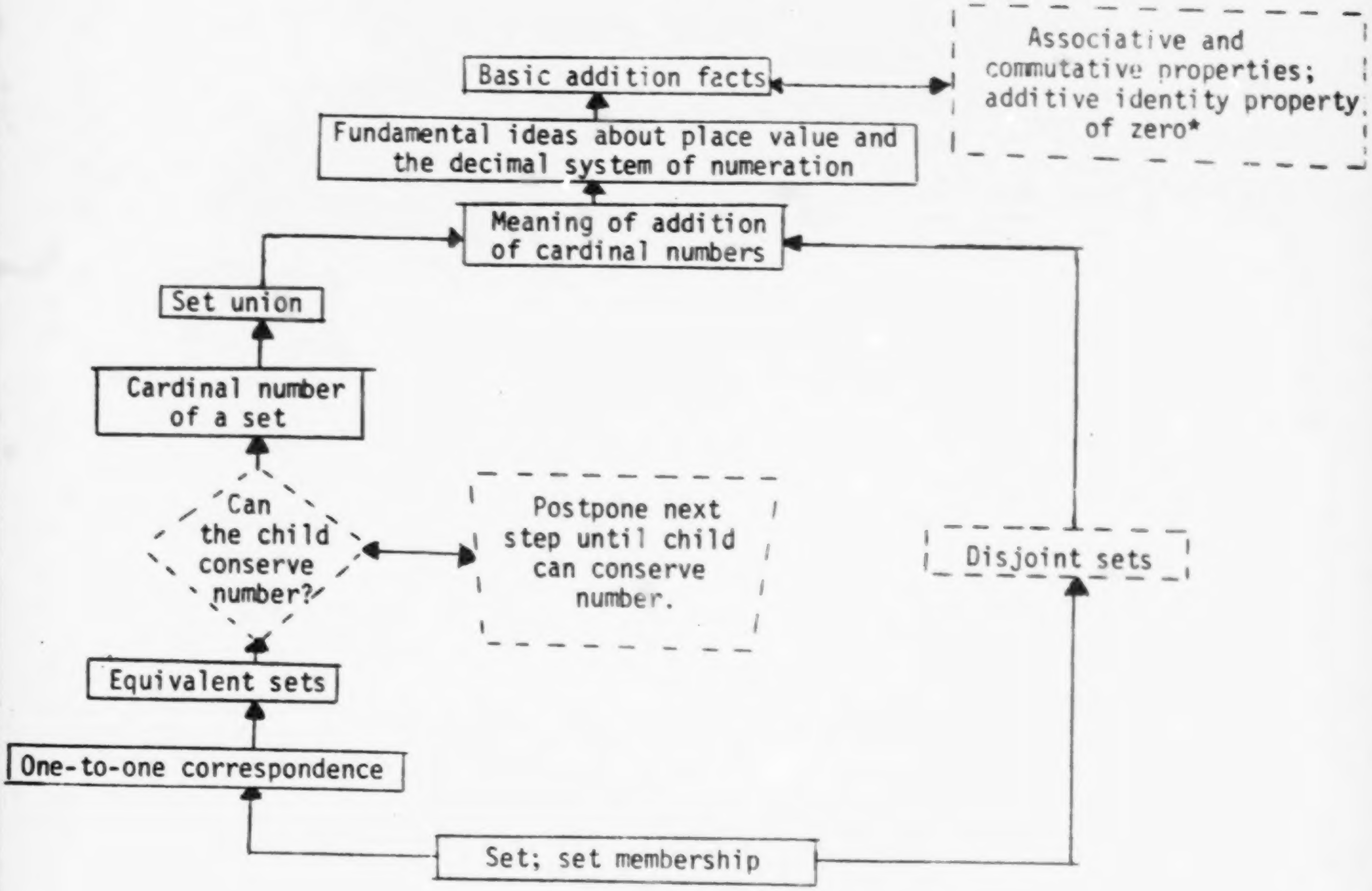
This workshop is designed to explore another area that, according to much research, causes the majority of students having difficulty to "get wrong answers." Throughout this workshop, emphasis will be placed on the analyzation of student procedures, primarily their written work. The workshop is designed to help classroom teachers become more sensitive to students' difficulties in learning mathematics and to give teachers the professional skills they need when working with these students. The participants will be engaged in a careful analysis of systematic errors in computation, errors familiar to most classroom teachers, and therefore quickly become aware of the subtlety and importance of such an analysis. The ability to identify the probable conceptual or procedural base for errors will be explicitly promoted, as well as the ability to decide what directions are appropriate for follow-up instruction.

Finally, this workshop should result in altering participating teachers' perspectives, and may even lead teachers to modify some of their long-ingrained teaching practices.

PARTICIPANT OBJECTIVES

AT THE CONCLUSION OF THIS WORKSHOP, EACH PARTICIPANT WILL BE ABLE TO:

1. DESCRIBE THE SEQUENCE OF ARITHMETIC FROM BEGINNING WORK IN NUMBERS TO THE BASIC OPERATION OF ADDITION.
2. PROPERLY SEQUENCE THE TEACHING OF THE FOUR BASIC OPERATIONS OF ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF WHOLE NUMBERS.
3. DISCUSS AND GIVE EXAMPLES OF ERRORS STUDENTS MAKE IN ELEMENTARY SCHOOL ARITHMETIC.
4. DEVISE A DIAGNOSTIC TEST OF ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF WHOLE NUMBERS.
5. DETERMINE PROCEDURAL ERRORS MADE BY STUDENTS ON A DIAGNOSTIC TEST OF ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF WHOLE NUMBERS.
6. DEMONSTRATE THE ABILITY TO RECOGNIZE ERROR PATTERNS IN STUDENTS' WRITTEN WORK.


Basic addition facts
Associative and commutative properties; additive identity property of zero*
Fundamental ideas about place value and the decimal system of numeration
Meaning of addition of cardinal numbers
Set union
Cardinal number of a set
Can the child conserve number?
Postpone next step until child can conserve number.
Disjoint sets
Equivalent sets
One-to-one correspondence
Set; set membership

STEPS OF DIFFICULTY IN ADDITION

1. The meaning of addition.

2. Sums to 10.

3. Single column with sums of 10 or less.

$$\begin{array}{ccc} 3 & 4 & 2 \\ 2 & 1 & 5 \\ \underline{3} & \underline{3} & \underline{2} \end{array}$$

4. Two-place numbers without carrying.

$$\begin{array}{cc} 43 & 52 \\ \underline{36} & \underline{55} \end{array}$$

5. Higher decade addition without bridging.

$$\begin{array}{rr} 5 + 1 = & 3 + 2 = \\ 15 + 2 = & 13 + 2 = \\ 25 + 3 = & 23 + 3 = \end{array}$$

6. Sums to 18.

7. Single column addition, bridging 10, but only basic facts involved.

$$\begin{array}{ccc} 6 & 5 & 7 \\ 3 & 3 & 2 \\ \underline{7} & \underline{8} & \underline{6} \end{array}$$

Higher decade addition with bridging.

$$\begin{array}{rr} 9 + 4 = & 8 + 5 = \\ 19 + 4 = & 28 + 5 = \\ 29 + 4 = & 48 + 5 = \end{array}$$

9. Single column, higher decade, without bridging.

$$\begin{array}{ccc} 7 & 5 & 6 \\ 8 & 7 & 9 \\ \underline{2} & \underline{4} & \underline{3} \end{array}$$

10. Adding 2 and 3-place numbers without carrying.

$$\begin{array}{ccc} 26 & 62 & 263 \\ \underline{33} & \underline{45} & \underline{124} \end{array}$$

11. Adding 2 and 3-place numbers with carrying.

a.) carry 10's

$$\begin{array}{cc} 26 & 267 \\ \underline{48} & \underline{127} \end{array}$$

b.) carry 100's

$$\begin{array}{cc} 254 & 463 \\ \underline{153} & \underline{281} \end{array}$$

c. carry 10's & 100's

$$\begin{array}{cc} 463 & 768 \\ \underline{159} & \underline{295} \end{array}$$

12. Single column, higher decade, with bridging.

$$\begin{array}{cc} 6 & 9 \\ 9 & 7 \\ \underline{8} & \underline{6} \end{array}$$

13, Carrying in Multiplication.

$$\begin{array}{rr} 57 & 87 \\ \underline{\times\ 8} & \underline{\times\ 6} \end{array}$$

STEPS OF DIFFICULTY IN SUBTRACTION

1. The meaning of subtraction.

2. Minuends of 10.

3. Two-place numbers without borrowing (or regrouping).

$$\begin{array}{r} 46 \\ -11 \\ \hline \end{array} \qquad \begin{array}{r} 54 \\ -42 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ -52 \\ \hline \end{array}$$

4. Minuends to 18.

5. Two and three-place numbers without borrowing.

$$\begin{array}{r} 109 \\ -54 \\ \hline \end{array} \quad \begin{array}{r} 124 \\ -63 \\ \hline \end{array} \quad \begin{array}{r} 138 \\ -78 \\ \hline \end{array} \quad \begin{array}{r} 463 \\ -123 \\ \hline \end{array} \quad \begin{array}{r} 783 \\ -721 \\ \hline \end{array} \quad \begin{array}{r} 47 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} 467 \\ -67 \\ \hline \end{array}$$

6. Two and three-place numbers with borrowing.

a.) borrow 10's

$$\begin{array}{r} 43 \\ -16 \\ \hline \end{array} \quad \begin{array}{r} 463 \\ -128 \\ \hline \end{array}$$

b.) borrow 100's

$$\begin{array}{r} 418 \\ -265 \\ \hline \end{array} \quad \begin{array}{r} 823 \\ -71 \\ \hline \end{array}$$

c.) borrow 10's & 100's

$$\begin{array}{r} 643 \\ -268 \\ \hline \end{array} \quad \begin{array}{r} 687 \\ -99 \\ \hline \end{array}$$

7. Zeros in the minuend.

$$\begin{array}{r} 460 \\ -279 \\ \hline \end{array} \quad \begin{array}{r} 500 \\ -368 \\ \hline \end{array} \quad \begin{array}{r} 704 \\ -268 \\ \hline \end{array} \quad \begin{array}{r} 9001 \\ -2648 \\ \hline \end{array} \quad \begin{array}{r} 8060 \\ -7483 \\ \hline \end{array}$$

STEPS OF DIFFICULTY IN MULTIPLICATION

1. Using such terms as two times, three times, two fours, three fives, twice as much, etc.

2. Counting by 10's, 5's, and 2's.

3. Learning that in the arrangement of objects, "3 twos make 6," "2 fours make 8," etc.

4. The meaning of multiplication.

5. Facts with products to 36.

6. Two-place multiplicands, no carrying.

   61 42
   x4 x3

7. Three-place multiplicands, no carrying.

   321 301
   x 3 x 4

8. Two and three-place multiplicands, with carrying.

   23 234
   x4 x 5

9. Facts with products to 81.

10. Two and three-place multiplicands with carrying.

   643 87
   x 8 x6

11. Two-place multipliers.

   20 24 24 113 172 800 605 $9.00
   x10 x30 x12 x43 x43 x25 xf0 x 30

STEPS OF DIFFICULTY IN DIVISION

1. Learning such number relationships as two 5's in 10, 1/2 of 6 is 3, etc.

2. The meaning of division.

3. Facts with dividents to 36.

4. One-place divisors, with and without remainders (measurement division)

   $4\overline{)12}$ $5\overline{)13}$

5. One-place divisors, 2 and 3-place dividends, no remainders, no carrying (partition division).

   $2\overline{)64}$ $3\overline{)159}$ $3\overline{)600}$

6. Facts with dividends to 81.

7. One-place divisors, 2 and 3-place dividends.

   a. with carrying, but no remainders

   $4\overline{)96}$ $5\overline{)175}$

   b. with carrying and with remainders

   $6\overline{)95}$ $8\overline{)117}$

   c. with zeros in the quotient

   1.) at the end

   $3\overline{)60}$ $8\overline{)880}$

   2.) in the middle

   $8\overline{)808}$ $9\overline{)937}$

8. Two-place divisors

   a. the apparent method:
      - -estimated quotient need not be corrected
      - -estimated quotient must be corrected

   b. increase-by-one method
      - -divisors ending in 0, 1, 2, 3, 4
      - -divisors ending in 5, 6, 7, 8, 9

C-1

ADDITION

Number each of the following (1-19) from least difficult to most difficult.

60	43	20	7	98
+74	+26	+6	+2	+76
653	40	56	8	15
+874	+30	+7	+4	+36
435	25	50	749	42
+263	+1	+80	+896	+6
75	836	43	27	
+62	+542	+7	+40	

ADDITION

1. $\begin{array}{r} 7 \\ +2 \\ \hline \end{array}$

2. $\begin{array}{r} 8 \\ +4 \\ \hline \end{array}$

3. $\begin{array}{r} 40 \\ +30 \\ \hline \end{array}$

4. $\begin{array}{r} 43 \\ +26 \\ \hline \end{array}$

5. $\begin{array}{r} 435 \\ +263 \\ \hline \end{array}$

6. $\begin{array}{r} 25 \\ +\ 1 \\ \hline \end{array}$

7. $\begin{array}{r} 42 \\ +\ 6 \\ \hline \end{array}$

8. $\begin{array}{r} 27 \\ +40 \\ \hline \end{array}$

9. $\begin{array}{r} 20 \\ +\ 6 \\ \hline \end{array}$

10. $\begin{array}{r} 50 \\ +80 \\ \hline \end{array}$

11. $\begin{array}{r} 75 \\ +62 \\ \hline \end{array}$

12. $\begin{array}{r} 60 \\ +74 \\ \hline \end{array}$

13. $\begin{array}{r} 43 \\ +\ 7 \\ \hline \end{array}$

14. $\begin{array}{r} 56 \\ +\ 7 \\ \hline \end{array}$

15. $\begin{array}{r} 15 \\ +36 \\ \hline \end{array}$

16. $\begin{array}{r} 98 \\ +76 \\ \hline \end{array}$

17. $\begin{array}{r} 836 \\ +542 \\ \hline \end{array}$

18. $\begin{array}{r} 653 \\ +874 \\ \hline \end{array}$

19. $\begin{array}{r} 749 \\ +896 \\ \hline \end{array}$

SUGGESTED ADDITION SEQUENCE

1. Basic facts with sums less than 11. (7 + 2)
2. Basic facts with sums greater than 10. (8 + 4)
3. Tens plus tens without regrouping. (40 + 30)
4. Two digits plus two digits without regrouping. (43 + 26)
5. Addition with three digit numbers and no regrouping. (435 + 263)
6. Two digits plus one more. (25 + 1)
7. Two digits plus one digit without regrouping. (42 + 6)
8. Tens plus two digits without regrouping. (27 + 40)
9. Tens plus one digit without regrouping. (20 + 6)
10. Tens plus tens with regrouping in tens place. (50 + 80)
11. Tens plus two digits with regrouping in tens place. (75 + 62)
12. Two digits plus two digits with regrouping in tens place. (60 + 74)
13. Two digits plus one digit with regrouping to tens. (43 + 7)
14. Two digits plus one digit with regrouping in ones place. (56 + 7)
15. Two digits plus two digits with regrouping in ones place. (15 + 36)
16. Two digits plus two digits with regrouping in ones and tens places. (98 + 76)
17. Addition of three digits with regrouping in the hundreds place. (836 + 542'
18. Addition of three digits with regrouping in the hundreds and tens places. (653 + 874)
19. Addition of three digits with regrouping in ones, tens, and hundreds places. (749 + 896)

SUBTRACTION

Number each of the following (1-16) from least difficult to most difficult.

$\begin{array}{r}74\\-51\\\hline\end{array}$	$\begin{array}{r}400\\-20\\\hline\end{array}$	$\begin{array}{r}8\\-5\\\hline\end{array}$	$\begin{array}{r}66\\-38\\\hline\end{array}$
$\begin{array}{r}701\\-95\\\hline\end{array}$	$\begin{array}{r}50\\-40\\\hline\end{array}$	$\begin{array}{r}80\\-9\\\hline\end{array}$	$\begin{array}{r}184\\-27\\\hline\end{array}$
$\begin{array}{r}14\\-6\\\hline\end{array}$	$\begin{array}{r}31\\-7\\\hline\end{array}$	$\begin{array}{r}932\\-87\\\hline\end{array}$	$\begin{array}{r}48\\-4\\\hline\end{array}$
$\begin{array}{r}500\\-46\\\hline\end{array}$	$\begin{array}{r}67\\-30\\\hline\end{array}$	$\begin{array}{r}246\\-85\\\hline\end{array}$	$\begin{array}{r}746\\-31\\\hline\end{array}$

SUBTRACTION

1. $\begin{array}{r} 8 \\ -5 \\ \hline \end{array}$

2. $\begin{array}{r} 14 \\ -6 \\ \hline \end{array}$

3. $\begin{array}{r} 50 \\ -40 \\ \hline \end{array}$

4. $\begin{array}{r} 74 \\ -51 \\ \hline \end{array}$

5. $\begin{array}{r} 67 \\ -30 \\ \hline \end{array}$

6. $\begin{array}{r} 48 \\ -4 \\ \hline \end{array}$

7. $\begin{array}{r} 746 \\ -31 \\ \hline \end{array}$

8. $\begin{array}{r} 31 \\ -7 \\ \hline \end{array}$

9. $\begin{array}{r} 66 \\ -38 \\ \hline \end{array}$

10. $\begin{array}{r} 80 \\ -9 \\ \hline \end{array}$

11. $\begin{array}{r} 184 \\ -27 \\ \hline \end{array}$

12. $\begin{array}{r} 246 \\ -85 \\ \hline \end{array}$

13. $\begin{array}{r} 400 \\ -20 \\ \hline \end{array}$

14. $\begin{array}{r} 932 \\ -87 \\ \hline \end{array}$

15. $\begin{array}{r} 500 \\ -46 \\ \hline \end{array}$

16. $\begin{array}{r} 701 \\ -95 \\ \hline \end{array}$

SUGGESTED SUBTRACTION SEQUENCE

1.	Basic facts with factors less than 11.	(8-5)
2.	Basic facts with one factor greater than 10.	(14 - 6)
3.	Tens minus tens without regrouping.	(50 - 40)
4.	Two digits minus two digits without regrouping.	(74 - 51)
5.	Two digits minus tens.	(67 - 30)
6.	Two digits minus one digit without regrouping.	(48 - 4)
7.	Three digits minus two digits without regrouping.	(746 - 31)
8.	Two digits minus one digit with regrouping.	(31 - 7)
9.	Two digits minus two digits with regrouping	(66 - 38)
10.	Tens minus one digit.	(80 - 9)
11.	Three digits minus two digits with regrouping from tens.	(184 - 27)
12.	Three digits minus two digits with regrouping from hundreds to tens.	(246 - 85)
13.	Hundreds minus tens.	(400 - 20)
14.	Three digits minus two digits with regrouping from hundreds to ones.	(932 - 87)
15.	Hundreds minus two digits.	(500 - 46)
16.	Three digits with zero in tens place minus two digits with regrouping.	(701 - 95)

MULTIPLICATION

Number each of the following problems (1-18) from least difficult to most difficult.

5 x 60
30 x 12
17 x 12
2 x 3

87 x 8
56 x 87
7 x 8
3 x 28

76 x 60
6 x 4
5 x 0
31 x 5

10 x 9
14 x 80
43 x 2
70 x 50

36 x 11
70 x 10

MULTIPLICATION

1. $\begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$
2. $\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$
3. $\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$
4. $\begin{array}{r} 5 \\ \times 0 \\ \hline \end{array}$
5. $\begin{array}{r} 10 \\ \times 9 \\ \hline \end{array}$
6. $\begin{array}{r} 43 \\ \times 2 \\ \hline \end{array}$
7. $\begin{array}{r} 31 \\ \times 5 \\ \hline \end{array}$
8. $\begin{array}{r} 60 \\ \times 5 \\ \hline \end{array}$
9. $\begin{array}{r} 28 \\ \times 3 \\ \hline \end{array}$
10. $\begin{array}{r} 87 \\ \times 8 \\ \hline \end{array}$
11. $\begin{array}{r} 70 \\ \times 10 \\ \hline \end{array}$
12. $\begin{array}{r} 70 \\ \times 50 \\ \hline \end{array}$
13. $\begin{array}{r} 30 \\ \times 12 \\ \hline \end{array}$
14. $\begin{array}{r} 14 \\ \times 80 \\ \hline \end{array}$
15. $\begin{array}{r} 36 \\ \times 11 \\ \hline \end{array}$
16. $\begin{array}{r} 17 \\ \times 12 \\ \hline \end{array}$
17. $\begin{array}{r} 76 \\ \times 60 \\ \hline \end{array}$
18. $\begin{array}{r} 56 \\ \times 87 \\ \hline \end{array}$

SUGGESTED MULTIPLICATION SEQUENCE

1.	Basic facts- 1,2,3, tables	(2 x 3)
2.	Basic facts- 4,5,6 tables	(6 x 4)
3.	Basic facts- 7,8,9 tables	(7 x 8)
4.	Basic facts with 0 as a factor	(5 x 0)
5.	One digit x tens without regrouping	(10 x 9)
6.	One digit x two digits without regrouping	(43 x 2)
7.	Tens x tens without regrouping	(70 x 10)
8.	One digit x ten with regrouping	(5 x 60)
9.	One digit x two digits with regrouping in tens digit	(31 x 5)
10.	One digit x two digits with regrouping in units	(3 x 28)
11.	One digit x two digits with two regroupings	(87 x 8)
12.	Tens x tens with regrouping	(70 x 50)
13.	Tens x two digits without regrouping	(30 x 12)
14.	Two digits x two digits without regrouping	(36 x 11)
15.	Tens x two digits with one regrouping	(14 x 80)
16.	Two digits x two digits with one regrouping	(17 x 12)
17.	Tens x two digits with two regroupings	(76 x 60)
18.	Two digits x two digits with multiple regrouping	(56 x 87)

DIVISION

Number each of the following (1-17) from least difficult to most difficult.

57 ÷ 3

72 ÷ 5

16 ÷ 2

70 ÷ 35

80 ÷ 4

88 ÷ 44

32 ÷ 4

46 ÷ 6

40 ÷ 32

24 ÷ 8

50 ÷ 4

90 ÷ 60

84 ÷ 4

37 ÷ 18

90 ÷ 10

60 ÷ 4

50 ÷ 6

DIVISION

1. $16 \div 2$
2. $32 \div 4$
3. $24 \div 8$
4. $84 \div 4$
5. $80 \div 4$
6. $57 \div 3$
7. $60 \div 4$
8. $46 \div 6$
9. $50 \div 6$
10. $72 \div 5$
11. $50 \div 4$
12. $90 \div 10$
13. $90 \div 60$
14. $70 \div 35$
15. $88 \div 44$
16. $40 \div 32$
17. $37 \div 18$

SUGGESTED DIVISION SEQUENCE

1. Basic facts with divisors of 1, 2, 3 (16 ÷ 2)
2. Basic facts with divisors of 4, 5, 6 (32 ÷ 4)
3. Basic facts with divisors of 7, 8, 9 (24 ÷ 8)
4. A two digit divided by a one digit without regrouping and without a remainder (84 ÷ 4)
5. A ten divided by a one digit without regrouping and without a remainder (80 ÷ 4)
6. A two digit divided by a one digit with regrouping and without a remainder (57 ÷ 3)
7. A ten divided by a one digit with regrouping and without a remainder (60 ÷ 4)
8. A two digit divided by a one digit without regrouping and with a remainder (46 ÷ 6)
9. A ten divided by a one digit without regrouping and with a remainder (50 ÷ 6)
10. A two digit divided by a one digit with regrouping and with a remainder (72 ÷ 5)
11. A ten divided by a one digit with regrouping and with a remainder (50 ÷ 4)
12. A ten divided by a ten without a remainder. (90 ÷ 10)
13. A ten divided by a ten with a remainder (90 ÷ 60)
14. A ten divided by a two digit without regrouping and without a remainder (70 ÷ 35)
15. A two digit divided by a two digit without regrouping and without a remainder (88÷ 44)
16. A ten divided by a two digit without regrouping and with a remainder (40 ÷ 32)
17. A two digit divided by a two digit without regrouping and with a remainder (37 ÷ 18)

CRITERIA FOR TEST DESIGN

1. Each problem type in the sequence of the operation tested should contain three problems--(With only one or two problems per problem type, the amount of data gathered will be insufficient to insure correct placement along the continuum.)
2. The items in each step must be arranged in order of difficulty, with the first problem being the easiest.
3. All basic combinations should be used at least once and no more than four times, if possible. This assures that a child's lack of mastery of any particular fact will not adversely affect his/her progression through the steps.
4. Problems must be designed so as to negate the possibility of a child making an error and still achieving the correct answer.

   Example:

   15 The child could regroup the wrong digit and still get
   +6 the correct answer.
5. Items must be designed in such a way as to prevent overlapping of incorrect responses as much as possible.

   Example:

   43 (a) Did the child regroup from tens to ones without
   -18 changing the tens digit?
   35 (b) Did the child subtract the smaller digit from the larger digit instead of regrouping?

TEST CONSTRUCTION -- Worksheet

	Description	Item 1	Item 2	Item 3
1.	Basic facts with sums less than 11.			
2.	Basic facts with sums greater than 10.			
3.	Tens plus tens without regrouping.			
4.	Two digits plus two digits without regrouping.			
5.	Addition with three digit numbers and no regrouping.			
6.	Two digits plus one more.			
7.	Two digits plus one digit without regrouping.			
8.	Tens plus two digits without regrouping.			
9.	Tens plus one digit without regrouping.			
10.	Tens plus tens with regrouping in tens place.			
11.	Tens plus two digits with regrouping in tens place.			
12.	Two digits plus two digits with regrouping in tens place.			
13.	Two digits plus one digit with regrouping to tens.			
14.	Two digits plus one digit with regrouping in ones place.			
15.	Two digits plus two digits with regrouping in ones place.			
16.	Two digits plus two digits with regrouping in ones and tens places.			
17.	Addition of three digits with regrouping in the hundreds place.			
18.	Addition of three digits with regrouping in the hundreds and tens places.			
19.	Addition of three digits with regrouping in ones, tens, and hundreds places.			

TEST CONSTRUCTION -- WORKSHEET

Description	Item 1	Item 2	Item 3
1. Basic facts with factors less than 11.			
2. Basic facts with one factor greater than 10.			
3. Tens minus tens without regrouping.			
4. Two digits minus two digits without regrouping.			
5. Two digits minus tens.			
6. Two digits minus one digit without regrouping.			
7. Three digits minus two digits without regrouping.			
8. Two digits minus one digit with regrouping.			
9. Two digits minus two digits with regrouping.			
10. Tens minus one digit.			
11. Three digits minus two digits with regrouping from tens to ones.			
12. Three digits minus two digits with regrouping from hundreds to tens.			
13. Hundreds minus tens.			
14. Three digits minus two digits with regrouping from hundreds and tens.			
15. Hundreds minus two digits.			
16. Three digits with zero in tens place minus two digits with regrouping.			

TEST CONSTRUCTION WORKSHEET

Description	Item 1	Item 2	Item 3
1. Basic facts - 1, 2, 3 tables			
2. Basic facts - 4, 5, 6 tables			
3. Basic facts - 7, 8, 9 tables			
4. Basic facts with 0 as a factor			
5. One digit x tens without regrouping			
6. One digit x two digits without regrouping			
7. One digit x two digits with regrouping in tens digit			
8. One digit x ten with regrouping			
9. One digit x two digits with regrouping in units			
10. One digit x two digits with two regroupings			
11. Tens x tens without regrouping			
12. Tens x tens with regrouping			
13. Two digits x tens without regrouping			
14. Tens x two digits with one regrouping			
15. Two digits x two digits without regrouping			
16. Two digits x two digits with one regrouping			
17. Tens x two digits with two regroupings			
18. Two digits x two digits with multiple regrouping			

TEST CONSTRUCTION -- WORKSHEET

Description	Item 1	Item 2	Item 3
1. Basic facts with divisors of 1, 2, 3			
2. Basic facts with divisors of 4, 5, 6			
3. Basic facts with divisors of 7, 8, 9			
4. A two digit divided by a one digit without regrouping and without a remainder			
5. A ten divided by a one digit without regrouping and without a remainder			
6. A two digit divided by a one digit with regrouping and without a remainder			
7. A ten divided by a one digit with regrouping and without a remainder			
8. A two digit divided by a one digit without regrouping and with a remainder			
9. A ten divided by a one digit without regrouping and with a remainder			
10. A two digit divided by one digit with regrouping and with a remainder			
11. A ten divided by a one digit with regrouping and with a remainder			
12. A ten divided by a ten without a remainder			
13. A ten divided by a ten with a remainder			
14. A ten divided by a two digit without regrouping and without a remainder			
15. A two digit divided by a two digit without regrouping and without a remainder			
16. A ten divided by a two digit without regrouping and with a remainder			
17. A two digit divided by a two digit without regrouping and with a remainder			

CATEGORIES OF ERROR
PATTERNS IN COMPUTATION

THE ERRORS IN COMPUTATION ARE NOT NECESSARILY THE RESULT OF CARELESSNESS OR NOT KNOWING HOW TO PROCEED. THE MAJORITY OF ERRORS STUDENTS MAKE CAN BE CATEGORIZED IN ONE OF FOUR GROUPS OR CATEGORIES. THEY ARE:

1. WRONG OPERATION. THE STUDENT ATTEMPTS TO RESPOND BY PERFORMING AN OPERATION OTHER THAN THE ONE THAT IS REQUIRED TO SOLVE THE PROBLEM.

2. OBVIOUS COMPUTATIONAL ERROR. THE PUPIL APPLIES THE CORRECT OPERATION, BUT HIS RESPONSE IS BASED ON ERROR IN RECALLING BASIC NUMBER FACTS.

3. DEFECTIVE ALGORITHM. THE PUPIL ATTEMPTS TO APPLY THE CORRECT OPERATION BUT MAKES ERRORS OTHER THAN NUMBER FACT ERRORS IN CARRYING THROUGH THE NECESSARY STEPS.

4. RANDOM RESPONSE. THE RESPONSE SHOWS NO DISCERNIBLE RELATIONSHIP TO THE GIVEN PROBLEM.

F-1

Addition Error Pattern Data

117 - 4th grade students who took the Addition Test.

80 - 4th grade students who showed no evidence of error patterns.

37 - 4th grade students who showed evidence of error patterns.

Common Error Patterns in Addition

1.	Student did not write the final regrouped digit.	65 +92 57	759 792 551
2.	Student did not carry-wrote entire digit in each column.	38 93 1211	989 589 141618
3.	Student did not carry-wrote units digit each time until reaching left-most column.	876 549 1315	759 792 1441
4.	Student added zero plus any number and always got zero for an answer.	60 + 7 60	70 +18 80
5.	Student did not carry in the middle digit.	11 864 748 1602	11 587 497 1074
6.	Student added all numbers together when he had a two digit plus a one digit problem.	26 + 5 13	58 + 8 21
7.	Student carried all numbers to left-most column and then added all to this column.	1 1 487 865 1442	1 1 587 497 1174
8.	Student wrote the tens digit and carried the tens digit in a one digit plus a two digit problem.	1 63 + 7 71	1 58 5 61
9.	Student wrote the tens digit and carried the units digit each time.	65 876 +549 1911	14 587 497 1021
10.	Student wrote the sum of only the ones digit in a two digit plus a one digit problem.	63 + 7 10	26 + 5 11

Subtraction Error Pattern Data

103 - 4th grade students who took the subtraction test.

20 - 4th grade students who showed no evidence of error patterns.

83 - 4th grade students who showed evidence of one, two or three subtraction error patterns.

Common Error Patterns in Subtraction

1.	Student did not grasp the concept of borrowing when a zero is present in the minuend.	70 - 8 78	30 -17 27	
2.	Student subtracted zero from any number or any number from zero and always got zero as the answer.	840 397 550	900 -543 400	
3.	Student did not borrow in the final digit.	67 - 9 68	760 -482 378	
4.	Student did not borrow in the middle digit.	790 263 537	694 396 308	
5.	Student subtracted any number from zero and the answer was always the number.	50 -24 34	500 357 257	
6.	Student always subtracted smaller number from the larger number no matter where the smaller number was located.	927 578 451	41 -27 26	83 -58 35
7.	Student did not understand borrowing when a zero was present in the minuend.	511 ~~6~~04 -387 227	411 ~~5~~00 -357 153	
8.	Student did not borrow in any digit.	694 -396 308	902 -749 263	
9.	Student subtracted from both digits in the minuend in a two digit minus a one digit problem.	46 - 4 02	57 - 3 24	25 - 5 30
10.	Student borrowed when not necessary.	46 - 4 32	57 - 3 44	376 -200 076

11. Student borrowed twice from the left most digit in the minuend.

$$\begin{array}{r} 91 \\ 500 \\ \underline{-357} \\ 043 \end{array} \qquad \begin{array}{r} 604 \\ \underline{-387} \\ 127 \end{array} \qquad \begin{array}{r} 902 \\ \underline{-749} \\ 063 \end{array}$$

12. Student increased the number borrowed from by one each time.

$$\begin{array}{r} 51 \\ \cancel{6}1 \\ \underline{-27} \\ 34 \end{array} \qquad \begin{array}{r} 7814 \\ \cancel{6}\cancel{9}\cancel{4} \\ \underline{-396} \\ 418 \end{array}$$

13. Student changed all zeroes to nine when borrowing.

$$\begin{array}{r} 49 \\ \cancel{5}\cancel{0} \\ \underline{-24} \\ 25 \end{array} \qquad \begin{array}{r} 499 \\ \cancel{5}\cancel{0}\cancel{0} \\ \underline{-357} \\ 142 \end{array}$$

14. Student always changed each regrouped number to ten.

$$\begin{array}{r} 310 \\ \cancel{4}\cancel{1} \\ \underline{-27} \\ 13 \end{array} \qquad \begin{array}{r} 10 \\ 69\cancel{4} \\ \underline{-396} \\ 304 \end{array}$$

Multiplication Error Pattern Data

141 students took the test.

73 showed error patterns.

68 showed no error patterns.

Common Error Patterns in Multiplication

1. The student does not move over the correct number of places in the product of a 3-digit times a 3-digit problem.

$$\begin{array}{r} 401\\ \text{x}512\\ \hline 802\\ 1401\\ 2005\\ \hline 3208 \end{array} \qquad \begin{array}{r} 675\\ \text{x}587\\ \hline 4725\\ 5100\\ 3355\\ \hline 3969725 \end{array} \qquad \begin{array}{r} 978\\ \text{x}849\\ \hline 8812\\ 3912\\ 7824\\ \hline 7871932 \end{array}$$

2. Zero in the middle digit-zero times a number was the number plus amount carried.

$$\begin{array}{r} 306\\ \text{x}\ \ 5\\ \hline 1580 \end{array} \qquad \begin{array}{r} 408\\ \text{x}\ \ 7\\ \hline 2926 \end{array}$$

3. The student multiplies ones by ones, tens by tens, and hundreds by hundreds in two- and three digit problems.

$$\begin{array}{r} 16\\ \text{x}35\\ \hline 330 \end{array} \qquad \begin{array}{r} 232\\ \text{x}323\\ \hline 666 \end{array}$$

4. In a three-digit with a zero in the middle digit, the student multiplies by the carried number.

$$\begin{array}{r} 306\\ \text{x}\ \ 5\\ \hline 1650 \end{array} \qquad \begin{array}{r} 408\\ \text{x}\ \ 7\\ \hline 3156 \end{array}$$

5. In two digits times two digits the student writes whole product when multiplying tens times ones in multiplier.

$$\begin{array}{r} 16\\ \text{x}35\\ \hline 80\\ 318\\ \hline 3260 \end{array} \qquad \begin{array}{r} 14\\ \text{x}67\\ \hline 98\\ 624\\ \hline 6338 \end{array} \qquad \begin{array}{r} 19\\ \text{x}42\\ \hline 38\\ 436\\ \hline 4398 \end{array}$$

6. The student did not carry in a three digit number with a zero in the middle digit--(answer was always zero).

$$\begin{array}{r} 306\\ \text{x}\ \ 5\\ \hline 1500 \end{array} \qquad \begin{array}{r} 408\\ \text{x}\ \ 7\\ \hline 2806 \end{array}$$

7. Multiplied in finding final answer rather than adding in two digits times two digits.

$$\begin{array}{r} 84\\ \text{x}21\\ \hline 84\\ 168\\ \hline 2244 \end{array} \qquad \begin{array}{r} 16\\ \text{x}35\\ \hline 80\\ 48\\ \hline 1040 \end{array} \qquad \begin{array}{r} 14\\ \text{x}67\\ \hline 98\\ 84\\ \hline 1168 \end{array} \qquad \begin{array}{r} 84\\ \text{x}21\\ \hline 84\\ 168\\ \hline 4664 \end{array}$$

8.	Tens multiplied by tens, student did not multiply through by tens digits.	50 x80	70 x90	60 x90	
9.	Adds all carried numbers in a two digit times a two digit	16 x35 80 78 860	14 x67 98 104 1138	19 x42 38 86 898	
10.	Two digits times a one digit student wrote whole product when multiplying ones by ones.	35 x 2 610	28 x 3 624	19 x 4 436	
11.	Two digits times one digit student brought down tens digit and multiplied.	34 x 2 368	40 x 3 420	21 x 5 205	
12.	One digits times two and three digits the student wrote the whole product rather than carrying.	306 x 5 15030	97 x 6 5442	809 x 9 72081	
13.	Student carries number adds it to tens digit then uses the sum of the two in multiplying in the tens place.	35 x 2 80	28 x 3 124	19 x 4 166	47 x 5 355
14.	Student mixed up order of multiplying in a two digit times a two digit.	21 x14 24 21 234	13 x32 36 39 426	42 x12 44 42 464	
15.	In a 3 digit times a 3 digit student multiplies by only one and tens digit.	401 x512 802 4010 4812	401 x502 802 0000 0802	302 x412 604 3020 3624	
16.	Adds all carried numbers in a two digit times a two digit.	16 x35 80 78 860	14 x67 98 104 1138	19 x42 38 86 898	
17.	Misplaces second digit in a two digit times a two digit.	47 x53 141 5 23 23191	64 x87 448 2 51 51468	19 x42 38 6 7 798	

Division Error Patterns Data

Most Common Error Patterns

1. Student divided by the smaller number each time.

$$\begin{array}{r} 63 \\ 6\overline{)362} \end{array} \qquad \begin{array}{r} 221 \\ 4\overline{)824} \end{array}$$

2. Student reversed the numbers in the quotient.

$$\begin{array}{r} 67 \\ 3\overline{)228} \\ \underline{210} \\ 18 \\ \underline{18} \end{array} \qquad \begin{array}{r} 46 \\ 4\overline{)256} \\ \underline{240} \\ 16 \\ \underline{16} \end{array}$$

3. Student did not remember to put place holders in the quotient.

$$\begin{array}{r} 94r2 \\ 6\overline{)5426} \\ \underline{54} \\ 26 \\ \underline{24} \\ 2 \end{array} \qquad \begin{array}{r} 48r3 \\ 4\overline{)1635} \\ \underline{16} \\ 35 \\ \underline{32} \\ 3 \end{array}$$

4. Student did not bring down one digit at a time and then left out place holders in the quotient

$$\begin{array}{r} 560r6 \\ 9\overline{)4560} \\ \underline{4500} \\ 60 \\ \underline{54} \\ 6 \end{array} \qquad \begin{array}{r} 460r6 \\ 7\overline{)2846} \\ \underline{2800} \\ 46 \\ \underline{42} \\ 4 \end{array}$$

5. Student understands the division process but lacks sufficient place value knowledge to get the problem correct

$$\begin{array}{r} 96 \\ 2\overline{)30} \\ \underline{-18} \\ 12 \\ \underline{-12} \\ 0 \end{array} \qquad \begin{array}{r} 95 \\ 7\overline{)98} \\ \underline{-63} \\ 35 \\ \underline{-35} \\ 0 \end{array} \qquad \begin{array}{r} 93 \\ 2\overline{)24} \\ \underline{-18} \\ 6 \\ \underline{-6} \\ 0 \end{array}$$

6. Student did not divide by all digits of the divedend.

$$\begin{array}{r} 6 \\ 6\overline{)360} \\ \underline{36} \\ 0 \end{array} \qquad \begin{array}{r} 7 \\ 7\overline{)490} \\ \underline{49} \\ 0 \end{array}$$

7. Student placed unnecessary digits in the quotient.

$$\begin{array}{r} 30r0 \\ 4\overline{)12} \\ \underline{12} \\ 0 \\ \underline{0} \\ 0 \end{array} \qquad \begin{array}{r} 10r14 \\ 56\overline{)70} \\ \underline{56} \\ 14 \\ \underline{0} \\ 14 \end{array}$$

8. Student failed to subtract correctly thus confusing her problem.

$$\begin{array}{r} 82 \\ 9\overline{)80} \\ \underline{72} \\ 18 \\ \underline{-18} \end{array} \qquad \begin{array}{r} 64 \\ 3\overline{)20} \\ \underline{-18} \\ 12 \\ \underline{-12} \end{array}$$

G-1 KEY

ADDITION

TEST 1

Add the following problems and show all your work.

11. $\begin{array}{r} 30 \\ +40 \\ \hline 70 \end{array}$

12. $\begin{array}{r} 30 \\ +26 \\ \hline 56 \end{array}$

13. $\begin{array}{r} 70 \\ +18 \\ \hline 88 \end{array}$

14. $\begin{array}{r} 23 \\ +45 \\ \hline 68 \end{array}$

15. $\begin{array}{r} 32 \\ +37 \\ \hline 69 \end{array}$

16. $\begin{array}{r} 22 \\ +8 \\ \hline 30 \end{array}$

17. $\begin{array}{r} 63 \\ +7 \\ \hline 70 \end{array}$

18. $\begin{array}{r} 26 \\ +5 \\ \hline 31 \end{array}$

19. $\begin{array}{r} 58 \\ +5 \\ \hline 63 \end{array}$

20. $\begin{array}{r} 50 \\ +70 \\ \hline 120 \end{array}$

21. $\begin{array}{r} 90 \\ +40 \\ \hline 130 \end{array}$

22. $\begin{array}{r} 20 \\ +84 \\ \hline 104 \end{array}$

23. $\begin{array}{r} 90 \\ +73 \\ \hline 163 \end{array}$

24. $\begin{array}{r} 65 \\ +92 \\ \hline 157 \end{array}$

25. $\begin{array}{r} 15 \\ +36 \\ \hline 51 \end{array}$

26. $\begin{array}{r} 56 \\ +28 \\ \hline 84 \end{array}$

27. $\begin{array}{r} 38 \\ +93 \\ \hline 131 \end{array}$

28. $\begin{array}{r} 76 \\ +97 \\ \hline 173 \end{array}$

29. $\begin{array}{r} 234 \\ +562 \\ \hline 796 \end{array}$

30. $\begin{array}{r} 627 \\ +162 \\ \hline 789 \end{array}$

31. $\begin{array}{r} 730 \\ +421 \\ \hline 1151 \end{array}$

32. $\begin{array}{r} 941 \\ +457 \\ \hline 1398 \end{array}$

33. $\begin{array}{r} 822 \\ +794 \\ \hline 1616 \end{array}$

34. $\begin{array}{r} 691 \\ +438 \\ \hline 1129 \end{array}$

35. $\begin{array}{r} 487 \\ +865 \\ \hline 1352 \end{array}$

36. $\begin{array}{r} 587 \\ +497 \\ \hline 1084 \end{array}$

37. $\begin{array}{r} 876 \\ +549 \\ \hline 1425 \end{array}$

38. $\begin{array}{r} 864 \\ +748 \\ \hline 1612 \end{array}$

39. $\begin{array}{r} 759 \\ +792 \\ \hline 1551 \end{array}$

40. $\begin{array}{r} 989 \\ +589 \\ \hline 1578 \end{array}$

ADDITION

TEST 1

Add the following problems and show all your work.

11. 30 +40 70	12. 30 +26 56	13. 70 +18 88	14. 23 +45 68	15. 32 +37 69
16. 22 +8 210	17. 63 +7 610	18. 26 +5 211	19. 58 +5 513	20. 50 +70 120
21. 90 +40 130	22. 20 +84 104	23. 90 +73 163	24. 65 +92 157	25. 15 +36 411
26. 56 +28 714	27. 38 +93 1211	28. 76 +97 1613	29. 234 +562 796	30. 627 +162 789
31. 730 +421 1131	32. 941 +457 1348	33. 822 +794 15116	34. 691 +438 10129	35. 487 +865 121412
36. 587 +497 91714	37. 876 +549 131113	38. 864 +748 151012	39. 759 +792 141411	40. 989 +589 141618

ADDITION

TEST 1

Add the following problems and show all your work.

11. 30 +40 70	12. 30 +26 56	13. 70 +18 88	14. 23 +45 68	15. 32 +37 69
16. 22 +8 31	17. 63 +7 61	18. 26 +5 31	19. 58 +5 63	20. 50 +70 20
21. 90 +40 30	22. 20 +84 14	23. 90 +73 63	24. 65 +92 57	25. 15 +36 41
26. 56 +28 74	27. 38 +93 21	28. 76 +97 63	29. 234 +562 796	30. 627 +162 789
31. 730 +421 151	32. 941 +457 398	33. 822 +794 516	34. 691 +438 129	35. 487 +865 242
36. 587 +497 977	37. 876 +549 315	38. 864 +748 512	39. 759 +792 741	40. 989 +589 468

ADDITION

TEST 1

Add the following problems and show all your work.

11. $\begin{array}{r} 30 \\ +40 \\ \hline 70 \end{array}$

12. $\begin{array}{r} 30 \\ +26 \\ \hline 56 \end{array}$

13. $\begin{array}{r} 70 \\ +18 \\ \hline 88 \end{array}$

14. $\begin{array}{r} 23 \\ +45 \\ \hline 68 \end{array}$

15. $\begin{array}{r} 32 \\ +37 \\ \hline 69 \end{array}$

16. $\begin{array}{r} 22 \\ +8 \\ \hline 30 \end{array}$

17. $\begin{array}{r} 1 \\ 63 \\ +7 \\ \hline 70 \end{array}$

18. $\begin{array}{r} 1 \\ 26 \\ +5 \\ \hline 31 \end{array}$

19. $\begin{array}{r} 1 \\ 58 \\ +5 \\ \hline 63 \end{array}$

20. $\begin{array}{r} 50 \\ +70 \\ \hline 120 \end{array}$

21. $\begin{array}{r} 90 \\ +40 \\ \hline 130 \end{array}$

22. $\begin{array}{r} 20 \\ +84 \\ \hline 104 \end{array}$

23. $\begin{array}{r} 90 \\ +73 \\ \hline 163 \end{array}$

24. $\begin{array}{r} 65 \\ +92 \\ \hline 157 \end{array}$

25. $\begin{array}{r} 1 \\ 15 \\ +36 \\ \hline 51 \end{array}$

26. $\begin{array}{r} 1 \\ 56 \\ +28 \\ \hline 84 \end{array}$

27. $\begin{array}{r} 1 \\ 38 \\ +93 \\ \hline 101 \end{array}$

28. $\begin{array}{r} 1 \\ 76 \\ +97 \\ \hline 173 \end{array}$

29. $\begin{array}{r} 234 \\ +562 \\ \hline 796 \end{array}$

30. $\begin{array}{r} 627 \\ +162 \\ \hline 789 \end{array}$

31. $\begin{array}{r} 730 \\ +421 \\ \hline 1151 \end{array}$

32. $\begin{array}{r} 941 \\ +457 \\ \hline 1398 \end{array}$

33. $\begin{array}{r} 822 \\ +794 \\ \hline \end{array}$

34. $\begin{array}{r} 1 \\ 691 \\ +438 \\ \hline 1129 \end{array}$

35. $\begin{array}{r} 1 \\ 1 \\ 487 \\ +865 \\ \hline 1442 \end{array}$

36. $\begin{array}{r} 1 \\ 1 \\ 587 \\ +497 \\ \hline 1174 \end{array}$

37. $\begin{array}{r} 1 \\ 1 \\ 876 \\ +549 \\ \hline 1515 \end{array}$

38. $\begin{array}{r} 1 \\ 1 \\ 864 \\ +748 \\ \hline 1702 \end{array}$

39. $\begin{array}{r} 1 \\ 1 \\ 759 \\ +792 \\ \hline 1641 \end{array}$

40. $\begin{array}{r} 1 \\ 1 \\ 989 \\ +589 \\ \hline 1668 \end{array}$

ADDITION

TEST 1

Add the following problems and show all your work.

11. 30 +40 70	12. 30 +26 56	13. 70 +18 88	14. 23 +45 68	15. 32 +37 69
16. 22 +8 12	17. 63 +7 16	18. 26 +5 13	19. 58 +5 18	20. 50 +70 120
21. 90 +40 130	22. 20 +84 104	23. 90 +73 163	24. 65 +92 157	25. 1 15 +36 51
26. 4 56 +28 111	27. 1 38 +93 121	28. 3 76 +97 191	29. 234 +562 796	30. 627 +162 789
31. 730 +421 1151	32. 941 +457 13[illegible]	33. 1 822 +794 1616	34. 2 691 +438 1119	35. 11 487 +865 2211
36. 14 587 +497 1021	37. 65 876 +549 1911	38. 22 864 +748 1711	39. 51 759 +792 1911	40. 09 989 +589 1981

ADDITION

TEST 1

Add the following problems and show all your work.

11. 30 +40 70	12. 30 +26 56	13. 70 +18 88	14. 23 +45 68	15. 32 +37 69
16. 22 +8 20	17. 63 +7 70	18. 26 +5 21	19. 58 +5 53	20. 50 +70 120
21. 90 +40 130	22. 20 +84 104	23. 90 +73 163	24. 65 +92 157	25. 15 +36 41
26. 56 +28 74	27. 38 +93 121	28. 76 +97 163	29. 234 +562 796	30. 627 +162 789
31. 730 +421 1151	32. 941 +457 1398	33. 822 +794 1516	34. 691 +438 1029	35. 487 +865 12442
36. 587 +497 964	37. 876 +549 1312	38. 864 +748 1502	39. 759 +792 1441	40. 989 +589 1468

G-2 KEY

SUBTRACTION

TEST 1

Subtract the following problems and show all your work.

5. 60
-20
40

6. 80
-30
50

7. 25
-5
20

8. 87
-7
80

9. 46
-4
42

10. 57
-3
54

11. 35
-10
25

12. 63
-30
33

13. 49
-26
23

14. 84
-41
43

15. 97
-62
35

16. 3 1
40
-6
34

17. 6 1
70
-8
62

18. 2 1
30
-17
13

19. 4 1
50
-24
26

20. 3 1
42
-8
34

21. 5 1
67
-9
58

22. 3 1
41
-27
14

23. 7 1
83
-58
25

24. 274
-153
121

25. 638
-314
324

26. 376
-200
176

27. 798
-400
398

28. 563
-301
262

29. 3 1
243
-129
114

30. 6
878
-549
329

31. [illegible]
694
-396
298

32. [illegible]
927
-578
349

33. 1 1
420
-114
306

34. 8 1
790
-263
527

35. [illegible]
840
-397
443

36. 6 15 1
760
-482
278

37. [illegible]
604
-387
217

38. [illegible]
902
-749
153

39. [illegible]
500
-357
143

40. [illegible]
900
-543
357

SUBTRACTION

TEST 1

Subtract the following problems and show all your work.

5. 60 -20 40	6. 80 -30 50	7. 25 -5 20	8. 87 -7 80
9. 46 -4 42	10. 57 -3 54	11. 35 -10 25	12. 63 -30 33
13. 49 -26 23	14. 84 -41 43	15. 97 -62 35	16. 40 -6 46
17. 70 -8 78	18. 30 -17 27	19. 50 -24 34	20. 42 -8 45
21. 67 -9 68	22. 41 -27 16	23. 83 -58 25	24. 274 -153 121
25. 638 -314 324	26. 376 -200 176	27. 798 -400 398	28. 563 -301 262
29. 243 -129 114	30. 878 -549 329	31. 694 -396 298	32. 927 -578 359
33. 420 -114 314	34. 790 -263 533	35. 840 -397 457	36. 760 -482 302
37. 604 -307 377	38. 902 -749 233	39. 500 -357 251	40. 900 -543 443

SUBTRACTION

TEST 1

Subtract the following problems and show all your work.

5. 60 -20 40	6. 80 -30 50	7. 25 -5 20	8. 87 -7 80
9. 46 -4 42	10. 57 -3 54	11. 35 -10 25	12. 63 -30 33
13. 49 -26 23	14. 84 -41 43	15. 97 -62 35	16. 10 40 -6 44
17. 10 70 -8 72	18. 10 30 -17 23	19. 10 50 -24 36	20. 12 42 8 44
21. 17 67 -9 68	22. 11 41 -27 24	23. 13 83 -58 35	24. 274 -153 121
25. 638 -314 324	26. 376 -200 176	27. 798 -400 348	28. 563 -301 262
29. 13 242 -129 126	30. 878 -549 339	31. 14 692 -396 308	32. 12 17 987 -578 459
33. 10 420 -114 316	34. 10 790 -263 587	35. 14 10 840 -397 663	36. 16 10 760 -402 388
37. 9 14 604 -387 317	38. 9 12 902 -749 253	39. 9 10 500 -357 243	40. 9 10 900 -543 456

SUBTRACTION

TEST 1

Subtract the following problems and show all your work.

5. 60
-20
40

6. 80
-30
50

7. 25
-5
25

8. 87
-7
87

9. 46
-4
42

10. 57
-3
54

11. 35
-10
25

12. 63
-30
33

13. 49
-26
23

14. 84
-41
43

15. 97
-62
35

16. 40
-6
40

17. 70
-8
70

18. 30
-17
20

19. 50
-24
30

20. 42
-8
46

21. 67
-9
69

22. 41
-27
27

23. 83
-58
35

24. 274
-153
121

25. 638
-314
324

26. 376
-200
176

27. 798
-400
398

28. 503
-301
262

29. 243
-129
126

30. 878
-549
331

31. 694
-396
342

32. 927
-578
451

33. 420
-114
310

34. 790
-263
630

35. 840
-397
550

36. 760
-482
320

37. 604
-387
303

38. 902
-749
207

39. 500
-357
200

40. 900
-543
400

SUBTRACTION

TEST 1

Subtract the following problems and show all your work.

5. $\begin{array}{r} 60 \\ -20 \\ \hline 40 \end{array}$

6. $\begin{array}{r} 80 \\ -30 \\ \hline 50 \end{array}$

7. $\begin{array}{r} 25 \\ -5 \\ \hline 20 \end{array}$

8. $\begin{array}{r} 87 \\ -7 \\ \hline 80 \end{array}$

9. $\begin{array}{r} 46 \\ -4 \\ \hline 42 \end{array}$

10. $\begin{array}{r} 57 \\ -3 \\ \hline 54 \end{array}$

11. $\begin{array}{r} 35 \\ -10 \\ \hline 25 \end{array}$

12. $\begin{array}{r} 63 \\ -30 \\ \hline 83 \end{array}$

13. $\begin{array}{r} 49 \\ -26 \\ \hline 23 \end{array}$

14. $\begin{array}{r} 84 \\ -41 \\ \hline 43 \end{array}$

15. $\begin{array}{r} 97 \\ -62 \\ \hline 35 \end{array}$

16. $\begin{array}{r} 40 \\ -6 \\ \hline 34 \end{array}$

17. $\begin{array}{r} 70 \\ -8 \\ \hline 62 \end{array}$

18. $\begin{array}{r} 30 \\ -17 \\ \hline 13 \end{array}$

19. $\begin{array}{r} 50 \\ -24 \\ \hline 26 \end{array}$

20. $\begin{array}{r} 42 \\ -8 \\ \hline 34 \end{array}$

21. $\begin{array}{r} 67 \\ -9 \\ \hline 58 \end{array}$

22. $\begin{array}{r} 41 \\ -27 \\ \hline 14 \end{array}$

23. $\begin{array}{r} 83 \\ -58 \\ \hline 25 \end{array}$

24. $\begin{array}{r} 274 \\ -153 \\ \hline 121 \end{array}$

25. $\begin{array}{r} 638 \\ -314 \\ \hline 324 \end{array}$

26. $\begin{array}{r} 376 \\ -200 \\ \hline 176 \end{array}$

27. $\begin{array}{r} 798 \\ -400 \\ \hline 398 \end{array}$

28. $\begin{array}{r} 563 \\ -301 \\ \hline 262 \end{array}$

29. $\begin{array}{r} 243 \\ -129 \\ \hline 126 \end{array}$

30. $\begin{array}{r} 878 \\ -549 \\ \hline 14 \end{array}$

31. $\begin{array}{r} 694 \\ -396 \\ \hline 208 \end{array}$

32. $\begin{array}{r} 927 \\ -578 \\ \hline 269 \end{array}$

33. $\begin{array}{r} 420 \\ -114 \\ \hline 216 \end{array}$

34. $\begin{array}{r} 790 \\ -263 \\ \hline 527 \end{array}$

35. $\begin{array}{r} 840 \\ -397 \\ \hline 463 \end{array}$

36. $\begin{array}{r} 760 \\ -482 \\ \hline 288 \end{array}$

37. $\begin{array}{r} 604 \\ -387 \\ \hline 219 \end{array}$

38. $\begin{array}{r} 902 \\ -749 \\ \hline 253 \end{array}$

39. $\begin{array}{r} 500 \\ -357 \\ \hline 143 \end{array}$

40. $\begin{array}{r} 900 \\ -543 \\ \hline 457 \end{array}$

SUBTRACTION

TEST 1

Subtract the following problems and show all your work.

5. 60 -20 40	6. 80 -30 50	7. 25 -5 20	8. 87 -7 80
9. 46 -4 42	10. 57 -3 54	11. 35 -10 25	12. 63 -30 33
13. 49 -26 23	14. 84 -41 43	15. 97 -62 45	16. 40 -6 46
17. 70 -8 78	18. 30 -17 27	19. 50 -24 34	20. 42 -8 46
21. 5 17 ~~67~~ -9 58	22. 3 11 41 -27 14	23. 7 13 ~~83~~ -58 25	24. 274 -153 121
25. 638 -314 324	26. 376 -200 176	27. 798 -400 398	28. 563 -301 262
29. 3 13 ~~243~~ -129 114	30. 6 18 8~~78~~ -549 329	31. 5 18 14 ~~694~~ -396 298	32. 10 11 17 ~~927~~ -578 549
33. 3 11 10 ~~420~~ -114 2106	34. 6 10 ~~790~~ -263 527	35. 7 13 10 ~~840~~ -397 443	36. 6 15 10 ~~760~~ -482 278
37. 5 9 14 ~~604~~ -387 217	38. 8 9 12 ~~902~~ -749 153	39. 4 9 10 ~~500~~ -357 143	40. 8 9 10 ~~900~~ -543 357

SUBTRACTION

TEST 1

Subtract the following problems and show all your work.

5. 60 -20 40	6. 80 -30 50	7. 25 -5 20	8. 87 -7 80
9. 46 -4 42	10. 57 -3 5544	11. 35 -10 25	12. 63 -30 33
13. 49 -26 23	14. 84 -41 43	15. 97 -62 35	16. 40 -6 46
17. 70 -8 78	18. 30 -17 27	19. 50 -24 34	20. 42 -8 34
21. 67 -9 62	22. 41 -27 24	23. 83 -58 35	24. 274 -153 121
25. 638 -314 324	26. 376 -200 176	27. 798 -400 398	28. 563 -301 262
29. 243 -129 126	30. 878 -549 331	31. 694 -396 302	32. 927 -578 459
33. 420 -114 314	34. 790 -263 533	35. 840 -387 557	36. 760 -482 322
37. 604 -387 018	38. 902 -749 247	39. 500 -357 143	40. 900 -543 357

G-3 KEY

MULTIPLICATION

TEST 1

Multiply the following problems and show all your work.

9. $\begin{array}{r} 34\\ \times\ 2\\ \hline 68 \end{array}$

10. $\begin{array}{r} 40\\ \times\ 3\\ \hline 120 \end{array}$

11. $\begin{array}{r} 60\\ \times\ 7\\ \hline 420 \end{array}$

12. $\begin{array}{r} 21\\ \times\ 5\\ \hline 105 \end{array}$

13. $\begin{array}{r} 42\\ \times\ 3\\ \hline 126 \end{array}$

14. $\begin{array}{r} 83\\ \times\ 2\\ \hline 166 \end{array}$

15. $\begin{array}{r} 302\\ \times\ 4\\ \hline 1208 \end{array}$

$\begin{array}{r} 501\\ \times\ 6\\ \hline 3006 \end{array}$

17. $\begin{array}{r} 803\\ \times\ 3\\ \hline 2409 \end{array}$

18. $\begin{array}{r} 306\\ \times\ 5\\ \hline 1530 \end{array}$

19. $\begin{array}{r} 408\\ \times\ 7\\ \hline 2856 \end{array}$

20. $\begin{array}{r} 809\\ \times\ 9\\ \hline 7281 \end{array}$

21. $\begin{array}{r} 35\\ \times\ 2\\ \hline 70 \end{array}$

22. $\begin{array}{r} 28\\ \times\ 3\\ \hline 84 \end{array}$

23. $\begin{array}{r} 19\\ \times\ 4\\ \hline 76 \end{array}$

24. $\begin{array}{r} 47\\ \times\ 5\\ \hline 235 \end{array}$

25. $\begin{array}{r} 86\\ \times\ 8\\ \hline 688 \end{array}$

26. $\begin{array}{r} 97\\ \times\ 6\\ \hline 582 \end{array}$

27. $\begin{array}{r} 50\\ \times 50\\ \hline 00\\ 250\\ \hline 2500 \end{array}$

28. $\begin{array}{r} 70\\ \times 90\\ \hline 00\\ 630\\ \hline 6300 \end{array}$

29. $\begin{array}{r} 60\\ \times 90\\ \hline 00\\ 540\\ \hline 5400 \end{array}$

30. $\begin{array}{r} 21\\ \times 14\\ \hline 84\\ 21\\ \hline 294 \end{array}$

31. $\begin{array}{r} 13\\ \times 32\\ \hline 26\\ 39\\ \hline 416 \end{array}$

32. $\begin{array}{r} 42\\ \times 12\\ \hline 84\\ 42\\ \hline 504 \end{array}$

33. $\begin{array}{r} 21\\ \times 61\\ \hline 21\\ 126\\ \hline 1{,}281 \end{array}$

34. $\begin{array}{r} 31\\ \times 62\\ \hline 62\\ 186\\ \hline 1{,}922 \end{array}$

35, $\begin{array}{r} 84\\ \times 21\\ \hline 84\\ 168\\ \hline 1{,}764 \end{array}$

36. $\begin{array}{r} 16\\ \times 35\\ \hline 80\\ 48\\ \hline 560 \end{array}$

37. $\begin{array}{r} 14\\ \times 67\\ \hline 98\\ 84\\ \hline 938 \end{array}$

38. $\begin{array}{r} 19\\ \times 42\\ \hline 38\\ 76\\ \hline 798 \end{array}$

MULTIPLICATION

G-3 KEY

(Page 2)

39. 26
x47
182
104
1,222

40. 47
x53
141
235
2,491

41. 64
x87
448
512
5,568

42. 324
x121
324
648
324
39,204

43. 232
x323
696
464
696
74,936

44. 132
x231
132
396
264
30,492

45. 302
x412
604
302
1208
124,424

46. 401
x512
802
401
2005
205,312

47. 201
x614
804
201
1206
123,414

48. 401
x502
802
2005
201,302

49. 802
x301
802
2406
241,402

50. 703
x201
703
1406
141,303

51. 967
x839
8703
2901
7736
811,313

52. 675
x587
4725
5400
3375
396,225

53. 978
x849
8808
3912
7824
830,322

MULTIPLICATION

TEST 1

Multiply the following problems and show all your work.

9. 34 x 2 68	10. 40 x 3 120	11. 60 x 7 420	12. 21 x 5 105
13. 42 x 3 126	14. 83 x 2 166	15. 302 x 4 1208	16. 501 x 6 3006
17. 803 x 3 2409	18. 306 x 5 15030	19. 408 x 7. 28056	20. 809 x 9 72090
21. 35 x 2 610	22. 28 x 3 624	23. 19 x 4 436	24. 47 x 5 2035
25. 86 x 8 108	26. 97 x 6 706		
27. 50 x50 250	28. 70 x90 630	29. 60 x90 540	30. 21 x14 24
31. 13 x32 36	32. 42 x12 44	33. 21 x61 122	34. 31 x62 182
35, 84 x21 164	36. 16 x15 330	37. 14 x67 628	38. 19 x42 418

MULTIPLICATION

(Page 2)

39. 26
x47
324

40. 47
x53
242

41. 64
x87
488

42. 324
x121
344
384
012910

43. 232
x323
543
457
897
01891

44. 132
x231
432
234
7007
0872
12072

45. 302
x412
402
243
341
30718

46. 401
x512
451
814
426
537
097319

47. 201
x614
304
400
390
305
091310

48. 401
x502
347
437
844
020
30781

49. 802
x301
879
468
579
104
0201

50. 703
x201
403
100
221
432
740017

51. 967
x839
439
457
459
970
30017

52. 675
x587
098
209
000
012041

53. 978
x849
987
111
213
438
5953

MULTIPLICATION

TEST 1

Multiply the following problems and show all your work.

9. 34
x 2
68

10. 40
x 3
120

11. 60
x 7
420

12. 21
x 5
105

13. 42
x 3
126

14. 83
x 2
616

15. 302
x 4
08
-12
110

16. 501
x 6
06
+36
312

17. 803
x 3
2509
+25
213

18. 306
x 5
15030

19. 408
x 7.
052
+ 28
0910

20. 809
x 9
88048

21. 35
x 2
610

22. 28
x 3
24

23. 19
x 4
436

24. 47
x 5
37
20
57

25. 86
x 8
48
64
108

26. 97
x 6
0642

27. 50
x50
250

28. 70
x90
720

29. 60
x90
700

30. 21
x14
84
21
-105

31. 13
x32
26
39
515

32. 42
x12
84
42
126

33. 21
x61
21
186
297

34. 31
x62
62
186
1148

35, 84
x21
84
+168
11412

36. 16
x15
30
5
18
3
416

345

57

37. 14
x67
28
7
30
6
521

38. 19
x42
18
2
36
4
420

M U L T I P L I C A T I O N

(Page 2)

39. 26
x47
1442
+ 248
16810

40. 47
x53
21
12
40
20
93

41. 64
x87
28
42
16
48
1124

42. 324
x121
324
648
324
12816

43. 232
x323
696
464
696
162416

44. 132
x231
132
346
264
61812

45. 302
x412
604
302
1208
21014

46. 401
x512
802
901
2005
21208

47. 201
x614
804
201
1806
118011

48. 401
x502
802
000
2005
2807

49. 802
x301
802
802
1606
32010

50. 703
x201
703
703
1406
28012

51. 967
x839
72
43
88
21
2310

52. 675
x587
7755
62
77117

53. 978
x849
88
72
92
36
88
70
78
4432

MULTIPLICATION

TEST 1

Multiply the following problems and show all your work.

9. 34 x 2 68	10. 40 x 3 120	11. 60 x 7 420	12. 21 x 5 105
13. 42 x 3 126	14. 83 x 2 166	15. 302 x 4 1208	16. 501 x 6 3,006
17. 803 x 3 2509	18. 13 306 x 5 1650	19. 35 408 x 7. 1,958	20. 98 809 x 9 7,101
21. 35 x 2 70	22. 2 28 x 3 84	23. 3 19 x 4 76	24. 3 47 x 5 235
25. 86 x 8 528	26. 4 97 x 6 422		
27. 50 x50 250	28. 70 x90 480	29. 60 x90 570	30. 21 x14 24
31. 13 x32 36	32. 42 x12 44	33. 21 x61 121	34. 31 x62 122
35, 84 x21 164	36. 3 16 x15 60	37. 2 14 x67 81	38. 1 19 x42 58

MULTIPLICATION

(Page 2)

39.
2
26
x47
101

40.
2
47
x53
224

41.
4
64
x87
292

42.
324
x121
344

43.
232
x323
666

44.
132
x231
292

45.
302
x412
1204

46.
401
x512
2,002

47.
201
x614
1,204

48.
401
x502
2,002

49.
302
x301
2,402

50.
703
x201
2,103

51.
1
967
x839
1,438

52.
3
675
x587
5

53.
978
x849

MULTIPLICATION

TEST 1

Multiply the following problems and show all your work.

9.
34
x 2
68

10.
40
x 3
120

11.
60
x 7
420

12.
21
x 5
105

13.
42
x 3
126

14.
83
x 2
166

15.
302
x 4
1208

16.
501
x 6
3006

17.
803
x 3
2409

18.
3
306
x 5
1580

19.
15
408
x 7.
2926

20.
8
809
x 9
171

21.
1
35
x 2
70

22.
2
28
x 3
84

23.
2
19
x 4
67

24.
3
47
x 5
235

25.
3
86
x 8
992

26.
4
97
x 6
980

27.
50
x50
200

28.
70
x90
6300

29.
60
x90
4200

30.
21
x14
24

31.
13
x32
36
29
65

32.
42
x12
44
22
66

33.
21
x61
121
261
382

34.
31
x62
182
362
444

35,
84
x21
164
881
1045

36.
3
16
x15
90
15
105

37.
4
14
x67
128
141
169

38.
1
19
x42
,58
142
200

MULTIPLICATION

(Page 2)

39.
1
26
x47
92
124
216

40.
2
47
x53
221
212
433

41.
2
64
x87
423
412
835

42.
324
x121
344
121
465

43.
232
x323
666
323
989

44.
132
x231
292
231
523

45.
302
x412
1204
413
617

46.
401
x512
2002
512
2,2514

47.
201
x614
1204
614
1,1818

48.
401
x502
2002
502
25,041

49.
302
x301
2702
301
3003
301
93,043

50.
703
x201
1403
201
1604
201
11,805

51.
15
967
x839
1064
839
1'90'3
839
27,142

52.
65
675
x587
33'8'3
587
3'9'70
587
45,527

53.
55
978
x849
'1'57'4
849
25,213

G-4 KEY

DIVISION

TEST 1

Divide the following problems and show all your work.

1. $\begin{array}{r} 3 \\ 4\,\overline{)\,12} \end{array}$

2. $\begin{array}{r} 4 \\ 7\,\overline{)\,28} \end{array}$

3. $\begin{array}{r} 9 \\ 8\,\overline{)\,72} \end{array}$

4. $\begin{array}{r} 5 \\ 2\,\overline{)\,10} \end{array}$

5. $\begin{array}{r} 5 \\ 8\,\overline{)\,40} \end{array}$

6. $\begin{array}{r} 7\ r\ 2 \\ 4\,\overline{)\,30} \end{array}$

7. $\begin{array}{r} 12 \\ 2\,\overline{)\,24} \end{array}$

8. $\begin{array}{r} 31 \\ 3\,\overline{)\,93} \end{array}$

9. $\begin{array}{r} 21 \\ 4\,\overline{)\,84} \end{array}$

10. $\begin{array}{r} 15 \\ 2\,\overline{)\,30} \end{array}$

11. $\begin{array}{r} 15 \\ 4\,\overline{)\,60} \end{array}$

12. $\begin{array}{r} 15 \\ 6\,\overline{)\,90} \end{array}$

13. $\begin{array}{r} 19 \\ 3\,\overline{)\,57} \\ \underline{3} \\ 27 \\ \underline{27} \end{array}$

14. $\begin{array}{r} 15 \\ 5\cdot\overline{)\,75} \\ \underline{5} \\ 25 \\ \underline{25} \end{array}$

15. $\begin{array}{r} 14 \\ 7\,\overline{)\,98} \\ \underline{7} \\ 28 \\ \underline{28} \end{array}$

16. $\begin{array}{r} 6\ r\ 2 \\ 3\,\overline{)\,20} \\ \underline{18} \\ 2 \end{array}$

17. $\begin{array}{r} 8\ r\ 4 \\ 7\,\overline{)\,60} \\ \underline{56} \\ 4 \end{array}$

18. $\begin{array}{r} 8\ r\ 8 \\ 9\,\overline{)\,80} \\ \underline{72} \\ 8 \end{array}$

19. $\begin{array}{r} 7\ r\ 4 \\ 6\,\overline{)\,46} \\ \underline{42} \\ 4 \end{array}$

20. $\begin{array}{r} 5\ r\ 2 \\ 8\,\overline{)\,42} \\ \underline{40} \\ 2 \end{array}$

21. $\begin{array}{r} 6\ r\ 4 \\ 9\,\overline{)\,58} \\ \underline{54} \\ 4 \end{array}$

22. $\begin{array}{r} 13\ r\ 1 \\ 3\,\overline{)\,40} \\ \underline{3} \\ 10 \\ \underline{9} \\ 1 \end{array}$

23. $\begin{array}{r} 12\ r\ 2 \\ 4\,\overline{)\,50} \\ \underline{4} \\ 10 \\ \underline{8} \\ 2 \end{array}$

24. $\begin{array}{r} 12\ r\ 6 \\ 7\,\overline{)\,90} \\ \underline{7} \\ 20 \\ \underline{14} \\ 6 \end{array}$

25. $\begin{array}{r} 13\ r\ 1 \\ 4\,\overline{)\,53} \\ \underline{4} \\ 13 \\ \underline{12} \\ 1 \end{array}$

26. $\begin{array}{r} 14\ r\ 2 \\ 5\,\overline{)\,72} \\ \underline{5} \\ 22 \\ \underline{20} \\ 2 \end{array}$

27. $\begin{array}{r} 12\ r\ 2 \\ 8\,\overline{)\,98} \\ \underline{8} \\ 18 \\ \underline{16} \\ 2 \end{array}$

28. $\begin{array}{r} 50 \\ 5\,\overline{)\,250} \end{array}$

29. $\begin{array}{r} 60 \\ 6\,\overline{)\,360} \end{array}$

30. $\begin{array}{r} 70 \\ 7\,\overline{)\,490} \end{array}$

31. $\begin{array}{r} 301 \\ 3\,\overline{)\,903} \end{array}$

32. $\begin{array}{r} 201 \\ 4\,\overline{)\,804} \end{array}$

DIVISION

G-4 KEY

(Page 2)

33. $\begin{array}{r}302\\2\overline{)604}\end{array}$

34. $\begin{array}{r}109\\6\overline{)654}\end{array}$

35. $\begin{array}{r}107\\7\overline{)749}\end{array}$

36. $\begin{array}{r}107\\8\overline{)856}\end{array}$

37. $\begin{array}{r}141\\3\overline{)423}\\\underline{3}\\12\\\underline{12}\\3\\\underline{3}\end{array}$

38. $\begin{array}{r}156\\4\overline{)624}\\\underline{4}\\22\\\underline{20}\\24\\\underline{24}\end{array}$

39. $\begin{array}{r}142\\7\overline{)994}\\\underline{7}\\29\\\underline{28}\\14\\\underline{14}\end{array}$

40. $\begin{array}{r}228\,r1\\2\overline{)457}\\\underline{4}\\5\\\underline{4}\\17\\\underline{16}\\1\end{array}$

41. $\begin{array}{r}124\,r5\\3\overline{)749}\\\underline{6}\\14\\\underline{12}\\29\\\underline{24}\\5\end{array}$

42. $\begin{array}{r}112\\8\overline{)896}\\\underline{8}\\9\\\underline{8}\\16\\\underline{16}\end{array}$

43. $\begin{array}{r}502\\3\overline{)1506}\\\underline{15}\\06\\\underline{6}\end{array}$

44. $\begin{array}{r}302\\4\overline{)1208}\end{array}$

45. $\begin{array}{r}201\\8\overline{)1608}\end{array}$

46. $\begin{array}{r}702\,r1\\2\overline{)1405}\end{array}$

47. $\begin{array}{r}701\,r1\\5\overline{)3506}\end{array}$

48. $\begin{array}{r}801\,r2\\7\overline{)5609}\end{array}$

49. $\begin{array}{r}766\,r2\\3\overline{)2300}\\\underline{21}\\20\\\underline{18}\\20\\\underline{18}\\2\end{array}$

50. $\begin{array}{r}616\,r4\\6\overline{)3700}\\\underline{36}\\10\\\underline{6}\\40\\\underline{36}\\4\end{array}$

51. $\begin{array}{r}575\\8\overline{)4600}\\\underline{40}\\60\\\underline{56}\\40\\\underline{40}\end{array}$

52. $\begin{array}{r}8\\10\overline{)80}\end{array}$

53. $\begin{array}{r}3\\30\overline{)90}\end{array}$

54. $\begin{array}{r}2\\40\overline{)80}\end{array}$

55. $\begin{array}{r}1\,r10\\20\overline{)30}\\\underline{20}\\10\end{array}$

56. $\begin{array}{r}1\,r20\\40\overline{)60}\\\underline{40}\\20\end{array}$

57. $\begin{array}{r}1\,r10\\70\overline{)80}\\\underline{70}\\10\end{array}$

58. $\begin{array}{r}2\\22\overline{)44}\end{array}$

59. $\begin{array}{r}3\\33\overline{)99}\end{array}$

60. $\begin{array}{r}2\\44\overline{)88}\end{array}$

DIVISION

TEST 1

Divide the following problems and show all your work.

1. $\begin{array}{r} 3 \\ 4\,\overline{)\,12} \\ -12 \\ \hline 00 \end{array}$

2. $\begin{array}{r} 4 \\ 7\,\overline{)\,28} \\ -28 \\ \hline 00 \end{array}$

3. $\begin{array}{r} 10\text{r}1 \\ 8\,\overline{)\,72} \\ -71 \\ \hline 01 \end{array}$

4. $\begin{array}{r} 5 \\ 2\,\overline{)\,10} \\ -10 \\ \hline 00 \end{array}$

5. $\begin{array}{r} 6\ \text{R}0 \\ 8\,\overline{)\,40} \\ -39 \\ \hline 01 \end{array}$

6. $\begin{array}{r} 7\ \text{R}2 \\ 4\,\overline{)\,30} \\ -28 \\ \hline 02 \end{array}$

7. $\begin{array}{r} 12 \\ 2\,\overline{)\,24} \\ -24 \\ \hline 00 \end{array}$

8. $\begin{array}{r} 3\ \text{R}3 \\ 3\,\overline{)\,93} \\ -9 \\ \hline 03 \end{array}$

9. $\begin{array}{r} 2\ \text{R}4 \\ 4\,\overline{)\,84} \\ -8 \\ \hline 04 \end{array}$

10. $\begin{array}{r} 15 \\ 2\,\overline{)\,30} \\ -2 \\ \hline 10 \\ -10 \\ \hline \end{array}$

11. $\begin{array}{r} 15 \\ 4\,\overline{)\,60} \\ -4 \\ \hline 20 \\ -20 \\ \hline \end{array}$

12. $\begin{array}{r} 15 \\ 6\,\overline{)\,90} \\ -6 \\ \hline 30 \\ -30 \\ \hline \end{array}$

13. $\begin{array}{r} 19 \\ 3\,\overline{)\,57} \\ -3 \\ \hline 27 \\ -27 \\ \hline 00 \end{array}$

14. $\begin{array}{r} 15 \\ 5\,\overline{)\,75} \\ -5 \\ \hline 25 \\ -25 \\ \hline 00 \end{array}$

15. $\begin{array}{r} 14 \\ 7\,\overline{)\,98} \\ -7 \\ \hline 28 \\ -28 \\ \hline 00 \end{array}$

16. $\begin{array}{r} 6\ \text{R}2 \\ 3\,\overline{)\,20} \\ -18 \\ \hline 02 \end{array}$

17. $7\,\overline{)\,60}$

18. $9\,\overline{)\,80}$

19. $6\,\overline{)\,46}$

20. $8\,\overline{)\,42}$

21. $9\,\overline{)\,58}$

22. $\begin{array}{r} 13\ \text{R}1 \\ 3\,\overline{)\,40} \\ -3 \\ \hline 010 \\ -9 \\ \hline 01 \end{array}$

23. $\begin{array}{r} 12\ \text{R}2 \\ 4\,\overline{)\,50} \\ -4 \\ \hline 010 \\ -8 \\ \hline 02 \end{array}$

24. $\begin{array}{r} 12\ \text{R}6 \\ 7\,\overline{)\,90} \\ -7 \\ \hline 120\ 10 \\ 14 \\ \hline 06 \end{array}$

25. $\begin{array}{r} 13\ \text{R}1 \\ 4\,\overline{)\,53} \\ -4 \\ \hline 13 \\ 12 \\ \hline 01 \end{array}$

26. $\begin{array}{r} 14\ \text{R}2 \\ 5\,\overline{)\,72} \\ -5 \\ \hline 22 \\ -20 \\ \hline 02 \end{array}$

27. $\begin{array}{r} 12 \\ 8\,\overline{)\,98} \\ -8 \\ \hline 18 \\ -16 \\ \hline 02 \end{array}$

28. $\begin{array}{r} 5 \\ 5\,\overline{)\,250} \\ -25 \\ \hline 000 \end{array}$

29. $\begin{array}{r} 60 \\ 6\,\overline{)\,360} \\ -36 \\ \hline 000 \end{array}$

30. $\begin{array}{r} 70 \\ 7\,\overline{)\,490} \\ 49 \\ \hline 000 \end{array}$

31. $\begin{array}{r} 30\ \text{R}3 \\ 3\,\overline{)\,903} \\ -9 \\ \hline 003 \end{array}$

32. $\begin{array}{r} 20\ \text{R}4 \\ 4\,\overline{)\,804} \\ -8 \\ \hline 004 \end{array}$

DIVISION

(Page 2)

33. $2\overline{)604}$ 34. $6\overline{)654}$ 35. $7\overline{)749}$ 36. $8\overline{)856}$

37. $3\overline{)423}$ 38. $4\overline{)624}$ 39. $7\overline{)994}$ 40. $2\overline{)856}$

41. $3\overline{)423}$ 42. $8\overline{)896}$ 43. $3\overline{)1506}$ 44. $4\overline{)1208}$

45. $8\overline{)1608}$ 46. $2\overline{)1405}$ 47. $5\overline{)3506}$ 48. $7\overline{)5609}$

49. $3\overline{)2300}$ 50. $6\overline{)3700}$ 51. $8\overline{)4600}$ 52. $10\overline{)80}$

53. $30\overline{)90}$ 54. $40\overline{)80}$ 55. $20\overline{)30}$ 56. $40\overline{)60}$

57. $70\overline{)80}$ 58. $22\overline{)44}$ 59. $33\overline{)99}$ 60. $44\overline{)88}$

D I V I S I O N

TEST 1

Divide the following problems and show all your work.

1\.
3 R0
4) 12
12
00

2\.
4 R0
7) 28
28
00

3\.
9 Ro
8) 72
72
00

4\.
5 R0
2) 10
10
00

5\.
5 R0
8) 40
40
00

6\.
9 R0
4) 30
30
00

7\.
12 R0
2) 24
24
00

8\.
3 R 3
3) 93
9
03

9\.
2 R4
4) 84
8
04

10\.
19R5
2) 30
35
05

11\.
1 R8
4) 60
4
20
20

12\.
15 R0
6) 90
6
30
30

13\.
17 R6
3) 57
3
27
21
06

14\.
15 Ro
5·) 75
5
25
25
00

15\.
14 Ro
7) 98
7
28
28

16\.
6 R7
3) 20
18
28
21
07

17\.
9 R3
7) 60
63
03

18\.
9 R1
9) 80
81
01

19\.
7 R4
6) 46
42
04

20\.
5 R2
8) 42
40
02

21\.
6 R4
9) 58
54
04

22\.
13 R1
3) 40
3
10
9
1

23\.
12 R 2
4) 50
4
10
8
2

24\.
12 R0
7) 90
7
20
14
14
14
00

25\.
13 R 1
4) 53
4
13
12
01

26\.
14 R 2
5) 72
5
22
20
02

27\.
12 R 2
8) 98
8
18
16
02

28\.
23 R0
5) 250
10
15
15
00
0
0

29\.
24 R 0
6) 360
12
24
24
00
0
0

30\.
7 R0
7) 490
49
00
0
0

31\.
3 R 3
3) 903
9
00
3
R 3

32\.
2 R 4
4) 804
8
00
4
4

DIVISION

(Page 2)

33. $\begin{array}{r} 3\text{ R }4 \\ 2\overline{)604} \\ \underline{6} \\ 00 \\ \underline{4} \\ 4 \end{array}$

34. $\begin{array}{r} 1\text{ R }1 \\ 6\overline{)654} \\ \underline{6} \\ 05 \\ \underline{-4} \\ 1 \end{array}$

35. $\begin{array}{r} 17\text{ R}0 \\ 7\overline{)749} \\ \underline{7} \\ 049 \\ \underline{49} \\ 00 \end{array}$

36. $\begin{array}{r} 17\text{ R }0 \\ 8\overline{)856} \\ \underline{8} \\ 056 \\ \underline{56} \\ 00 \end{array}$

37. $\begin{array}{r} 14\text{R}3 \\ 3\overline{)423} \\ \underline{3} \\ 12 \\ \underline{12} \\ 07 \end{array}$

38. $\begin{array}{r} 1\text{R}23 \\ 4\overline{)624} \\ \underline{4} \\ 22 \\ \underline{20} \\ 023 \end{array}$

39. $\begin{array}{r} 1\text{ R }14 \\ 7\overline{)994} \\ \underline{7} \\ 29 \\ \underline{28} \\ 014 \end{array}$

40. $2\overline{)856}$

41. $3\overline{)423}$

42. $8\overline{)896}$

43. $3\overline{)1506}$

44. $4\overline{)1208}$

45. $8\overline{)1608}$

46. $2\overline{)1405}$

47. $5\overline{)3506}$

48. $7\overline{)5609}$

49. $3\overline{)2300}$

50. $6\overline{)3700}$

51. $8\overline{)4600}$

52. $10\overline{)80}$

53. $30\overline{)90}$

54. $40\overline{)80}$

55. $20\overline{)30}$

56. $40\overline{)60}$

57. $70\overline{)80}$

58. $22\overline{)44}$

59. $33\overline{)99}$

60. $44\overline{)88}$

DIVISION

TEST 1

Divide the following problems and show all your work.

1. $\begin{array}{r} 3 \\ 4\overline{)12} \\ \underline{12} \end{array}$ $\begin{array}{r} 3 \\ \times 4 \\ \hline 12 \end{array}$	2. $\begin{array}{r} 4 \\ 7\overline{)28} \\ \underline{28} \end{array}$ $\begin{array}{r} 4 \\ \times 7 \\ \hline 28 \end{array}$	3. $\begin{array}{r} 9 \\ 8\overline{)72} \\ \underline{72} \end{array}$ $\begin{array}{r} 9 \\ \times 8 \\ \hline 72 \end{array}$	4. $\begin{array}{r} 5 \\ 2\overline{)10} \\ \underline{10} \end{array}$ $\begin{array}{r} 5 \\ \times 2 \\ \hline 10 \end{array}$
5. $\begin{array}{r} 5 \\ 8\overline{)40} \\ \underline{40} \end{array}$ $\begin{array}{r} 5 \\ \times 8 \\ \hline 40 \end{array}$	6. $\begin{array}{r} 80\,R2 \\ 4\overline{)30} \\ \underline{32} \\ 2 \\ \underline{2} \\ \underline{2} \end{array}$ $\begin{array}{r} 80 \\ \times 4 \\ \hline 320 \\ +2 \\ \hline 323 \end{array}$	7. $\begin{array}{r} 66 \\ 2\overline{)24} \\ \underline{12} \\ 12 \\ \underline{12} \end{array}$ $\begin{array}{r} 66 \\ \times 2 \\ \hline 132 \end{array}$	8. $\begin{array}{r} 31 \\ 3\overline{)93} \\ \underline{9} \\ 3 \\ \underline{3} \end{array}$ $\begin{array}{r} 31 \\ \times 3 \\ \hline 93 \end{array}$
9. $\begin{array}{r} 21 \\ 4\overline{)84} \\ \underline{8} \\ 4 \\ \underline{4} \end{array}$ $\begin{array}{r} 21 \\ \times 4 \\ \hline 84 \end{array}$	10. $\begin{array}{r} 15 \\ 2\overline{)30} \\ \underline{2} \\ 10 \\ \underline{10} \end{array}$ $\begin{array}{r} 15 \\ \times 2 \\ \hline 30 \end{array}$	11. $\begin{array}{r} 15 \\ 4\overline{)60} \\ \underline{4} \\ 20 \\ \underline{20} \end{array}$ $\begin{array}{r} 15 \\ \times 4 \\ \hline 60 \end{array}$	12. $\begin{array}{r} 15 \\ 6\overline{)90} \\ \underline{6} \\ 30 \\ \underline{30} \end{array}$ $\begin{array}{r} 15 \\ \times 6 \\ \hline 90 \end{array}$
13. $\begin{array}{r} 19 \\ 3\overline{)57} \\ \underline{3} \\ 27 \\ \underline{27} \end{array}$ $\begin{array}{r} {}^{2}19 \\ \times 3 \\ \hline 57 \end{array}$	14. $\begin{array}{r} 15 \\ 5\overline{)75} \\ \underline{5} \\ 25 \\ \underline{25} \end{array}$ $\begin{array}{r} {}^{2}15 \\ \times 5 \\ \hline 75 \end{array}$	15. $\begin{array}{r} 14 \\ 7\overline{)98} \\ \underline{7} \\ 28 \\ \underline{28} \end{array}$ $\begin{array}{r} 14 \\ \times 7 \\ \hline 98 \end{array}$	16. $\begin{array}{r} 92\,R\,1 \\ 3\overline{)20} \\ \underline{27} \\ 7 \\ \underline{6} \\ 1 \end{array}$ $\begin{array}{r} 92 \\ \times 3 \\ \hline 276 \end{array}$
17. $\begin{array}{r} 82\,R\,2 \\ 7\overline{)60} \\ \underline{56} \\ 16 \\ \underline{14} \\ R2 \end{array}$ $\begin{array}{r} 82 \\ \times 7 \\ \hline 574 \\ +2 \\ \hline 576 \end{array}$	18. $9\overline{)80}$	19. $6\overline{)46}$	20. $8\overline{)42}$
21. $0\overline{)58}$	22. $3\overline{)40}$	23. $4\overline{)50}$	24. $7\overline{)90}$
25. $\begin{array}{r} 13\,R\,1 \\ 4\overline{)53} \\ \underline{4} \\ 13 \\ \underline{12} \\ R1 \end{array}$ $\begin{array}{r} 13 \\ \times 4 \\ \hline 52 \\ +1 \end{array}$	26. $\begin{array}{r} 14\,R2 \\ 5\overline{)72} \\ \underline{5} \\ 22 \\ \underline{20} \\ R2 \end{array}$ $\begin{array}{r} 14 \\ \times 5 \\ \hline 70 \\ +2 \\ \hline 72 \end{array}$	27. $\begin{array}{r} 12 \\ 8\overline{)98} \\ \underline{8} \\ 18 \\ \underline{16} \\ R2 \end{array}$ $\begin{array}{r} 12 \\ \times 8 \\ \hline 96 \\ +2 \\ \hline 98 \end{array}$	28. $\begin{array}{r} 50 \\ 5\overline{)250} \\ \underline{25} \\ 0 \\ \underline{0} \end{array}$
29. $6\overline{)360}$	30. $7\overline{)490}$	31. $3\overline{)903}$	32. $4\overline{)804}$

DIVISION

(Page 2)

33. 2) 604 34. 6) 654 35. 7) 749 36. 8) 856

37. 3) 423 38. 4) 624 39. 7) 994 40. 2) 856

41. 3) 423 42. 8) 896 43. 3) 1506 44. 4) 1208

45. 8) 1608 46. 2) 1405 47. 5) 3506 48. 7) 5609

49. 3) 2300 50. 6) 3700 51. 8) 4600 52. 10) 80

53. 30) 90 54. 40) 80 55. 20) 30 56. 40) 60

57. 70) 80 58. 22) 44 59. 33) 99 60. 44) 88

DIVISION

TEST 1

Divide the following problems and show all your work.

1. 4) 12
30 Ro
12
00
0
0

2. 7) 28
40 Ro
28
00

3. 8) 72
90 Ro
72
00

4. 2) 10
50 R O
10
00
0
0

5. 8) 40
50 R O
40
00

6. 4) 30
73 Ro
28
12
12

7. 2) 24
12 R8
2
4
4
0

8. 3) 93
31 R3
9
3
3
3

9. 4) 84
21
8
4
4
0

10. 2) 30
15 R O
2
10
10

11. 4) 60
15 Ro
4
20
20

12. 6) 90
15 R O
6
30
30

13. 3) 57
19 R O
3
27
27

14. 5) 75
15 R O
5
25
25

15. 7) 98
14 R O
7
28
28

16. 3) 20
66 R O
18
18
18

17. 7) 60
82 R 2
56
16
14
2

18. 9) 80
81 R 3
72
42
9
3

19. 6) 46
70 R 2
42
2
0
2

20. 8) 42
60
48
6
0
6

21. 0) 58
60 R 4
54
4
0
4

22. 3) 40
13 R 1
3
10
9
1

23. 4) 50
12 R 2
40
10
10
8
2

24. 7) 90
12 R 14

25. 4) 53
13 R 1
4
13
12
1

26. 5) 72
14 R 2
5
22
20
2

27. 8) 98
92 R 1
81
17
16
1

28. 5) 250
50 R O
25
00
0
0

29. 6) 360
60 R O
36
00

30. 7) 490
70 R O
49 0
0

31. 3) 903
31 R 6
9
03
3
6

32. 4) 804
21 R 8
8
04
4
8

D I V I S I O N

(Page 2)

32 R 6
33. 2) 664
61
64
2
6

19 Ro
34. 6) 654
6
54
54
0

17 R o
35. 7) 749
7
49
49
00

18 R O
36. 8) 856
8
56
56

14 R3
37. 3) 423
3
12
12
3

15 R24
38. 4) 624
4
22
20
24

14 R14
39. 7) 994
7
29
28
14

2
40. 2) 457
4

12 R 29
41. 3) 749
6
14
12
29

190 R 6
42. 8) 896
8
096
90
6

52 Ro
43. 3) 1506
15
06
06
0

32 RO
44. 4) 1208
12
08
8
0

21 Ro
45. 8) 1608
16
08
8
0

72 R1
46. 2) 1405
14
05
5
0

71 R1
47. 5) 3506
35
06
5
1

81 R2
48. 7) 5609
56
09
7
2

76 R 18
49. 3) 2300
21
20
18
18

61 R 16
50. 6) 3700
36
10
6
16

101 R 18
51. 8) 4600
45
10
8
18

80 Ro
52. 10) 80
80
0
0
0

33 R 30
53. 30) 90
30
60
30
30

40 R O
54. 40) 80
80
00
00
00

55. 20) 30

56. 40) 60

57. 70) 80

58. 22) 44

59. 33) 99

60. 44) 88

IDENTIFYING PATTERNS OF ERROR IN STUDENTS' WRITTEN WORK

Students often make careless or random errors in computation. More often than not, their mistakes are systematic. That is, there is a pattern to their thinking---a pattern based on a misconception of one sort or another. The following examples of students' work display some common systematic or patterned errors. Can you identify the "system" they are using? Demonstrate your understanding of the pattern they are following by working the last two exercises in each case exactly the way the student would work them.

ADDITION - - WHOLE NUMBERS

Student #1

$$\begin{array}{r} {}^{1}86 \\ +45 \\ \hline 131 \end{array} \quad \begin{array}{r} {}^{1}73 \\ +24 \\ \hline 107 \end{array} \quad \begin{array}{r} {}^{1}38 \\ +51 \\ \hline 99 \end{array} \quad \begin{array}{r} {}^{1}3{}^{1}47 \\ +48 \\ \hline 495 \end{array} \quad \begin{array}{r} 74 \\ +63 \\ \hline \end{array} \quad \begin{array}{r} 286 \\ +57 \\ \hline \end{array}$$

Student #2

$$\begin{array}{r} 174 \\ +25 \\ \hline 199 \end{array} \quad \begin{array}{r} 64 \\ +78 \\ \hline 132 \end{array} \quad \begin{array}{r} 367 \\ +186 \\ \hline 443 \end{array} \quad \begin{array}{r} 208 \\ +9 \\ \hline 207 \end{array} \quad \begin{array}{r} 36 \\ +45 \\ \hline \end{array} \quad \begin{array}{r} 217 \\ +97 \\ \hline \end{array}$$

Student #3

$$\begin{array}{r} 74 \\ +56 \\ \hline 1210 \end{array} \quad \begin{array}{r} 35 \\ +92 \\ \hline 127 \end{array} \quad \begin{array}{r} 67 \\ +18 \\ \hline 715 \end{array} \quad \begin{array}{r} 56 \\ +97 \\ \hline 1413 \end{array} \quad \begin{array}{r} 43 \\ +68 \\ \hline \end{array} \quad \begin{array}{r} 88 \\ +39 \\ \hline \end{array}$$

EPS-1

SUBTRACTION - - WHOLE NUMBERS

Student #1

$$\begin{array}{r} 32 \\ -16 \\ \hline 14 \end{array} \quad \begin{array}{r} 245 \\ -137 \\ \hline 112 \end{array} \quad \begin{array}{r} 524 \\ -298 \\ \hline 374 \end{array} \quad \begin{array}{r} 135 \\ -67 \\ \hline 132 \end{array} \quad \begin{array}{r} 458 \\ -372 \\ \hline \end{array} \quad \begin{array}{r} 241 \\ -96 \\ \hline \end{array}$$

Student #2

$$\begin{array}{r} 1\overset{8}{\cancel{9}}\overset{1}{7} \\ -43 \\ \hline 1414 \end{array} \quad \begin{array}{r} 1\overset{6}{\cancel{7}}\overset{1}{6} \\ -23 \\ \hline 1413 \end{array} \quad \begin{array}{r} 3\overset{7}{\cancel{8}}\overset{1}{4} \\ -59 \\ \hline 325 \end{array} \quad \begin{array}{r} 273 \\ -38 \\ \hline \end{array} \quad \begin{array}{r} 285 \\ -63 \\ \hline \end{array}$$

Student #3

$$\begin{array}{r} 147 \\ -20 \\ \hline 120 \end{array} \quad \begin{array}{r} 624 \\ -323 \\ \hline 301 \end{array} \quad \begin{array}{r} 527 \\ -304 \\ \hline 203 \end{array} \quad \begin{array}{r} 805 \\ -201 \\ \hline 604 \end{array} \quad \begin{array}{r} 446 \\ -302 \\ \hline \end{array} \quad \begin{array}{r} 760 \\ -230 \\ \hline \end{array}$$

Student #4

$$\begin{array}{r} 7\overset{9}{\cancel{0}}\overset{1}{8} \\ -329 \\ \hline 479 \end{array} \quad \begin{array}{r} 2\overset{5}{\cancel{6}}\overset{1}{1} \\ -38 \\ \hline 223 \end{array} \quad \begin{array}{r} 3\overset{9}{\cancel{0}}\overset{1}{6} \\ -187 \\ \hline 219 \end{array} \quad \begin{array}{r} 3\overset{6}{\cancel{7}}\overset{1}{5} \\ -166 \\ \hline 209 \end{array} \quad \begin{array}{r} 805 \\ -136 \\ \hline \end{array} \quad \begin{array}{r} 344 \\ -127 \\ \hline \end{array}$$

EPM-1

MULTIPLICATION - - WHOLE NUMBERS

Student #1

$$\begin{array}{r} \overset{2}{4}6 \\ \times 24 \\ \hline 184 \\ 102 \\ \hline 1204 \end{array} \qquad \begin{array}{r} \overset{1}{7}6 \\ \times 32 \\ \hline 152 \\ 228 \\ \hline 2432 \end{array} \qquad \begin{array}{r} \overset{5}{4}8 \\ \times 57 \\ \hline 336 \\ 250 \\ \hline 2836 \end{array} \qquad \begin{array}{r} 98 \\ \times 56 \\ \hline \end{array} \qquad \begin{array}{r} 86 \\ \times 45 \\ \hline \end{array}$$

Student #2

$$\begin{array}{r} 34 \\ \times 2 \\ \hline 68 \end{array} \qquad \begin{array}{r} \overset{2}{2}7 \\ \times 4 \\ \hline 88 \end{array} \qquad \begin{array}{r} \overset{2}{1}8 \\ \times 3 \\ \hline 34 \end{array} \qquad \begin{array}{r} \overset{1}{2}4 \\ \times 4 \\ \hline 86 \end{array} \qquad \begin{array}{r} 35 \\ \times 3 \\ \hline \end{array} \qquad \begin{array}{r} 28 \\ \times 4 \\ \hline \end{array}$$

Student #3

$$\begin{array}{r} 304 \\ \times 506 \\ \hline 1824 \\ 1520 \\ \hline 17024 \end{array} \qquad \begin{array}{r} 236 \\ \times 14 \\ \hline 944 \\ 236 \\ \hline 3304 \end{array} \qquad \begin{array}{r} 236 \\ \times 104 \\ \hline 944 \\ 236 \\ \hline 3304 \end{array} \qquad \begin{array}{r} 718 \\ \times 42 \\ \hline \end{array} \qquad \begin{array}{r} 718 \\ \times 402 \\ \hline \end{array}$$

EPD-1

DIVISION - - WHOLE NUMBERS

Student #1

$\begin{array}{r} 233 \\ 2\overline{)176} \end{array}$ $\quad \begin{array}{r} 221 \\ 4\overline{)824} \end{array}$ $\quad \begin{array}{r} 231 \\ 3\overline{)713} \end{array}$ $\quad 3\overline{)639}$ $\quad 4\overline{)518}$

Student #2

$\begin{array}{r} 44 \\ 2\overline{)88} \\ \underline{80} \\ 8 \\ \underline{8} \end{array}$ $\quad \begin{array}{r} 14 \\ 4\overline{)164} \\ \underline{160} \\ 4 \\ \underline{4} \end{array}$ $\quad \begin{array}{r} 67 \\ 3\overline{)228} \\ \underline{210} \\ 18 \\ \underline{18} \end{array}$ $\quad \begin{array}{r} 39 \\ 5\overline{)465} \\ \underline{450} \\ 15 \\ \underline{15} \end{array}$ $\quad 3\overline{)75}$ $\quad 6\overline{)510}$

Student #3

$\begin{array}{r} 65 \text{ r.1} \\ 7\overline{)456} \\ \underline{42} \\ 36 \\ \underline{35} \\ 1 \end{array}$ $\quad \begin{array}{r} 94 \text{ r.2} \\ 6\overline{)5426} \\ \underline{54} \\ 26 \\ \underline{24} \\ 2 \end{array}$ $\quad \begin{array}{r} 67 \text{ r. 4} \\ 8\overline{)4860} \\ \underline{48} \\ 60 \\ \underline{56} \\ 4 \end{array}$ $\quad 9\overline{)2721}$ $\quad 6\overline{)4232}$

Student #4

$\begin{array}{r} 50 \text{ r.4} \\ 5\overline{)254} \\ \underline{250} \\ 4 \end{array}$ $\quad \begin{array}{r} 560 \text{ r.6} \\ 9\overline{)4560} \\ \underline{4500} \\ 60 \\ \underline{54} \\ 6 \end{array}$ $\quad \begin{array}{r} 730 \\ 8\overline{)5840} \\ \underline{5600} \\ 240 \\ \underline{240} \end{array}$ $\quad 6\overline{)4818}$ $\quad 7\overline{)3525}$

PROBLEM SOLVING

Dr. Rose Strahan

PROBLEM SOLVING

Overview and Rationale

For many years teachers have agreed that problem solving is a very important part of mathematics and of life. The major responsibility of schools is to help children learn skills and concepts that will enable them to function in everyday life. Certainly problem solving ability is essential in facing and solving problems encountered as adults.

Although we all agree to the importance of problem solving, it is a neglected area in many elementary school mathematics curriculums. Why do we spend so little time teaching problem solving? The major reason is that problem solving is difficult to teach. As teachers we emphasize the learning of basic facts and the use of those facts in doing many different types of computational activities. We hope that problem solving skills develop along the way and that students will be able to apply the computational skills in problem solving situations. If problem solving is so important, should we leave the development of those skills to chance? We are doing students a disservice if we do not emphasize problem solving as a vital part of the elementary school mathematics curriculum.

The purpose of this workshop is to assist you in developing techniques to be used in your classroom to help students acquire and improve their problem solving abilities. Maybe you will also improve your ability and, perhaps, acquire a more positive attitude toward the teaching of problem solving. As we discuss how to teach problem solving, you will be solving several different types of problems. Since problem solving means many different things to people, let's begin our study by defining problem solving and then we will solve some problems.

Objectives for Workshop Participants. Workshop participants will:

1) Define problem solving and discuss the role of problem solving in the elementary school mathematics curriculum.
2) Discuss types of problems and work in groups of two or three with all groups solving various types of problems.
3) Discuss problem solving strategies and all participants will solve a problem demonstrating each strategy.
4) Discuss techniques to be used by teachers to improve problem solving abilities of students.
5) Write several problems appropriate for elementary school students.
6) Describe methods of evaluating problem solving.
7) Identify sources of problems and problem solving materials.
8) Receive a collection of problems and a list of sources of problems.

Workshop Leader Behaviors. Workshop leaders will:

1) **Provide several definitions of a problem and problem solving.**
2) **Discuss factors that influence the problem solving process.**
3) **Present the National Council of Teachers of Mathematics's recommendation regarding problem solving and lead a discussion of the role of problem solving as an integral part of the elementary school mathematics curriculum.**
4) **Identify several types of problems and present an example of many of the types.**
5) **Divide participants into groups and provide problems of various types for the groups to solve.**
6) **Lead a discussion of the problems and the processes used in solving them.**
7) **Present several problem solving strategies and demonstrate the use of each in solving a problem.**
8) **Provide problems and allow time for the participants to solve the problems using the strategies discussed.**
9) **Lead a discussion of the strategies used in solving the problems, emphasizing the process used rather than the result.**
10) Discuss techniques for teaching problem solving.
11) Provide time for each participant to write problems appropriate for classroom use.
12) Have participants share the problems that they have written with the entire group.
13) Provide guidelines for evaluation of problem solving.
14) Distribute sample problems and a list of sources of problems.
15) Discuss some implications of the research on problem solving.

Preparing for the Workshop. Prepare the following materials for the workshop. Before you begin preparing materials read the section on workshop content and activities.

Transparencies - PS-1 through PS-7, PS-10, PS-11, PS-13, PS-14, PS-17, PS-25

Handouts - PS-3, PS-4, PS-8, PS-10, PS-11, PS-12, PS-14 through PS-20, PS -22, PS-23, PS-24

5 books of approximately the same size

Several copies of the Arithmetic Teacher containing "Problem Solving: Tips for Teachers"

Blank transparencies and a pen

A picture and/or a calendar to use as motivation in writing a problem

Large manila envelope per participant containing the following: 10 coins or cardboard circular regions, at least 17 toothpicks, 7 lima beans, 2 paper plates, a strip of poster board for a log, and a sheet of grid paper.

Workshop Content and Activities

1) Definitions of a Problem and Problem Solving. People use the word problem to describe everything from basic facts in addition to very complex word problems in analysis. In primary grades after we have taught the concepts and meaning of addition and multiplication, we want students to practice on the facts until they have instant recall. Some teachers refer to all of this practice as working problems. Other teachers call these exercises. There is no clear-cut agreement as to what constitutes a problem. What may be a problem for one person is an exercise for another. As we grow mathematically, those things that were problems initially may become routine exercises.

Let's look at some definitions of a problem (Transparency PS-1). According to Webster's New Twentieth Century Dictionary, a problem is"1) a question proposed for solution or consideration, 2) a question, matter, situation, or person that is perplexing or difficult, 3) in mathematics, anything required to be done, or requiring the doing of something." Funk and Wagnalls' New Standard Dictionary of the English Language defines as problem as "a question for discussion and settlement; a matter of uncertainty requiring further light to determine the truth, especially when difficult or uncertain of solution." According to the American Heritage Dictionary, a problem is "a question or situation that presents uncertainty, perplexity or diffitulty."

Stephen Krulik and Jesse Rudnick in their book, Problem Solving: A Handbook for Teachers, define a problem as "a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent or obvious means or path to obtaining the solution" (p. 3). According to Randall Charles and Frank Lester in Teaching Problem Solving: What, Why, and How, a problem is a task for which:

1) The person confronting it wants or needs to find a solution.
2) The person has no readily available procedure for finding the solution.
3) The person must make an attempt to find a solution.

As we study all of these definitions, we find common elements. Let's agree that a problem is a question or situation that confronts us and we want to find a solution but we do not know exactly what to do to find the solution.

Now that we have defined a problem, let's define problem solving (Transparency PS-2). Krulik and Rudnick define problem solving as "the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation" (p.4). The National Council of Supervisors of Mathematics (1977) defined problem solving as "the process of applying previously acquired knowledge to new and unfamiliar situations." "Problem solving is the process of attacking . . . a problem: accepting the challenge, formulating the questions, clarifying the goal, defining and executing the plan of action, and evaluating the solution" (NCTM, 1983 Yearbook, p. 10). Let's agree that problem solving is a process -- for any given problem, problem solving is the process used in finding a solution.

George Polya is a master problem solver and teacher of problem solving. In his book How To Solve It, he identifies and describes four stages in problem solving (Transparency PS-3). [Discuss the four stages as the transparency is shown. Provide a copy for each participant (Handout PS-3).]

2) Factors that Influence the Problem Solving Process. Our definition of problem solving indicates that a problem solver must have the ability to take previously learned information and previous experience and use the two in a new situation. The person must also have the motivation and perseverance to allow progress toward a solution. Randall Charles and Frank Lester have identified three sets of interacting factors that influence the problem solving process - affective factors, cognitive factors, and experience. Problem solving is a very complex activity which is influenced by attitude and experience as well as computational ability and reasoning ability. Experience factors include both environmental and personal factors. Affective factors include attitude, interest, anxiety, motivation, and perseverance. The set of cognitive factors includes reading ability, reasoning ability, visual ability, and computational skills.

As you can see by studying these factors, the activities and experiences that we provide for students in our classroom can have a great impact on their problem solving ability.

3) NCTM's Recommendations and the Role of Problem Solving. In 1979-80 as a result of the back-to-basics movement, the National Council of Teachers of Mathematics published a list of recommendations for school mathematics of the 1980s (Transparency PS-4, Handout PS-4). To implement the first recommendation NCTM recommended six specific forms of action. They are as follows.

1) The mathematics curriculum should be organized around problem solving.
2) The definition and language of problem solving in mathematics should be developed and expanded to include a broad range of strategies, processes,

and modes of presentation that encompass the full potential of mathematical applications.

3) Mathematics teachers should create classroom environments in which problem solving can flourish.
4) Appropriate curricular materials to teach problem solving should be developed for all grade levels.
5) Mathematics programs of the 1980s should involve students in problem solving by presenting applications at all grade levels.
6) Researchers and funding agencies should give priority in the 1980s to investigations into the nature of problem solving and to effective ways to develop problem solvers. (An Agenda for Action, NCTM, 1980)

[Lead participants in a discussion of how and to what extent the first recommendation has been implemented in their own school.]

4) Types of Problems. Problems are frequently placed in categories according to type, however, there is no well-defined classification system. Charles and Lester partitioned mathematical problems into six sets (Transparency PS-5). [Discuss each type while the transparency is shown.] Thomas Butts (NCTM, 1983 Yearbook, p.24) used five subsets. (Transparency PS-6). [Discuss each type.] A third scheme for classifying problems makes use of seven categories (Transparency PS-7). [Discuss each type.]

Although there is no agreement on the classification system, educators agree that a wide variety of problems is available and should be a part of the curriculum.

5) Divide Participants Into Groups of Two or Four. [Use any convenient method to divide the participants into groups of two or four per group. Give each participant a copy of Handout PS-8. Allow time for the groups to solve the problems. Pages 6 and 7 contain the solutions for PS-8.]

6) Discussion of the Problems. [Lead a discussion of the problems that have been solved. Discuss the type of problem and the process used in the solution. Stress the following two points: 1) a problem may be classified as more than one type and being able to label each as a specific type is not important, and 2) the process used in solving the problem is more important than the result.]

SOLUTIONS FOR PS-8

1. 19 Information about Steve was not needed.

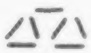
2. Not enough information to find Ruth's weight.

3. 3, 8 Look at pairs whose sum is 11.

11,0	9,2	7,4
10,1	8,3	6,5

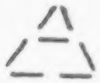
Check the product of each pair.

4. a)



Take off 2 toothpicks from the top.

b)



Take out 2 toothpicks from lower section.

5. 10 Make a table.

Nickels	Dimes	Quarters	
10	0	0	
9	-	-	(cannot be done)
8	1	0	
7	-	-	(cannot be done)
6	2	0	
5	0	1	
4	3	0	
3	1	1	
2	4	0	"Total of 10 Ways"
1	2	1	
0	5	0	
0	0	2	

6. 156 and 157. The two pages are consecutive and the sum is 313. Try pairs of 3 digit numbers.

7.

$$\begin{array}{r} \text{DONALD} \\ +\ \text{GERALD} \\ \hline \text{ROBERT} \end{array} \qquad \begin{array}{r} 5\ 2\ 6\ 4\ 8\ 5 \\ +\ 1\ 9\ 7\ 4\ 8\ 5 \\ \hline 7\ 2\ 3\ 9\ 7\ 0 \end{array}$$

Since D = 5, T is 0. Start with O + E = O.
E has to be 9 and O + E + 1 = 10 + O.
If E = 9, then A = 4.

Now you have

$$\begin{array}{r} 5\ \text{O}\ \text{N}\ 4\ \text{L}\ 5 \\ \text{G}\ 9\ \text{B}\ 4\ \text{L}\ 5 \\ \hline \text{R}\ \text{O}\ \text{B}\ 9\ \text{R}\ \text{O} \end{array}$$

Since 2L + 1 = R, R is 1, 3, or 7.
Try R = 7. Then L = 8 and G = 1.

$$\begin{array}{r} 5\ \text{O}\ \text{N}\ 4\ 8\ 5 \\ 1\ 9\ 7\ 4\ 8\ 5 \\ \hline 7\ \text{O}\ \text{B}\ 9\ 7\ 0 \end{array}$$

O must be 2.

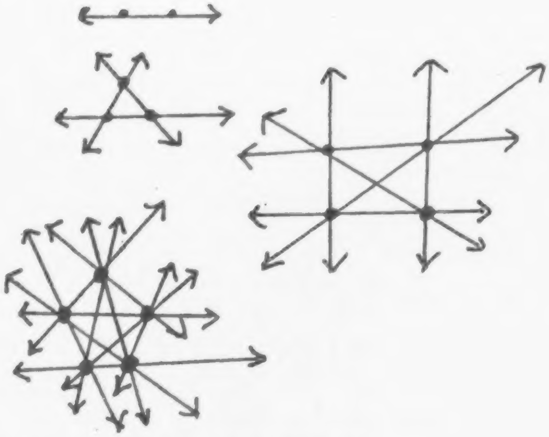
Then N = 6 and B = 3.

Check:

$$\begin{array}{r} 5\,2\,6\,4\,8\,5 \\ \underline{1\,9\,7\,4\,8\,5} \\ 7\,2\,3\,9\,7\,0 \end{array} \qquad \text{Okay.}$$

8.

Number of points		Number of lines
2		1
	2	
3		3
	3	
4		6
	4	
5		10
n		$\frac{n(n-1)}{2}$



9. The driver had to hear the lady when she gave him her destination.

10. Tic's horse won. (See statements 3, 5, and 8.) Tic did not own Speedy. (See statements 4 and 6.) Tic did not own Steady since Steady did not win (See statement 7). Therefore, Tic owned Slowpoke.

7) Present Problem Solving Strategies. What process should be used in solving a problem? Many attempts have been made over the years to define problem solving and develop an algorithm for solving any given problem. Although these efforts have not produced a step-by-step procedure that guarantees a correct solution, many useful techniques have been developed. There is no one correct procedure for solving a problem. Teachers must allow and encourage a wide variety of approaches, ideas, questions, solutions, and discussions.

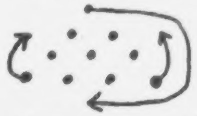
To provide a starting point and to give some measure of security for students suggest a general framework or an overall procedure such as following Polya's stages. The second stage requires the student to devise a plan. Students need to be familiar with several different strategies or plans so they can choose and apply the one that they feel is appropriate for a given problem. In initial problem solving activities the teacher may need to suggest a strategy. Many times you will find it beneficial to combine several strategies in solving a problem. After you have worked together using a variety of strategies, students should be able to suggest and choose their own strategy. They will not choose a strategy that they do not know how to use so the teacher must plan instructional activities to acquaint students with some strategies. Let's discuss some specific strategies and apply them in solving appropriate problems (Transparency PS-10, Handout PS-10).

The first four are appropriate for young children in grades one and two. The other strategies are appropriate for use in grades three through eight, however, draw a picture or guess and check might be used in any grade. [Briefly describe each strategy while the transparency is shown.]

Now let's apply each strategy as we solve problems (Transparency PS-11, Handout PS-11).

We will solve the first problem by acting out the problem. [Have five participants act out the problem as others record results: 10 hand shakes.]

Use objects in solving the second problem. [Use an overhead and arrange the 10 coins as shown. Rearrange in the following way.]



Draw a picture to solve the third problem. [Allow participants a few seconds to draw a picture and get a solution. Have someone show their drawing and correct solution: 3 birds]

Guess and check is also called trial and error. As you use this strategy in the classroom encourage students to use a calculator to check. Solve problem four by guessing and checking to see whether your guess is correct. [Allow a few seconds for participants to guess before giving them the solution : ${}_{2}{}^{6}\overset{1}{{}_{4}}{}^{5}{}_{3}$]

Too often we have our mind set on only one way of thinking. Solve problem five by looking at it another way. Consider the following alternatives: could be thinking only 28 days; could be thinking at least 28 days. [Solution: All months have 28 days.]

To find the sum of the first n odd numbers, let's begin with the first odd number and extend the sum. $1 = 1$, $1 + 3 = 4$, $1 + 3 + 5 = 9$, $1 + 3 + 5 + 7 = 16$, $1 + 3 + 5 + 7 + 9 = 25$. Is there a pattern? $1 + 3 + 5 + \dots + m = n^2$.

We can write all of the three-digit numbers formed using 4, 7, and 5 (problem 7). An organized list will make our work easier. 444, 445, 454, 455, 447, 474, 457, 475, 477, ..., 777. [27 three-digit numbers.]

Making a table will help in solving problem eight.

	Vegas	Washington	Sierra	Tampa
Bill	X	X	0	X
Minnie	X	0	X	X
Dawn	X	X	X	0
Chuck	0	X	X	X

Now mark each box in the table using the information given. Use as "X" if the person doesn't come from that town. Use an "0" if the person comes from that town.

Working backwards is an appropriate strategy for solving problem nine. Consider what the result will look like.

	0s	1s	2s	...	7s	8s	9s
Digits							

There can't be many 9s in our ten-digit number, for each time a 9 appears in a slot there must then be nine of that slot's digits in the number. For example, if 9 were the third digit and the fourth digit of the number, then the number must have nine 2s and nine 3s among its digits. That makes eighteen digits, which exceeds our limit of ten. Indeed, even one 9 would imply that our number repeats nine of some digit, and so could only be of the form

0s	1s	2s	3s	...	9s
x	x	x	x	...	1

for some fixed digit x $(0 \leq x \leq 8)$. We see that $x = 0$ won't work, since we already have a 1 (in the number-9 slot); so the number of 1s is greater than 0. If $x > 0$, then the number of 0s + 1s + ... + 8s = 9 (digits), and so $9x = 9$, or $x = 1$.

But

0s	1s	2s	...	8s	9s
1	1	1	...	1	1

yields the number 1 111 111 111, which certainly doesn't have one 0, one 1, one 2, and so on, for its digits. So there cannot be any 9s in our number. In a similar way, by working backward from the number-8 slot you can show that the number of 8s is 0.

Continue working backward and show that the number of 7s is 0. Now you have reached the number 6 slot. You will find the following solution.

0s	1s	2s	3s	4s	5s	6s	7s	8s	9s
6	2	1	0	0	0	1	0	0	0

Identifying and solving cases involves consideration of exclusive possibilities. There are two cases in the solution of problem ten. Case 1 - Both numbers are even or Case 2 - Both numbers are odd.

Since bringing a log into the classroom and cutting it into pieces is not practical, make a model to solve problem eleven. Represent the log with a strip of paper and cut it with scissors. To cut it into 3 pieces, you make 2 cuts - 3 minutes per cut. To get 6 pieces, you would make 3 additional cuts so it takes 15 minutes.

All problem solving involves some reasoning so we have used logical reasoning in solving other problems. Solving problem 12 requires use of logical reasoning. Think - the bottle costs a dollar more than the cork. So the bottle costs $1.05 and the cork costs $0.05.

8) Participants Solve Problems. We have worked some problems together, and you are familiar with several strategies. Choose a strategy and solve each problem. (Handout PS-12) You may work together if you wish. [Decide on a time period and announce the time limit.]

9) Discussion of Strategies Used. [Lead a discussion of the strategies used for each problem. Emphasize the following points: no one correct strategy but a choice of several equally correct possibilities and the process is the important thing. Use Transparency PS-13 to illustrate one method of solving each problem.]

10) Discuss Techniques for Teaching Problem Solving. (Transparency PS-14. Give each participant a copy of Handout PS-14.)

    1. Create an atmosphere of success. "Nothing succeeds like success." Choose your problems carefully to ensure a reasonable degree of success. Success means more than a correct answer; success means making a sustained attempt to find a solution.

2. Encourage your students to solve problems. For students to become good problem solvers, they must be constantly exposed to and involved in problem solving. Ability to solve problems is developed by solving problems.

3. Take advantage of problem situations which arise normally in and out of the classroom. If you want to have students work in groups, let them decide how many groups. Planning a field trip or a class project can be used as a problem solving activity.

4. Present problem situations in a variety of ways not just in written form. Present some problems by using pictures. (Handouts 15 and 16) Use your bulletin board to present problems. (See the examples at the end of this manual, pages 64 through 74 .)

5. Teach students how to read a problem. Mathematics is not always read from left to right. Have students identify key words in problems. Many words have a special meaning in mathematics. Make a list of those words. Reading rate for mathematics is much lower than for prose. Teach students to read critically and carefully for meaning.

6. Encourage students to make up problems for the class to solve. Students learn a great deal about problem solving by making up their own problems. Students usually have a greater desire to solve problems if they are written by classmates. Some student created problems will have too much information and some insufficient information. Students have valuable problem solving experience as they try to solve each type. Remember the process is the important thing!

7. Have children dramatize situations described in problems. This causes students to be a part of the problem. They become actively involved in the problem solving process.

8. Rewrite textbook problems using the names of teachers from your school or the names of the children in your classroom. Students are more interested in solving problems containing their own names or the names of teachers that they know.

9. Make problem solving a class activity. Students will learn by interacting with each other and the teacher. The questions asked, the possible solutions, and even the brainstorming will help students learn to express their thoughts and provide insight into the problem solving process.

10. Raise creative, constructive questions. Use questions to provide some guidance. "What if ..." questions help extend and guide students thinking.

11. Make your problems meaningful for the children in your classroom. Know your students well enough to be aware of their interests and try to create realistic problems involving their interests. Textbook problems may involve themes that are of no interest to your students. Students will work more enthusiastically on a problem that interests them.

12. Develop a problem solving framework. To help students keep their work organized for communication decide on a general problem solving framework and use it. Polya's four stages or some modification of them would be useful for any problem.

13. Encourage your students to use a calculator. By using a calculator for the computation, students can work with problems that are interesting even though the computations may be beyond their level. The focus is on the problem solving; strategies and processes can be emphasized with less time devoted to computation.

14. Incorporate problem solving throughout the learning of a skill and concept. Problem solving activities help give meaning to skills and concepts. Seeing the usefulness of an idea motivates students to learn by giving a purpose to the learning.

15. Use problem solving to review concepts. This will help students remember concepts and see the usefulness of them. They will also learn how previously learned concepts are related to other concepts.

16. Solve several problems each week. Do not spend long periods of time on problem solving alone. Try using a "problem a day" or a "problem a week." Do not spend even one entire week on problem solving exclusively. Do problem solving throughout the year instead of covering a unit on problem solving.

17. Try to keep the mathematics involved in your problems well within the students ability level. If students cannot do the mathematics or do not understand the mathematics, they will not be able to solve the problems. In some cases, give problems containing little mathematics so the problem solving process will clearly be the goal of the activity.

18. Encourage students to find alternative methods of solution to a given problem. Avoid having a one-track mind regarding a solution. There is much to be learned by using several methods of solution for a problem and discussing the merits of each method. Insisting that students use the correct method has no place in the teaching of problem solving.

19. Provide children with problems of varying degrees of difficulty. Everyone enjoys the feeling of being successful. Provide some problems that even the slowest student will be able to solve. Also provide some problems that will challenge your best students.

20. Do not expect every child to excel in problem-solving, but expect every child to participate in problem-solving activities. Create an atmosphere where the threat of failure will not keep a student from attempting to solve problems. Be enthusiastic about problem solving but be realistic. Some people will not be problem solvers so group or team work is necessary.

11) Provide Time for Each Participant to Write Problems. [Briefly discuss guidelines for Writing Problems (Transparency PS-17, Handout PS-17].

Students are more likely to try to solve problems that are interesting to them, so write problems that will interest your students. Consider writing several problems of the same type but make each problem deal with a different topic. Let students choose the one they wish to solve.

A major consideration in writing word problems is reading ability of your students. If you teach primary grades, use few words and only words that are on the students' reading level.

Do not overemphasize one type of problem and neglect other types. For example, in writing subtraction problems, consider all subtraction situations - take away, additive, comparative, partitioning.

Present up-to-date real-world information in your problems. Make sure that you are asking students to find an answer that is unknown. The benefit of the problem solving experience is lost if the students know the answer in advance. Do not expect students to work to find a solution if they cannot see any reason for ever wanting or using the solution.

Seldom do we encounter a problem in everyday life where we have exactly the data needed available. Analyzing the information to determine what facts are needed is an important part of problem solving. Rejecting facts that are not useable in finding a solution is valuable experience for students. Try having students gather their own data to solve a problem. Example - How much paper does your school use in a week?

Problems that require guessing or searching require higher level cognitive skills than application problems. Example - Find all the primes that are one less than a fourth power.

Pose some problems in a picturesque manner rather than in a dry, mathematical style. The use of illustrative examples helps increase the number of students who attempt to solve the problem.

In his essay "Posing Problems Properly," Thomas Butts refers to the posing of problems as an art. [See #10 on Transparency PS-17]

Have each participant write problems appropriate for use in their classroom. [Determine the number of problems to be written by the amount of time you have available.] To help you get started, write a problem based on this picture. (Display a picture).

12) Have Participants Share Their Problems. Ask for volunteers to read their problems to the group. If time permits, discuss strategies for solving each problem presented.

13) Provide Guidelines for Evaluation of Problem Solving. As we make problem solving activities an integral part of our curriculum, evaluation of these activities will be necessary. There is no agreement regarding the method of evaluation, however, everyone seems to agree that students must be evaluated by some method other than an analysis of their written work. Evaluation that is limited to checking for answer tells a teacher very little about how the student got the answer. The answer is not nearly as important as the process used to get the answer. Sometimes students get correct answers through use of incorrect procedures. Perhaps the most accurate evaluation could be achieved by talking individually with students while they are solving a problem, however, it may not be possible for teachers to conduct such evaluations. It is possible for teachers to combine an analysis of the written work with notes made while observing and questioning students.

We are going to discuss a method of evaluating problem solving by using a point system and two methods of recording observations. The point system we are going to discuss was published in the January 1983 Arithmetic Teacher and was developed by Randall Charles. (Handout PS-18). [Discuss the system and the examples.]

Randall Charles also presented two methods of recording observations of problem solving activities (Handout PS-19).

Some teachers use a notebook to record informal evaluative comments. These comments could then be shared with the student and/or the parents. The observation checklist is a more structured method of recording observations. [Discuss the checklist.]

Choose the method that works best for you. The important point is that a record of observations should be a part of the evaluation of problem solving. As you evaluate problem solving performance, consider all aspects of the activity - the written work, the process used, and the responses made to oral questions.

14) Distribute Sample Problems, Answers, and a List of Sources for Problems. [Give each participant a copy of handouts PS-20 through PS-23.] This is a collection of problems covering a variety of topics and varying in difficulty. Some of the problems are for students in the primary grades and have holiday themes. This is another way to motivate your students to solve problems. Take the textbook problems and rewrite them for holidays and special activities.

There are many problems available for use in your classroom. This list of sources will help you in beginning your own collection of problems (Handout PS-24). Stress the use of the annotated bibliography from NCTM 1980 Yearbook which contains entries from books and journals. Distribute among the participante several copies of the Arithmetic Teacher and call their attention to "Problem Solving: Tips for Teachers."

15) Discuss Some Implications of the Research on Problem Solving. Marilyn Suydam has prepared several summaries of the research on problem solving and has provided many teaching suggestions based on the research results. Let's look at a few of the instructional implications (Transparency PS-25).

Research indicates that "children can solve problems even before they come to school." Young children frequently use counting strategies and manipulative materials to solve problems. Do not rush into the writing of number sentences to solve problems. Children sometimes feel that they have to write a number sentence and will write incorrect sentences because they cannot coordinate their modeling of problems with a mathematical sentence.

Provide opportunities for children to become familiar with various strategies. Many studies have shown that the heuristic type of problem solving instruction was the most effective. The heuristic method stresses the teaching of general problem solving skills which help a problem solver to approach and understand a problem and, then, efficiently use their resources in solving it.

Children have difficulty with problems that cannot be solved by a routine application of a single arithmetic operation. Do we overemphasize one-step problems? We may be teaching students that they do not have to think without realizing what we are doing. We need to insure that students have to think about problems; therefore, try to present problems in ways so that they cannot be solved mechanically.

Lack of reading skill is frequently given as a reason for poor problem solving ability. This is an over-simplification. Instruction on reading skills in mathematics can lead to increased problem solving ability, however, in the mathematics assessments conducted by NAEP, students were unable to solve problems even when reading difficulty was not a concern. Children must learn to interpret the words in the problem.

Students have to know how to go about solving the problem when they use the calculator to perform the computation. They are less afraid to attempt difficult problems if they are allowed to use the calculator. The calculator does not solve the problem; the student must do that. Use of calculators gives the students more time to concentrate on ways of approaching a problem. The fear of doing the computation has been removed.

Teach problem solving by solving problems.

16) Suggestions for a Workshop in Your Local District.

The material in this manual could be broken into small segments for local workshops. You may want to divide your teachers into groups according to the grades they teach, such as K - 3 and 4 - 6. There is enough material here for several two-hour sessions on each level or a full-day session on each level.

To conduct several short workshops, you might want to pick a couple of strategies and work on those for the entire session. Choose the strategies that seem most appropriate for your teachers or the strategies that you especially like. Remember there are many strategies that are not included in this manual, so feel free to select some of those instead of using the ones presented here.

All of the problems in this manual are not appropriate for every grade level. When you plan your workshop, try to select appropriate sections of the manual and limit your material to what you will be able to cover reasonably well during the time available for your workshop.

WHAT IS A PROBLEM

Webster's New Twentieth Century Dictionary --

A problem is:

1) a question proposed for solution or consideration,
2) a question, matter, situation, or person that is perplexing or difficult,
3) in mathematics, anything required to be done, or requiring the doing of something.

Funk and Wagnalls' New Standard Dictionary of the English Language --

A problem is a question for discussion and settlement; a matter of uncertainty requiring further light to determine the truth, especially when difficult or uncertain of solution.

American Heritage Dictionary --

A problem is a question or situation that presents uncertainty, perplexity or difficulty.

Stephen Krulik and Jesse Rudnick in Problem Solving: A Handbook for Teachers --

A problem is a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent or obvious means or path to obtaining the solution.

Randall Charles and Frank Lester in Teaching Problem Solving: What, Why, and How --

A problem is a task for which:

1) the person confronting it wants or needs to find a solution.
2) the person has no readily available procedure for finding the solution.
3) the person must make an attempt to find a solution.

PROBLEM SOLVING

Krulik and Rudnick --

> Problem solving is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation.

National Council of Supervisors of Mathematics --

> Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations.

National Council of Teachers of Mathematics, 1983 Yearbook --

> Problem solving is the process of attacking ... a problem: accepting the challenge, formulating the questions, clarifying the goal, defining and executing the plan of action, and evaluating the solution.

POLYA'S FOUR STAGES IN PROBLEM SOLVING

I. UNDERSTANDING THE PROBLEM

What is the unknown? What are the data? What is the condition?
Is it possible to satisfy the condition?
Separate the various parts of the condition. Can you write them down?

II. DEVISING A PLAN

Have you seen the same problem in a slightly different form?
Do you know a related problem?
Look at the unknown. Try to think of a familiar problem having the same or a similar unknown.
Could you use a previously solved problem that is related to your problem?
Could you restate the problem?
If you cannot solve the proposed problem, try to solve first some related problem or some part of your problem.
Did you use all the necessary data? Have you taken into account all of the essential ideas in the problem?

III. CARRYING OUT THE PLAN

Check each step.
Can you see clearly that the step is correct?

IV. LOOKING BACK

Can you check the result? Can you check the argument?
Can you derive the result differently?
Can you use the result, or the method, for some other problem?

AN AGENDA FOR ACTION

Recommendations for

School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that --

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population ;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

TYPES OF PROBLEMS

1. DRILL EXERCISE

$$\begin{array}{r} 346 \\ \underline{\text{X}\ \ 28} \end{array}$$

2. SIMPLE TRANSLATION PROBLEM

Jenny has 7 tropical fish in her aquarium. Tommy has 4 tropical fish in his aquarium. How many more fish does Jenny have than Tommy?

3. COMPLEX TRANSLATION PROBLEM

Ping-Pong balls come in packs of 3. A carton holds 24 packs. Mr. Collins, the owner of a sporting goods store, ordered 1800 Ping-Pong balls. How many cartons did Mr. Collins order?

4. PROCESS PROBLEM

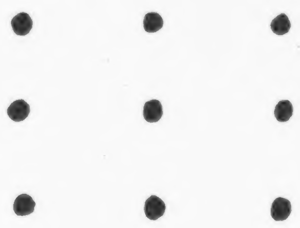
A chess club held a tournament for its 15 members. If every member played one game against each other member, how many games were played?

5. APPLIED PROBLEM

How much paper of all kinds does your school use in a month?

6. PUZZLE PROBLEM

Draw 4 straight line segments, without picking up your pencil, to pass through all 9 dots in the figure below. Each segment must be connected to an endpoint of at least one other line segment.



TYPES OF PROBLEMS

1. RECOGNITION EXERCISES

Give an example of a proper fraction greater than $\frac{3}{4}$.

2. ALGORITHMIC EXERCISES

Choose two whole numbers. Find their sum and nonnegative difference. Add these results. Any observations?

3. APPLICATION PROBLEMS

If an 8-inch pizza serves two, how many should two 12-inch pizzas serve?

4. OPEN-SEARCH PROBLEMS

Susan wanted to buy a candy bar that costs 25 cents. The machine would take pennies, nickels, and dimes in any combination. List the different conbinations of coins she could use to pay for her candy.

5. PROBLEM SITUATIONS

Design a parking lot. Possible problems to consider include the following.

a) How large is the lot?

b) How large should each parking space be?

c) At what angle should each space be placed?

d) How much should be charged per car per hour?

PROBLEMS

Too Much Information

In last night's basketball game 7 players fouled out and only 3 boys from our team scored. Joe scored 12 points, Tom scored 15 points, and Gary scored 9 points. What was the total number of points scored by the three boys?

Too Little information

Judy planted tulip bulbs in her garden this morning. She planted 80 bulbs. How many rows did she plant?

No Numerical Solution

Betty wanted to buy some tropical fish. She wanted to buy 10 guppies and 5 angelfish. How could she find out the cost of the 15 fish?

No Questions Asked

On her math test Jill answered 22 of 25 problems correctly. Her friend Beth answered 17 of 25 problems correctly. Make up some questions which could be asked about the situation.

Problems Without Numbers

A sporting goods store has some baseballs and some softballs on hand. A baseball and a softball are not worth the same amount of money. What is the total value of the baseballs and the softballs on hand?

Mathematical Sentence Identification Only

Billy had 10 baseball cards. His brother gave him 15 more cards. How could you find out how many baseball cards Billy has now? Write a mathematical sentence.

Word Problems constructed from a Given Mathematical Sentence

Write a word problem to be solved by ____ + 4 = 10.

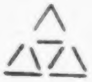
PROBLEMS

1. Bill had 95 cents. Steve had 55 cents. Bill spent 76 cents. How much does Bill have left?

2. Ruth weighs 15 kilograms less than her older brother. She is 8 centimeters shorter than he. How much does Ruth weigh?

3. Find two numbers whose sum is 11 and whose product is 24.

4. Arrange 9 toothpicks as shown.



   a) Remove 2 toothpicks and leave 3 triangles.

   b) Remove 2 toothpicks and leave 2 triangles. (Go back to the original formation before removing 2 toothpicks.)

5. How many different ways can you make change for a 50¢ piece without using pennies?

6. Where would you open a book so the sum of the two page numbers is 313?

7. D = 5. Other letters stand for a unique digit. Identify the digits represented by each letter.

$$\begin{array}{r} \text{DONALD} \\ +\ \text{GERALD} \\ \hline \text{ROBERT} \end{array}$$

8. How many straight lines can be drawn through 2 points? 3 non-collinear points? 4 non-collinear points? 5 non-collinear points? n non-collinear points?

9. One day this lady in New York City hailed a passing taxicab. On the way to her destination the lady talked so much that the driver got quite annoyed. Driver: I'm sorry lady, but I can't hear a word you're saying. I'm deaf as a post, and my hearing aid hasn't work all day. When the lady heard this, she stopped yakking. But after she left the cab she suddenly realized that the cabbie had lied to her. How did she know?

10. Which horse won the race and who was the owner of the winning horse?

    1) Three horses named Steady, Speedy, and Slowpoke ran in a race.

    2) Tic, Tac, and Toe each owned one of the horses.

    3) Tac's horse nearly won.

    4) This was Speedy's fifth race.

    5) The black horse was owned by Toe.

    6) The horse owned by Tic had not previously raced.

    7) Steady broke an ankle after the start of the race.

    8) The horse that won was brown.

ANSWERS

1. 19 cents.

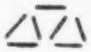
2. Not enough information to find Ruth's weight.

3. 3, 8 Look at pairs whose sum is 11.

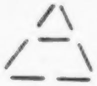
11,0 9,2 7,4
10,1 8,3 6,5

Check the product of each pair.

4. a)



b)



5. 10

6. Pages 156 and 157

7. D = 5, O = 2, N = 6, A = 4, G = 1, E = 9, R = 7, B = 3, T = 0, L = 8

8. 1, 3, 6, 10, $\frac{n(n - 1)}{2}$

9. He heard her when she gave her destination.

10. Slowpoke, Tic

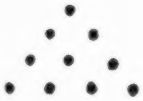
PROBLEM SOLVING STRATEGIES

1. Act out the problem.
2. Use objects.
3. Draw a picture.
4. Guess and check.
5. Look at it another way.
6. Identify a pattern.
7. Make an organized list.
8. Make a table.
9. Work backwards.
10. Identify and solve cases.
11. Make a model.
12. Use logical reasoning.

PROBLEMS

1. There are 5 people at a party. If each person shakes hands with the other guests, how many handshakes will there be?

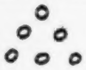
2. Ten coins are arranged to form a triangle as shown in the figure. By rearranging only three of the coins, form a new triangle that points in the opposite direction from the one shown.



3. What is the smallest number of birds that could fly in this formation:

   2 birds in front of a bird,
   2 birds behind a bird,
   1 bird between 2 birds?

4. Arrange the digits 1 through 6 in the circles so that the sum of each side of the triangle is 9.



5. Some months have 31 days; some have 30. How many have 28 days?

6. Find the sum of the first n odd numbers.

7. The number 475 is a three-digit number that uses only the three digits 4, 5, and 7. How many three-digit numbers can be formed using these 3 digits, if repeated digits are allowed?

8. Four students--Bill, Minnie, Dawn, and Chuck--met at a rally. They are from the towns of Vegas, Washington, Tampa, and Sierra. Chuck is from Vegas and Minnie is not from Sierra. If Dawn is from Tampa, where is Bill from?

9. Write a ten-digit number whose first digit tells the number of 0s in that number, the second digit tells the number of 1s in the number, ..., and the tenth digit tells the number of 9s in the number.

10. If two numbers have a sum which is an even number, what do you know about the two numbers?

11. Bryan can cut a log into 3 pieces in 6 minutes. How long will it take him to cut a log into 6 pieces?

12. If a cork and a bottle together cost $1.10 and the bottle costs $1 more than the cork, what is the cost of each?

1. A sleepy koala bear wants to climb to the top of a fruit tree that is 10 m tall. Each day the bear climbs up 5 m, but at night, while asleep, slides back 4 m. At this rate, how many days will it take the bear to reach the top of the tree?

2. Reversing the digits of 132 produces 231. When these numbers are added, the sum is a palindrome (a number that reads the same backward or forward). 132 + 231 = 363. Find some other numbers that produce a palindrome. Give a rule for finding such numbers.

3. Peggy is writing the numbers from 1 to 1000. She stops to rest after writing a total of 630 digits. What is the last number she wrote?

4. Jill had 8 more Smurfs than Kathy. When they counted all their Smurfs, they had 24. How many Smurfs did Kathy have?

5. What is the greatest number of pieces you can get if you cut a pizza using 5 straight cuts?

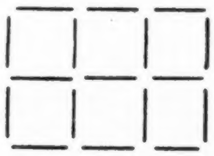
6. How many ways can you put 7 lima beans on 2 plates?

7. Barbara, Debbie, and Lynne all attend Lincoln High School. They presently hold the positions of cheerleader, yearbook editor, and class president, not necessarily in that order. Last Friday, the editor looked for the president after the game, but the president had gone off with the cheerleader. The president was livelier than the editor. Debbie was livelier than Barbara. Lynne had never met Debbie. Can you identify each of the lovely girls?

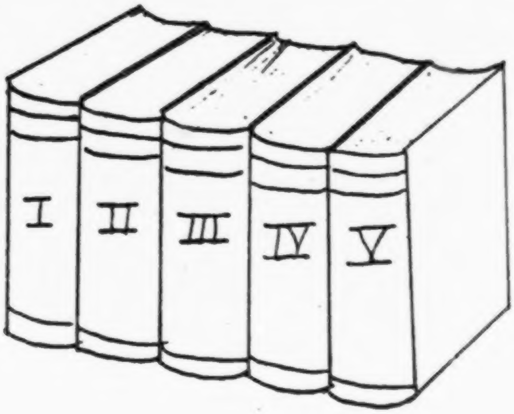
8. In the division problem below, each letter represents a different one-digit number. Determine the number represented by each letter.

```
     BEST
A ) SAND
    A
    BA
    BT
     TN
     TN
       D
       D
```

9. Here are 17 toothpicks. Remove 6 to leave just 2 squares.



10. The pages of each book are 1" thick and its covers are 1/5" thick. If a bookworm begins eating at the first page of Volume I, and eats through to the last page of Volume V, how many inches has he eaten through?

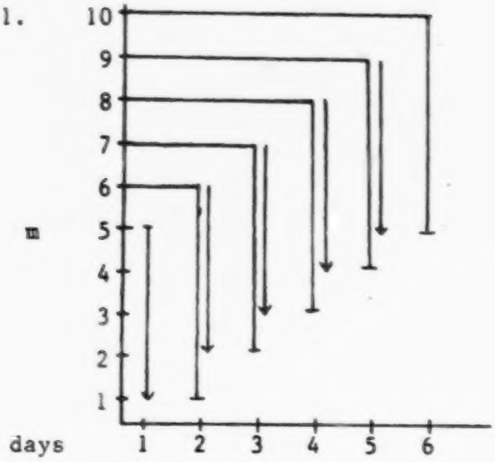


11. How many different squares are there in the 10-by-10 grid?

12. Three boxes are labeled "apples", "oranges", and "apples and oranges". Each label is incorrect. How can you relabel each box correctly by looking at only one piece of fruit from one box?

SOLUTIONS

1.



Strategies: Draw a picture.
Look for a pattern.
Work backwards.

2.

$$\begin{array}{r} 132 \\ +\ 231 \\ \hline 363 \end{array} \qquad \begin{array}{r} 243 \\ +\ 342 \\ \hline 585 \end{array} \qquad \begin{array}{r} 124 \\ +\ 421 \\ \hline 545 \end{array} \qquad \begin{array}{r} 314 \\ +\ 413 \\ \hline 727 \end{array}$$

Hint: Look at the sum of the digits and the value of the middle digit.
Rule: If the sum of the digits is less than ten and the middle digit is less than five, the number will always produce a palindrome after one step.

Strategies: Look for a pattern.
Guess and check.

3.

Numbers	Number of digits used	Total digits used
1-9	9	9
10-99	180	189
100-199	300	489
200-299	300	789

So the number is between 199 and 299.

Numbers	Number of digits used	Total digits used
200-224	75	564
225-234	30	594
235-244	30	624
245-246	6	630

So the last number written is 246.

Strategy: Make an organized list.

4. **Strategy:** Use objects.

Start with 24 and make a set for Kathy and a set for Jill. Place 8 in Jill's set, then one for Kathy and one for Jill until all are placed.

5. **Strategies:** Guess and check.
Draw a picture.
Look for a pattern.
Look at it a different way.

Number of Cuts	Number of Pieces
1	2
2	4
3	7
4	11
5	16

6. **Strategies:** Draw a picture.
Use objects.
Make an organized list.

7,0
6,1
5,2
4,3
3,4
2,5
1,6
0,7

Total: 8 ways

7. **Strategy:** Make a table.

	President	Cheerleader	Editor
Debbie	X	0	X
Barbara	0	X	X
Lynne	X	X	0

Case 1: Lynne = Cheerleader, Debbie = Editor, Barbara = President.

Check: P livelier than E → B livelier than D. NO!

Case 2: Lynne = Editor, Debbie = Cheerleader, Barbara = President.

Check: P livelier than E → B livelier than L. OK
D livelier than B → C livelier than P. OK
P met C → B met D. OK
E met P → L met B. OK
L not met D → E not met C. OK

8. Strategy: Make an organized list.

$A \times B = A \rightarrow B = 1$

$A \times E = BT \rightarrow 1T$

$A \times S = TN$

$A \times T = D$

$S - A = 1$

$A - T = T \rightarrow A = 2T$

Start with the A and T.

If $T = 2, 3, 4$ then $A = 4, 6, 8$.

$A \times T = D = 8, \cancel{18}$

So $D = 8$, $T = 2$, and $A = 4$.

$S - A = 1 \rightarrow S = 5$

$A \times E = BT \rightarrow A \times E = 12$ so $E = 3$.

$A \times S = TN \rightarrow N = 0$

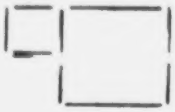
Check:

```
         1352
     4 ) 5408
         4
         14
         12
          20
          20
            8
            8
            0    OK
```

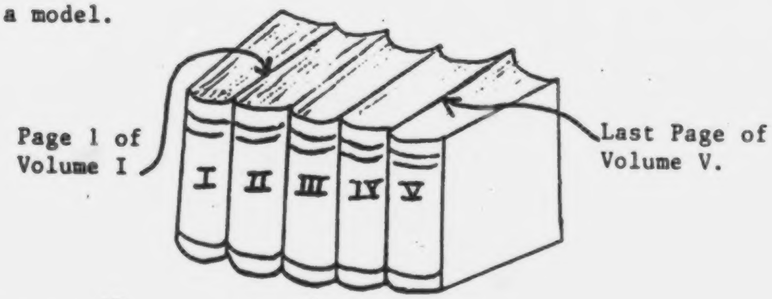
9. Strategy: Look at it a different way.

Must the 2 squares be the same size? No, the problem did not say that the 2 were the same size.

Then



10. Strategy: Make a model.

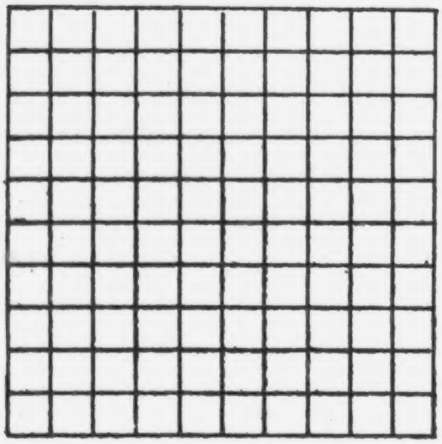


8 covers at $\frac{1}{5}''$ each $= 1\frac{3}{5}''$

3 sets of pages at 1" each = 3"

Total: $4\frac{3}{5}''$

11.



Strategies: Look for a pattern.
Make a table.
Make an organized list.

	SQUARES						
GRID	1 X 1	2 X 2	3 X 3	4 X 4	5 X 5	...	TOTAL
1 X 1	1						1
2 X 2	4	1					5
3 X 3	9	4	1				14
4 X 4	16	9	4	1			
5 X 5	25	16	9	4	1		
⋮							
10 X 10	100	81	64	49	36		385

12. **Strategies:** Draw a picture.
Make a model.

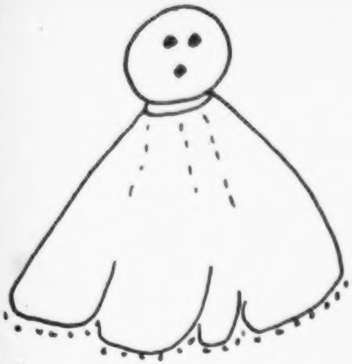
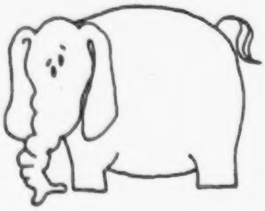
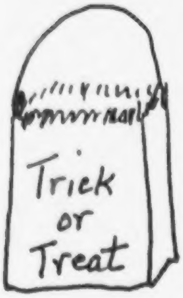
apples
and
oranges

All labels are incorrect. So take a piece of fruit from apples and oranges. You can relabel all boxes by looking at that one piece.

TECHNIQUES FOR TEACHING PROBLEM-SOLVING

1. Create an atmosphere of success.
2. Encourage your students to solve problems
3. Take advantage of problem situations which arise normally in and out of the classroom.
4. Present problem situations to the child in a variety of ways, not just in written form.
5. Teach students how to read a problem.
6. Encourage children to make up problems for the class to solve.
7. Have children dramatize situations described in problems.
8. Rewrite textbook problems using the names of teachers from your school or the names of the children in your classroom.
9. Make problem-solving a class activity.
10. Raise creative, constructive questions.
11. Make your problems meaningful for the children in your classroom.
12. Develop a problem-solving framework: identify the problem, plan a method for solution, write a mathematical sentence, solve, interpret the answer in terms of the problem.
13. Encourage your students to use a calculator.
14. Incorporate problem-solving throughout the learning of each skill and concept.
15. Use problem-solving to review concepts.
16. Solve several problems each week. Do not spend long periods of time (several weeks) on problem-solving. Try using "problem of the day" technique.
17. Try to keep the mathematics involved in your problems well within the students' ability level.
18. Encourage children to find alternate methods of solution to given problems.
19. Provide the children with problems of varying degrees of difficulty.
20. Do not expect every child to excel in problem-solving, but expect every child to participate in problem-solving activities.

PS-15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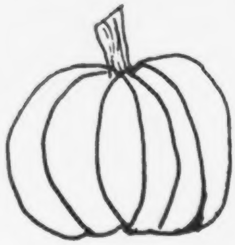
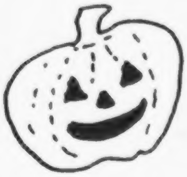
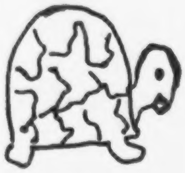
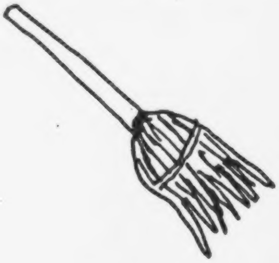
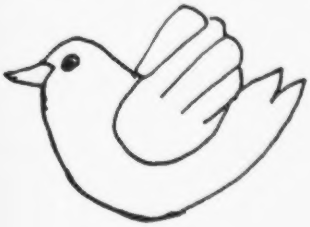
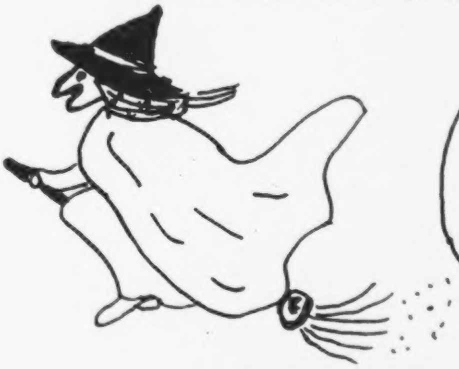
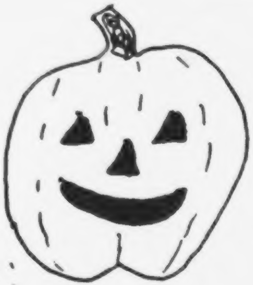
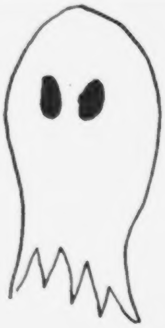
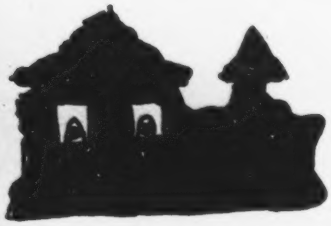
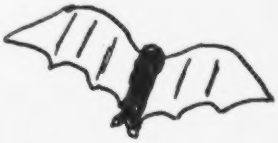
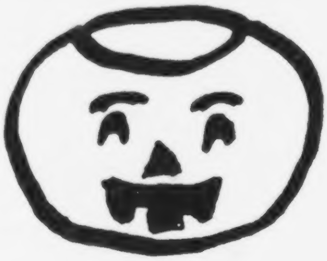
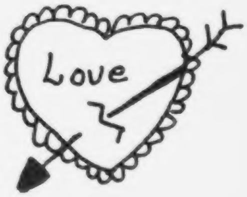




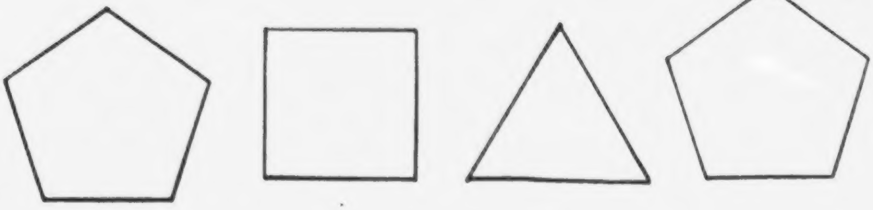



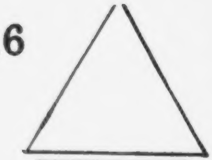





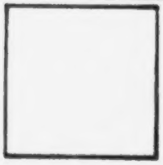





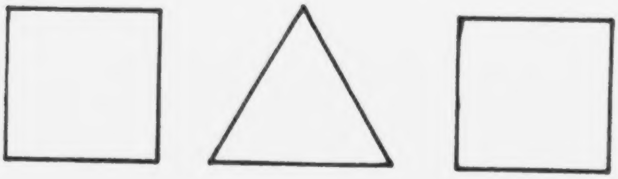
MARK THE OBJECT IN EACH GROUP THAT DOES NOT BELONG.

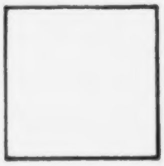
PS-16

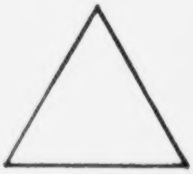


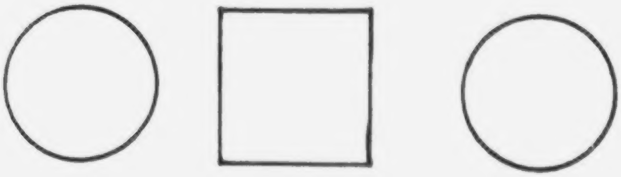


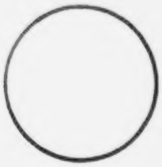


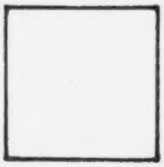


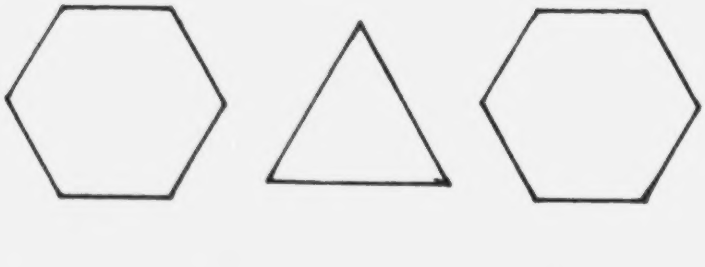


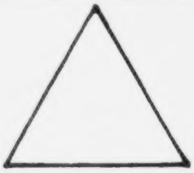


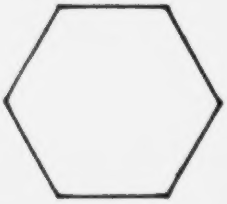












MARK THE GEOMETRIC SHAPE THAT WOULD BE NEXT IN EACH GROUP.

SUGGESTIONS FOR WRITING PROBLEMS

1. Write problems that will interest your students.
2. Consider the reading ability of the students who will be solving the problem.
3. Do not concentrate on one type of problem to the extent that you exclude other types.
4. The information given should be realistic.
5. The "unknown" should reasonably be expected to be unknown in reality.
6. The answer should be a quantity someone might have a reason to seek.
7. Some problems should contain insufficient or extraneous data since a critical analysis of the data is necessary in real-world problem solving.
8. Use some open-search problems since they require higher-level thinking than an application type problem.
9. Brevity is not necessarily a virture in writing a problem.
10. Problem posing requires "the creativity of an artist to pose a problem so that the potential solver will -
    1) be motivated to solve the problem;
    2) understand and retain the concept involved in the solution of the problem;
    3) learn something about the art of solving problems" (NCTM, 1980 Yearbook, p. 33).

Arithmetic Teacher, Jan. 1983, p. 6-7

PS-18

Fig. 1

A Point System for Scoring Written Work

Understanding the problem

0 completely misinterprets the problem
1 misinterprets part of the problem
2 complete understanding of the problem

Choosing and implementing a solution strategy

0 no attempt or a totally inappropriate strategy
1 partly correct strategy based on part of the problem interpreted correctly
2 a strategy that could lead to a correct solution if used without error

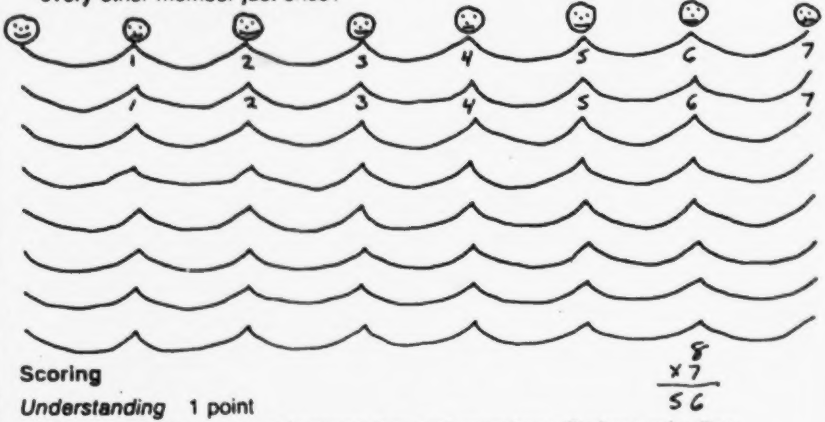
Getting the answer

0 no answer or wrong answer based on an inappropriate solution strategy
1 copying error, computational error, partial answer for a problem with multiple answers, answer labeled incorrectly
2 correct solution

Fig. 2

An Example Showing How to Use the Scoring System

Problem: Bill's ping pong club is planning a tournament. If there are 8 people in the club, how many games must be played in order to have every member play every other member just once?



Scoring

Understanding 1 point

The student does not understand that each member will play each other member just once.

Strategy 1 point

The student selected an appropriate strategy but used it incorrectly because of misunderstanding.

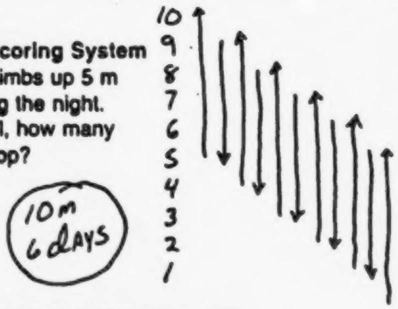
Answer 0 points

The answer is wrong because of the incorrect use of the solution strategy.

Fig. 3

An Example Showing How to Use the Scoring System

Problem: A well is 10 m deep. A frog climbs up 5 m during the day, but slips back 4 m during the night. If the frog starts at the bottom of the well, how many days does it take the frog to get to the top?



Scoring

Understanding 2 points

The student appears to have understood all parts of the problem.

Strategy 2 points

The student selected an appropriate strategy and implemented it correctly.

Answer 2 points

The answer is correct.

Arithmetic Teacher, Jan. 1983, p. 7

Fig. 4

Some Observations Recorded in a Problem-Solving Notebook

Jack Fulton

10/2 Very willing to try problems and help others.
11/3 Learned to use strategies very easily
Gives good explanations

Mary Cummings

10/8 Tries hard but has trouble selecting a strategy
10/20 Knows how to accurately complete tables.
11/1 Continues to forget to check her work
11/2 Getting better at suggesting solution strategies.

Fig. 5

Problem-Solving Observation Checklist

Student J. Usilton
Date Oct. 31

	Frequently	*Sometimes*	*Never*
1. Selects appropriate solution strategies		✓	
2. Accurately implements solution strategies		✓	
3. Tries a different solution strategy when stuck (without help from the teacher)			✓
4. Approaches problems in a systematic manner (clarifies the question, identifies needed data, selects and implements a solution strategy, checks solution)		✓	
5. Shows a willingness to try problems	✓		
6. Demonstrates self confidence	✓		

PS-20

COLLECTION OF PROBLEMS

1. The head of a fish is $\frac{1}{3}$ as long as its body. The tail of the fish is as long as its head and body combined. The total length of the fish is 48 cm. How long is each part of the fish?

2. Given four digits, where in the top four boxes should you write them to obtain the largest product?

$$\begin{array}{r} \square\square \\ \times\ \square\square \\ \hline \square\square\square\square \end{array}$$

3. Adult tickets cost \$6 and student tickets cost \$4. Jenny sold 13 tickets for \$66. How many adult tickets did she sell?

4. Eight pennies are placed in a row on a table top. Every other coin is then replaced by a nickel. Every third coin is replaced with a dime. Finally every fourth coin is replaced with a quarter. What is the value of the 8 coins now on the table?

5. Eric and 6 friends spent a day at the amusement park. At the end of the day, they decided to pair up for roller-coaster rides so that each friend would ride with each of the other friends exactly once. How many rides must be taken?

6. Draw 3 straight lines to cut the pie into 7 pieces.

7. How can you get exactly 6 liters of water from a river by using two containers with capacities of 9 liters and 4 liters? The containers do not have markings of any kind on them.

8. A man had 26 cows. All but 9 died. How many lived?

9. Jim's mother paid him \$1.60 in quarters, dimes, and nickels. He received 17 coins in all. How many of each coin could he have received?

10. James went to the store to buy 3 cartons of Coke. If each carton contains 6 bottles, how many bottles of Coke did James buy?

11. Beth saw 18 chickens and pigs in a farmyard. If she counted 52 legs, how many chickens and how many pigs were in the farmyard?

12. At Barry's school there are 6 basketball teams. They want to plan an after school tournament so that each team will play every other team once. How many games must be played?

13. Each letter represents a single digit only. Solve the cryptarithms.

a) $(HE)^2 = SHE$

b)
$$\begin{array}{r} \text{WRONG} \\ +\ \text{WRONG} \\ \hline \text{RIGHT} \end{array}$$

14. I am thinking of two 2-digit numbers. They have the same digits. The sum of the digits of each number is 10 and the difference between the numbers is 18. What are the numbers?

15. Write expressions for the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 using four 4s and any combination of the four basic operations. Each expression must involve four 4s.

16. Mr. Walt owns the local gas station. This morning 7 cars came in and each car needed 4 new tires. How many new tires were needed?

17. If 32 pencils are available and 8 children are each to receive the same number of pencils, how many should each receive?

18. Two boys are ahead of Bill in a line. Three girls are behind Bill. How many children are in line?

19. John, Alex, and Maryann live on the same road. John lives 10 km from Alex. Maryann lives 2 km from Alex. How far does John live from Maryann?

20. Ninety-six children are to be placed in rows, with eight children in each row. How many rows will there be?

21. There are three men named Tom, Dick, and Harry who are going on an archaeological expedition. Each of the men has two occupations. The occupations are geologist, photographer, archaeologist, writer, sociologist, and paleontologist. From the list of facts below see if you can find the two occupations of each man.

    1. Both the paleontologist and sociologist used to go hunting with Tom.

    2. The photographer borrowed a book from the archaeologist.

    3. Harry beat both Dick and the photographer at chess.

    4. The geologist offended the paleontologist by laughing at his mustache.

    5. The geologist courted the photographer's sister.

    6. Dick owed the sociologist some money.

22. There are 16 teams in the Mid-South Football League. To conduct their annual draft of players, teams in each city must have direct telephone lines to each of the other cities. How many direct phone lines must be installed by the telephone company to accomplish this?

23. Divide 30 by one-half and add 10. What is the answer?

24. Sue wanted to buy a candy bar that costs 35 cents. The machine would take nickels, dimes, and quarters in any combination. List the different coins she could have used to pay for her candy.

25. I am a proper fraction. The sum of my numerator and denominator is one less than a square. Their difference is one more than a square. Their product is one less than a square. What's my name?

ANSWERS FOR COLLECTION OF PROBLEMS

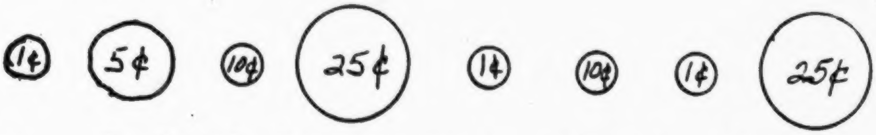
1. body = 18 cm
   head = 6 cm
   tail = 24 cm

2. Choose any 4 digits. From the digits pick two pairs of two-digit numbers that you think will produce the two largest products. The factors in a pair with the smallest difference have the largest product.

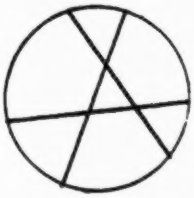
   Example: Suppose the digits are 7, 4, 8, 9.

   97 X 84 or 94 X 87
   97 - 84 = 13 94 - 87 = 7
   97 X 84 = 8148 94 X 87 = 8178

3. 7 adult tickets and 6 student tickets.

4. 78¢ 

5. 21 rides. The formula for n persons is $\frac{n(n-1)}{2}$ rides.

6. 

7. Fill the 4 liter container and pour in 9 liter container. Repeat this until 9 liter container is full leaving 3 liters in the 4 liter container. Empty the 9 liter container. Pour the 3 liters in the 9 liter container. Fill the 4 liter and pour into the 9 liter container until the 9 liter is full leaving 2 liters in the 4 liter container. Empty the 9 liter container. Pour the 2 liters from the 4 liter container into the 9 liter container. Fill the 4 liter container and pour into the 9 liter container. The 9 liter container now contains 6 liters.

8. 9 Since "all but 9 died," 9 lived.

9. Several solutions.

Quarters	Dimes	Nickels
3	3	11
2	7	8
1	11	5
0	15	2

10. 18 bottles

11. 10 chickens, 8 pigs

12. 15 games

13. a) $(25)^2 = 625$ b) $\begin{array}{r} 24153 \\ \underline{24153} \\ 48306 \end{array}$

14. 46 and 64

15. 0: $44 - 44$
1: $44 \div 44$
2: $(4 \div 4) + (4 \div 4)$
3: $(4 + 4 + 4) \div 4$
4: $4 + 4\ (4 - 4)$
5: $(4 \times 4 + 4) \div 4$
6: $4 + \left(\frac{4 + 4}{4}\right)$
7: $44 \div 4 - 4$
8: $(4 \times 4) - 4 - 4$
9: $4 + 4 + (4 \div 4)$

Other solutions are possible.

16. 28

17. 4

18. 6

19. 8 km or 12 km

20. 12 rows

21. Tom is the photographer and writer.
Dick is the archaeologist and paleontologist.
Harry is the sociologist and geologist.

22. 120

23. 70

24.

Quarters	Dimes	Nickels
1	1	0
1	0	2
0	3	1
0	2	3
0	1	5
0	0	7

25. $\frac{3}{5}$

PROBLEMS FOR PRIMARY GRADES

1. Two bats were hanging on the ceiling. Two more bats came to hang from the ceiling. Now how many bats are hanging from the ceiling?

2. Tina had 3 pieces of halloween candy. She ate 2 of them. How many did she have left?

3. Carlos has 5 pet black cats. He has 3 pet gray cats. How many cats does Carlos have?

4. There were 5 pumpkins on the fence. 3 disappeared. How many were left?

5. 7 candy apples are on the table. 4 candy apples are in the trick or treat bag. How many candy apples are there in all?

6. 3 ghosts are in the attic. 5 ghosts are in the basement. How many ghosts are there in all?

7. Wendy Witch has 4 long brooms. She has 6 short brooms. How many brooms does Wendy have?

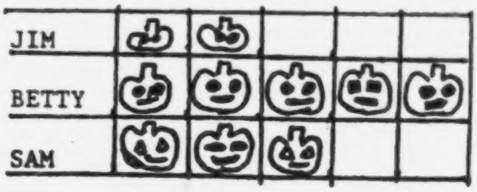
8. Wendy had 10 black cats. 2 of the cats were stolen by Wendy's rival on October 29th. How many did Wendy have left for Halloween night?

9. Balloons are 5¢ each. How much would 6 balloons cost?

10. Space monster stickers cost 50¢. Ghost stickers cost 60¢. What is the cost to buy one of each? What is the cost to buy 2 space monsters and 3 ghost stickers?

11. Bat wings cost 25¢ each. Jeremy would like to buy 3 pairs. He has $1.60. Does he have enough money?

12.



Jim has _________ pumpkins. Betty has _________ pumpkins. Sam has _________ pumpkins. Who has the most? ___________ How many more than _____________? Who has the least? ___________

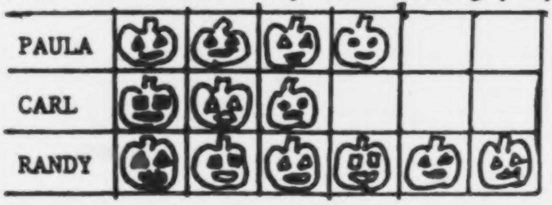
13. Wendy drinks 1/4 of her bowl of witches brew. Wilma drinks 1/3 of her bowl of witches brew. Which witch drinks more brew?

14. 6 dogs are in my driveway. 4 dogs are on the grass. How many more dogs are in the driveway?

15. 3 groups of children go trick or treating. There are 3 children in each group. How many children are there in all?

16. Steve buys 4 tickets to the haunted house. Each ticket costs 10¢. How much does Steve spend?

17. Paula, Carl, and Randy are selling pumpkins for halloween.



Who sold the most (fewest)?
Who sold 5?
Who sold more (fewer) pumpkins, Carl or Randy?
How many more (fewer)?

18. There are 40 legs in all. Each spider has 8 legs. How many spiders are there?

19. Luke bought 5 tickets to the Halloween carnival. He paid for them with a 20-dollar bill and a 10-dollar bill. How much was each ticket?

20. Some children picked 34 apples. They wanted to divide the apples equally into 4 bags. How many apples will they put in each bag? How many will be left?

21. Betsy bought 42 cans of juice. There were 6 cans in each box. How many boxes of juice did Betsy buy?

22. The Halloween carnival begins at 7:00 p.m. It lasts 1 hour and 30 minutes. What time will it be over?

23. Wendy Witch was 15 minutes late for the carnival. What time did Wendy arrive? (The carnival started at 7:00 p.m.)

24. Amy makes Halloween sandwiches with either cheese, jam, or meat. She uses white bread or rye bread. How many different sandwiches can she make?

25. Mrs. Smith bought 27 Halloween books for her class. Each book cost $4.95. How much did all the books cost?

PROBLEMS FOR PRIMARY GRADES

1. Two leprechauns were playing in the back yard. Two more leprechauns arrive to join the fun. Now how many leprechauns are playing in the yard?

2. Tina had 3 green lollipops. She ate 2 of them. How many does she have left?

3. Carl has 5 shamrocks. He also has 3 four-leaf clovers. How many does he have altogether?

4. Leprechauns often carry a bag of gold. Patrick is a leprechaun who has a bag containing 10 pieces of gold. As he plays, Patrick loses 3 pieces of gold. How many are left?

5. Shamrock stickers cost 30¢ each. Leprechaun stickers cost 60¢ each. What is the cost to buy one sticker of each kind? What is the cost to buy 3 shamrocks and 2 leprechauns?

6. Thirty shamrocks are hidden in the room. Fifteen have just been found by the students. How many are still hidden?

7. St. Patrick's Day balloons are 5¢ each. How much would 6 balloons cost.

8. Patrick has two types of gold pieces in his treasure bag. If he has 3 pieces of each type, how many pieces of gold does he have in his bag?

9. Have students use green watercolors or crayons to make a bar graph showing:

   a. the number of students wearing green.

   b. the number of green pencils being used in the classroom.

   c. the number of students who use a green toothbrush.

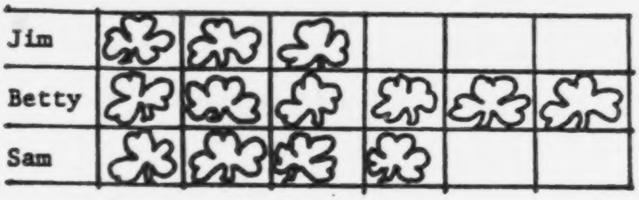
   d. the number of students whose favorite color is green.

   e. the number of students who have a green room at home.

   f. the number of students whose favorite lollipop is lime.

   After the graph has been completed, ask students questions that can be answered by looking at the graph.

10.



Jim has ______ shamrocks. Betty has _______ shamrocks. Sam has _______ shamrocks.

Who has the most? _____________ How many more than __________? Who has the least?______________

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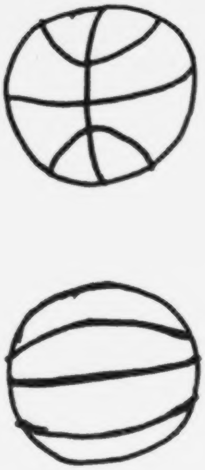
IMPLICATIONS FROM RESEARCH

1. Children can solve problems even before they come to school.
2. Knowing problem-solving strategies or heuristics promotes children's ability to solve problems.
3. Children have much more difficulty with multistep problems than with one-step problems.
4. Reading skill is related to problem solving achievement, but children may be unable to solve problems even if they know all the words.
5. Calculators can aid children by expanding their repertoire of problem solving strategies.

(Marilyn N. Suydam, "Update on Research on Problem Solving: Implications for Classroom Teaching," Arithmetic Teacher; February 1982, p. 56-59.)

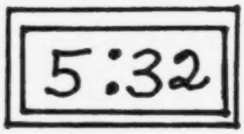
PROBLEM SOLVING TASK CARDS

FOR PRIMARY GRADES



THE 10 MEMBERS OF THE WACKY COMEDY BASKETBALL TEAM ALWAYS HAVE A WARM-UP DRILL IN WHICH EACH PLAYER TOSSES THE BALL TO EACH OF THE OTHER PLAYERS. HOW MANY TIMES IS THE BALL TOSSED DURING THIS DRILL?

HOW MANY DIFFERENT OUTFITS CAN YOU MAKE IF YOU HAVE A YELLOW, A RED, AND A BROWN SHIRT AND BLUE SHORTS AND WHITE SHORTS?



IN A 12-HOUR PERIOD, FOR HOW MANY MINUTES IS THE SUM OF THE DIGITS DISPLAYED ON A DIGITAL CLOCK EQUAL TO 10?

HAVE STUDENTS PRETEND THAT THEY ARE WORKING IN AN ICE-CREAM PARLOR. HAND OUT 2 DIFFERENT TYPES OF CONES MADE OUT OF BROWN PAPER AND FOUR FLAVORS OF ICE CREAM MADE FROM PAPER CIRCLES OF DIFFERENT COLORS. QUESTION: HOW MANY DIFFERENT FLAVORS OF SINGLE-DIP CONES ARE POSSIBLE WITH THE MATERIALS YOU HAVE BEEN GIVEN?

GRADES: 1 - 4

SUGGESTED STRATEGY: MAKE AN ORGANIZED LIST.

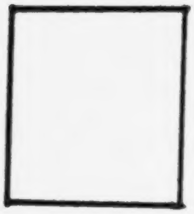
SOLUTION: 62 MINUTES.

GRADES: 2 - 4

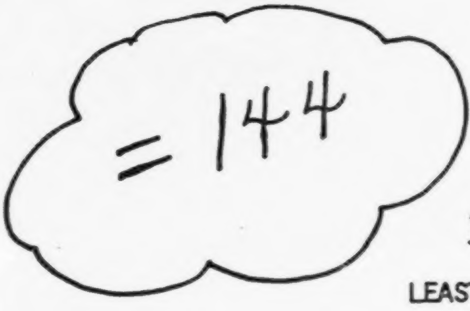
MATERIALS: 2 DIFFERENT TYPES OF CONES MADE OF BROWN PAPER.
4 FLAVORS OF ICE CREAM MADE FROM DIFFERENT COLORED PAPER CIRCLES.

SUGGESTED STRATEGY: USE THE OBJECTS AND MAKE ALL POSSIBLE COMBINATIONS.

SOLUTION: 8 SINGLE-DIP CONES ARE POSSIBLE.



HOW MANY WAYS CAN YOU FOLD A PIECE OF PAPER IN HALF?



WRITE 10 COMPUTATIONAL EXERCISES THAT HAVE THE ANSWER 144. USE +, -, X, AND ÷, AT LEAST ONCE IN EACH EXERCISE.

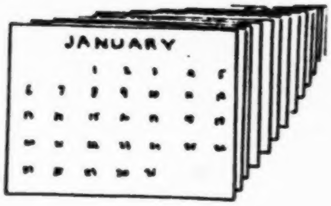
GRADES: 3 - 4

SOLUTION: LOCATE THE CENTER OF THE PAPER, P. ANY FOLD THROUGH POINT P WILL FOLD THE PAPER IN HALF. THERE IS AN ENDLESS NUMBER OF WAYS.

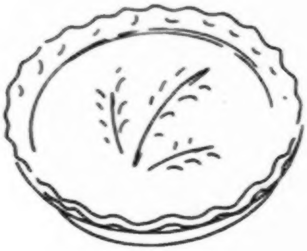
GRADES: 3 - 6

SOLUTION: MANY EXAMPLES ARE POSSIBLE, SUCH AS

$100 \div 2 \times 3 + 10 - 16 = 144.$



HOW MANY BIRTHDAYS WILL YOUR CLASS HAVE DURING THE NEXT YEAR?



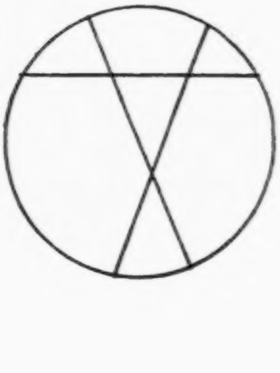
DRAW 3 STRAIGHT LINES TO CUT THE PIE INTO 7 PIECES.

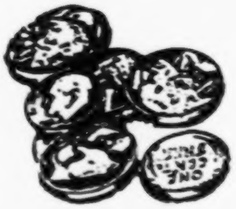
GRADES: 1 - 3

SOLUTION: ONE BIRTHDAY FOR EACH STUDENT (UNLESS YOU HAVE ONE ON FEBRUARY 29).

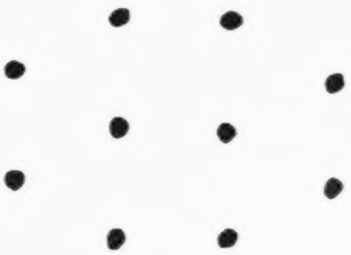
GRADES: 1 - 4

SOLUTION:





HOW MANY DIFFERENT WAYS CAN YOU HAVE 25¢?



YOU HAVE 10 POINTS. HOW MANY LINE SEGMENTS ARE NEEDED TO CONNECT EACH PAIR?

Grades: 3 - 4

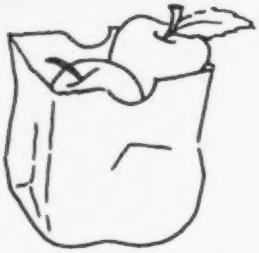
Suggested strategy: Make an organized list.

Solution: 8 ways.

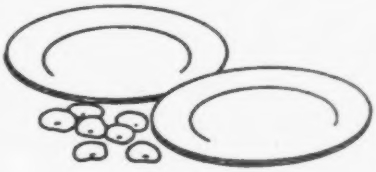
Grades: 3 - 5

Suggested strategies: Make a table and look for a pattern.

Soution: 45 line segments.



THERE ARE 3 APPLES IN THE SACK. HOW CAN YOU GIVE ONE APPLE TO EACH OF 3 GIRLS AND YET LEAVE ONE APPLE IN THE SACK?



HOW MANY WAYS CAN YOU PUT 7 LIMA BEANS ON TWO PLATES?

GRADES: 1 - 3

SOLUTION: GIVE THE 3RD GIRL THE APPLE IN THE SACK.

GRADES: K - 3

MATERIALS: PAPER PLATES AND DRIED BEANS.

SOLUTION: 6 WAYS: (6,1), (5,2), (4,3), (3,4), (2,5), (1,6)
8 WAYS IS YOU INCLUDE (7,0) AND (0,7).

PROBLEM SOLVING TASK CARDS
FOR UPPER ELEMENTARY GRADES

HOW MANY DIFFERENT BIKE LICENSE PLATES CAN YOU MAKE WITH THE NUMBERS 1, 2, 3, AND 4? DO NOT REPEAT DIGITS IN MAKING A LICENSE PLATE.

TINA WANTED TO SEAT GUESTS AT TABLES AT AN OUTDOOR BARBECUE. HOW MANY PEOPLE CAN SHE SEAT BY PUSHING TOGETHER 27 CARD TABLES THAT SEAT 1 PERSON ON EACH SIDE TO MAKE ONE LONG TABLE?

GRADES: 4 - 6

SUGGESTED STRATEGY: MAKE AN ORGANIZED LIST.

SOLUTION: LIST ALL POSSIBILITIES THAT START WITH 1.

1234 1243 1432

1324 1342 1423

CONTINUE UNTIL ALL POSSIBILITIES HAVE BEEN GIVEN.

24 LICENSE PLATES.

GRADES: 3 - 6

SUGGESTED STRATEGIES: SOLVE A SIMPLER PROBLEM.

MAKE A TABLE.

LOOK FOR A PATTERN.

SOLUTION:

TABLES	1	2	3	4 ...	27
PERSONS	4	6	8	10 ...	2(N+1) OR 56

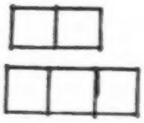
FIFTY SQUARES, EACH WITH EDGE 1 UNIT LONG, ARE PLACED SIDE BY SIDE IN A ROW. WHAT IS THE PERIMETER OF THE RESULTING FIGURE?

SUE COUNTED THE AMOUNT OF MONEY SHE HAD EARNED DURING THE WEEK AND WROTE IT ON A SLIP OF PAPER. THEN SHE BOUGHT SOME POSTCARDS FOR $0.56 AND A PEN THAT COST $2.80. AFTER HER SHOPPING SHE HAD $6.89 LEFT, BUT SHE HAD LOST THE SLIP OF PAPER. WHAT WAS THE AMOUNT WRITTEN ON THE PAPER?

GRADES: 4 - 8

SUGGESTED STRATEGIES: SOLVE A SIMPLER PROBLEM.
MAKE A TABLE.
LOOK FOR A PATTERN.

SOLUTION:



2 SQUARES - (2 x 2) + 2 = 6
3 SQUARES - (2 x 3) + 2 = 8
⋮
50 SQUARES - (2 x 50) + 2 = 102

PERIMETER - 102 UNITS

GRADES: 4 - 8

SUGGESTED STRATEGY: WORK BACKWARDS.

SOLUTION: FINAL   BEFORE PEN   BEFORE CARDS   ORIGINAL

$$\boxed{\$6.89} \rightarrow \boxed{\$6.89 + 2.80} \rightarrow \boxed{\$9.69 + 0.56} \rightarrow \boxed{\$10.25}$$

CARLOTTA READ THAT 2 OUT OF EVERY 5 PEOPLE HAVE BLUE EYES. SHE DECIDED TO USE THIS IDEA TO PREDICT HOW MANY PUPILS IN HER CLASS OF 30 WOULD HAVE BLUE EYES. HOW MANY DO YOU THINK SHE PREDICTED?

ANN, BEN, CARLOS, AND DINA LIKE DIFFERENT TYPES OF BOOKS: HUMOR, MYSTERY, SPORTS, AND ADVENTURE. ONE OF ANN'S CLASSMATES IN THE GROUP LIKES MYSTERY BOOKS BEST. CARLOS AND DINA DO NOT LIKE ADVENTURE BOOKS. BEN'S FAVORITE TYPE OF BOOK IS SPORTS. DINA DID LIKE HUMOR BOOKS BUT HAS CHANGED HER FAVORITE. WHAT IS DINA'S FAVORITE BOOK?

GRADES: 4 - 6

SUGGESTED STRATEGY: MAKE A TABLE.

SOLUTION: SHE PREDICTED THAT 12 PUPILS IN HER CLASS HAVE BLUE EYES.

GRADES: 5 - 8

SUGGESTED STRATEGIES: USE LOGICAL REASONING.
MAKE A TABLE AND USE ELIMINATION.

SOLUTION: DINA'S FAVORITE BOOKS ARE MYSTERY BOOKS.

A RICH NEIGHBOR GAVE SAMANTHA A CHOICE OF $600 FOR A 16-DAY HOUSE-PAINTING JOB OR 1 CENT THE 1ST DAY, TWICE AS MUCH THE 2ND DAY, AND SO ON, DOUBLING THE AMOUNT EACH DAY. WHICH ARRANGEMENT SHOULD SAMANTHA CHOOSE?

TENNIS BALLS ARE PACKED TIGHTLY WITH 3 IN A CAN. IS THE CAN TALLER THAN THE DISTANCE AROUND IT OR THE REVERSE?

GRADES: 4 - 6

SUGGESTED STRATEGIES: MAKE A TABLE.
LOOK FOR A PATTERN.

SOLUTION: SAMANTHA SHOULD CHOOSE THE SECOND METHOD OF PAYMENT. TOTAL FOR THAT METHOD WILL BE $655.35.

GRADES: 5 - 8

SUGGESTED STRATEGY: EXPERIMENTATION

SOLUTION: THE DISTANCE AROUND IS GREATER THAN THE HEIGHT OF THE CAN.

$$\pi d > 3d$$

HOW LONG A PIECE OF WIRE IS NEEDED TO MAKE ALL OF THE PAPER CLIPS IN A NEW BOX OF PAPER CLIPS?

JIM GOT INTO AN ELEVATOR. HE WENT DOWN 5 FLOORS, UP 6 FLOORS, AND DOWN 7 FLOORS. HE WAS THEN ON THE SECOND FLOOR. WHERE DID HE GET ON?

GRADES: 4 - 8

SUGGESTED STRATEGY: EXPERIMENTATION

SOLUTION: MEASURE ONE OF THE PAPER CLIPS IN YOUR CLASSROOM AND MULTIPLY BY THE NUMBER IN A NEW BOX.

GRADES: 4 - 8

SUGGESTED STRATEGY: DRAW A PICTURE OR DIAGRAM.

SOLUTION: JIM GOT ON THE ELEVATOR ON THE 8TH FLOOR.

SOME CHILDREN ARE SEATED AT A LARGE ROUND TABLE. THEY PASS AROUND A BOX OF CANDY CONTAINING 25 PIECES. TED TAKES THE FIRST PIECE. EACH CHILD TAKES ONE PIECE OF CANDY AS THE BOX IS PASSED AROUND. TED ALSO GETS THE LAST PIECE OF CANDY, AND HE MAY HAVE MORE THAN THE FIRST AND LAST PIECES. HOW MANY CHILDREN COULD BE SEATED AROUND THE TABLE?

AT WHAT TIME IS THE SUM OF THE DIGITS ON A DIGITAL CLOCK THE GREATEST?

GRADES: 5 - 9

SUGGESTED STRATEGIES: DRAW A PICTURE.
USE OBJECTS.
ACT IT OUT.

SOLUTION: 2, 3, 4, 6, 8, 12, OR 24 CHILDREN.

GRADES: 3 - 6

SUGGESTED STRATEGY: GUESS AND CHECK.

SOLUTION: 9:59

PS-28

BULLETIN BOARDS

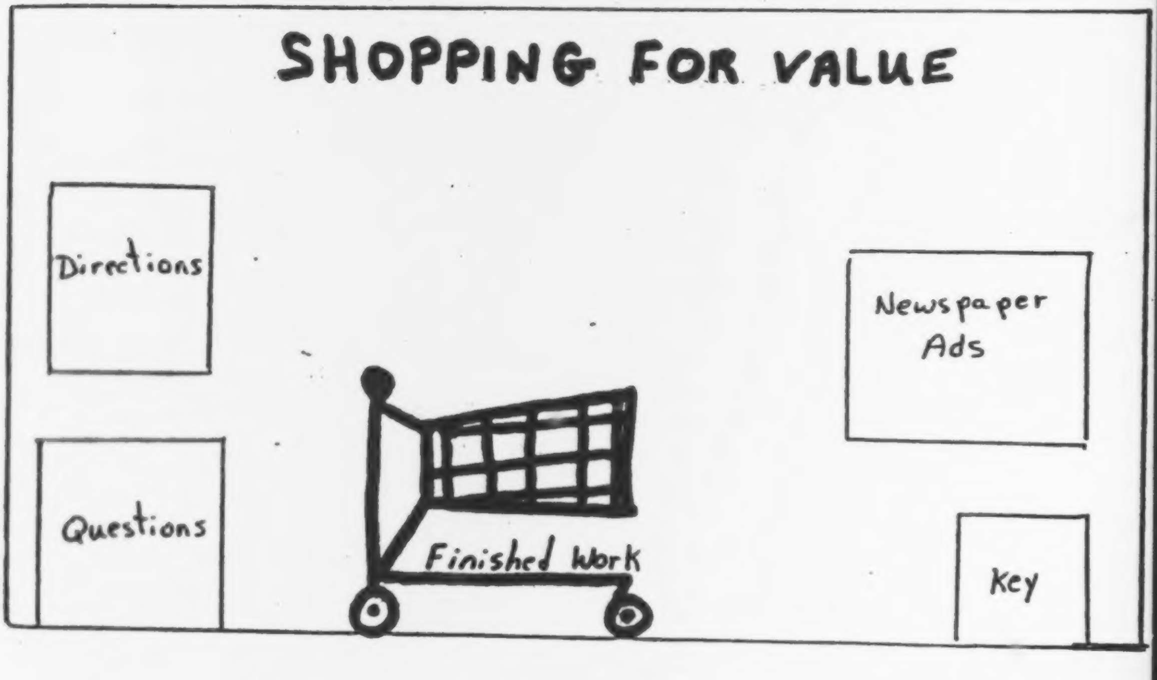
SHOPPING FOR VALUE

PURPOSE: To solve reading problems using information found in a newspaper ad. To use the calculator to solve reading problems involving money.

MATERIALS:
1. construction paper pockets or envelopes labeled Directions, Newspaper Ads, Key, and Questions
2. Picture of shopping card drawn on tag board and cut out. This should be labeled "Finished Work."
3. newspaper ads from local stores and restaurants
4. calculators

DIRECTIONS: Put a set of directions in the proper pocket on the board. Put question sheets in the proper pocket. Place local ads in the proper pocket. Make out a key for your questions so that the student may check his own work. This goes in the pocket labeled "KEY."

ADAPTATIONS: Since this is an activity-oriented board, it should be made as sturdy as possible. Instead of making this a bulletin board, you could put it on heavy poster board and put it in a learning center.



MATH MADNESS

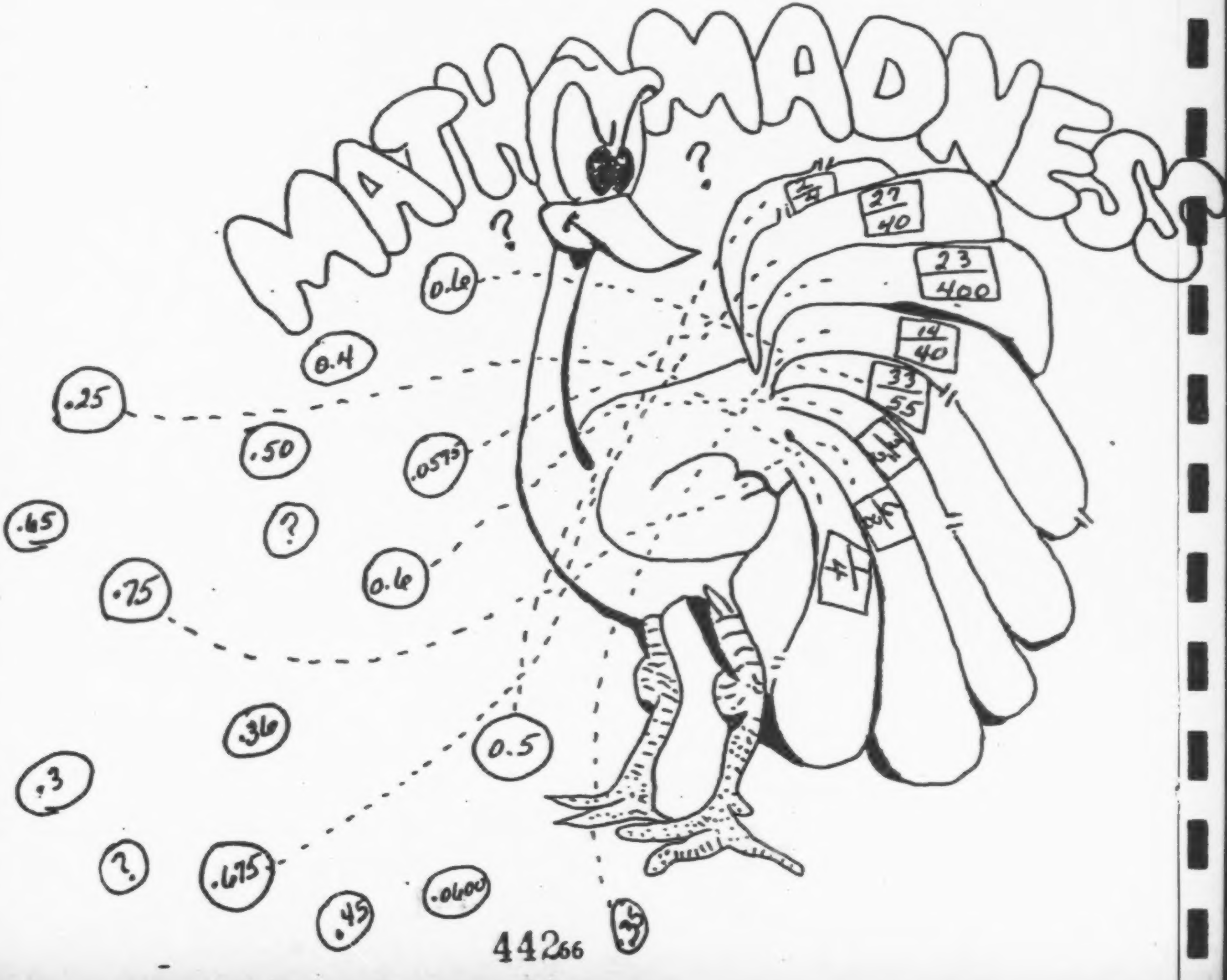
PURPOSE: To give practice in solving word problems.

MATERIALS:

1. Picture of crazy looking bird with different colored feathers (see sample below).
2. Colored yarn to match tail feathers.
3. Strips of construction paper with word problems written on them. These are placed on tail feathers and can be changed often.
4. Circles made from tag board with answers to problems (these are placed all over the board).

DIRECTIONS: The student connects all the word problems to their answers with yarn. After he has finished, he has tied the crazy bird up. The problems on the tail feathers and answers may be changed often.

NOTE: Since this is an activity-oriented bulletin board, it should be made as sturdy as possible.



THE TWELVE DAYS OF CHRISTMAS

PURPOSE: To reinforce problem solving using addition and subtraction.
To use a calculator in problem solving.

MATERIALS:
1. Christmas tree (can be made out of tagboard, paper, or wood)
2. cut outs of the gifts mentioned in the song (see following page)
3. copies of the song
4. sheet with questions concerning the song (see examples below)
5. calculators

DIRECTIONS: After cutting out the figures, arrange them so that they are in the shape of a Christmas. Give the students a calculator and a sheet of questions. Also give the class copies of the song so that they can sing along together after the questions are completed and checked.

Sample Questions

1. How many more "ladies dancing" are there than "maids-a-milking?"_______

2. How many more "swans-a-swimming" are there than "turtle doves?"________

3. How many gifts did the lady receive on: a) the third day ____________

b) the sixth day ____________

c) the ninth day ____________

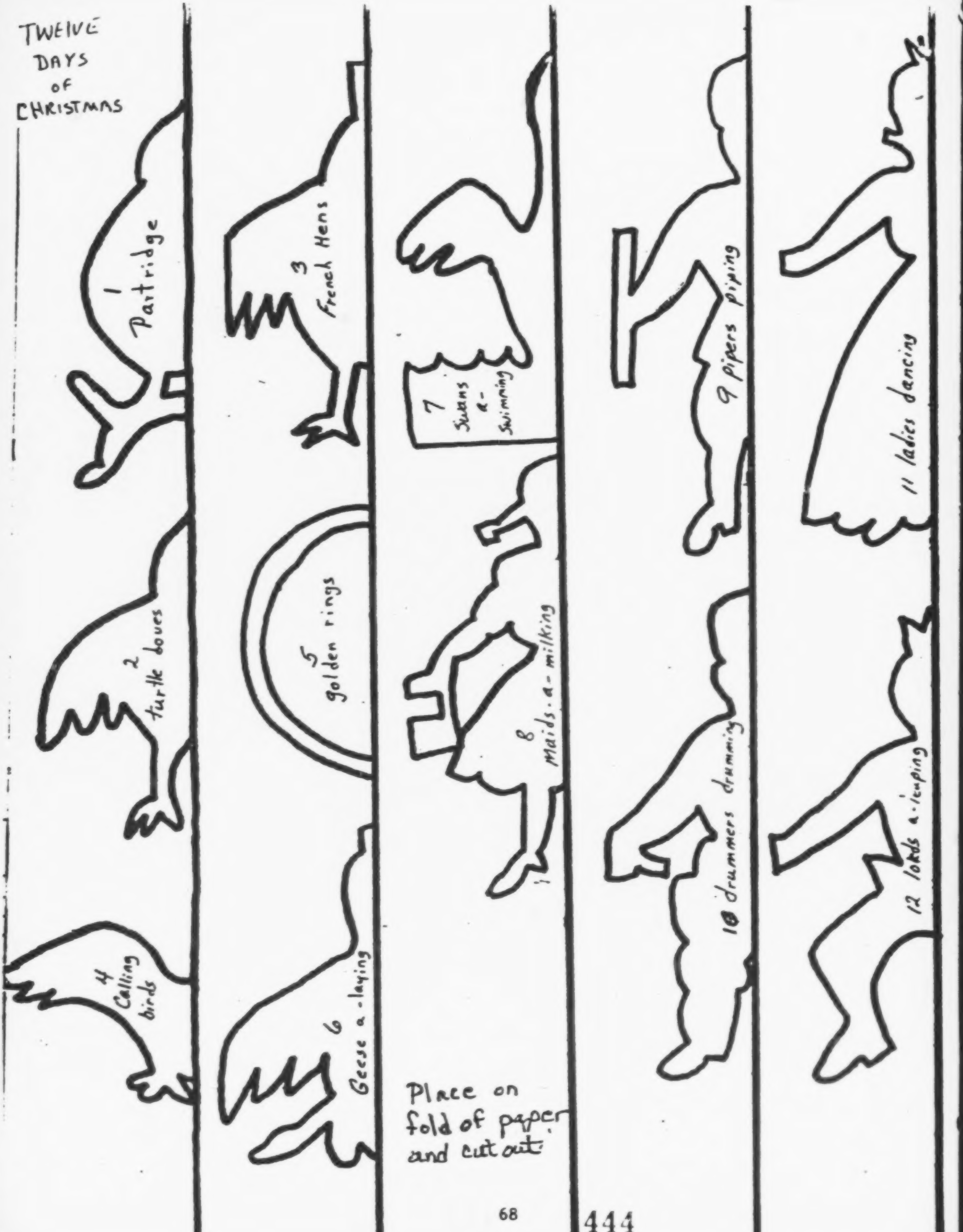
d) the twelfth day __________

5. How many gifts did the lady receive in all? __________________________

6. If the lady returned one gift a day, starting January 1, 1987, when would she return the last gift? ____

NOTE: You may make your questions easier or more difficult depending on the grade level of your students.

TWELVE DAYS OF CHRISTMAS


1 Partridge
2 turtle doves
3 French Hens
4 Calling birds
5 golden rings
6 Geese a-laying
7 Swans a-swimming
8 Maids-a-milking
9 pipers piping
10 drummers drumming
11 ladies dancing
12 lords a-leaping

Place on
fold of paper
and cut out

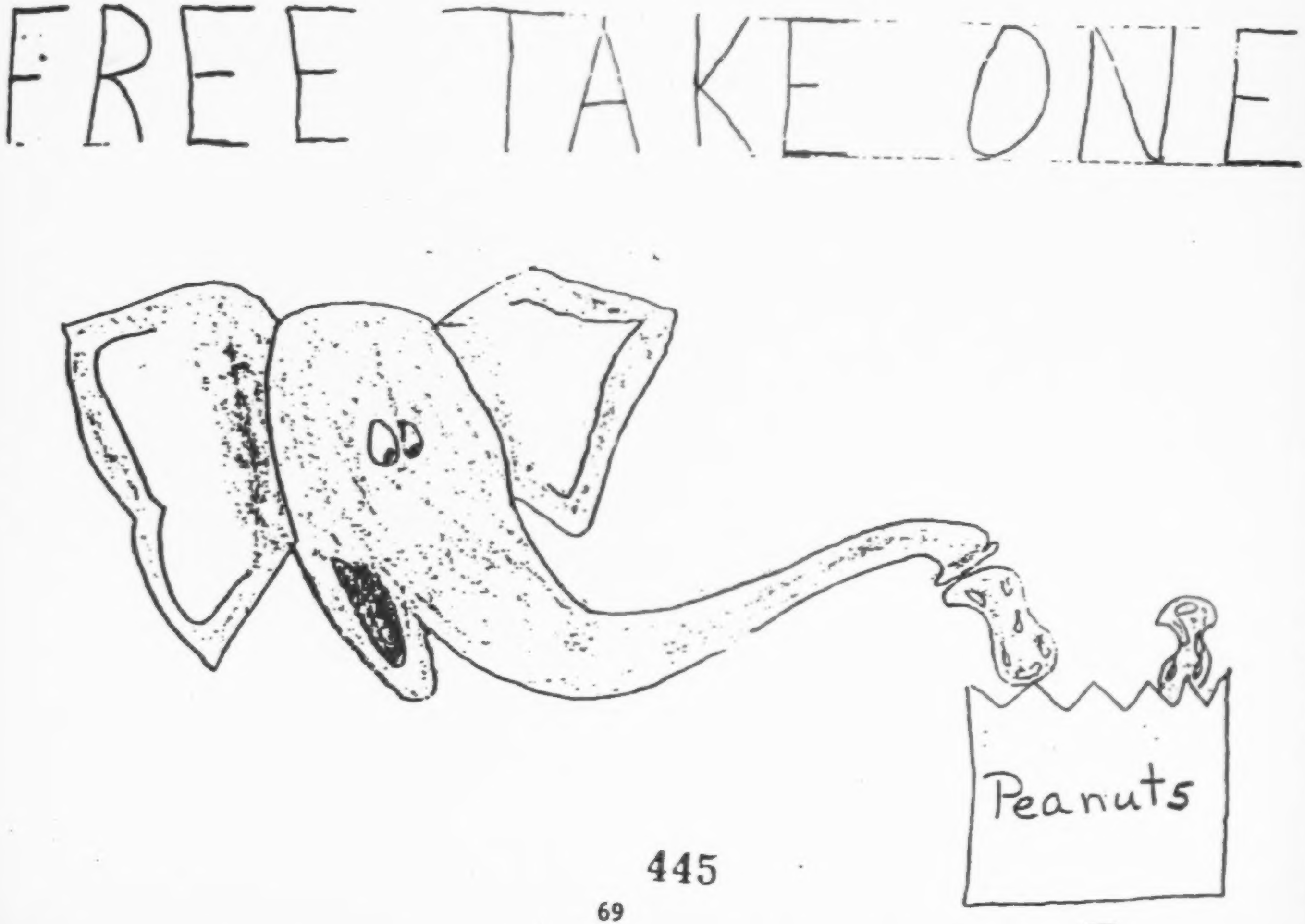
FREE TAKE ONE

PURPOSE: To reinforce problem solving skills.
To give practice in writing word problems with all the essential information.

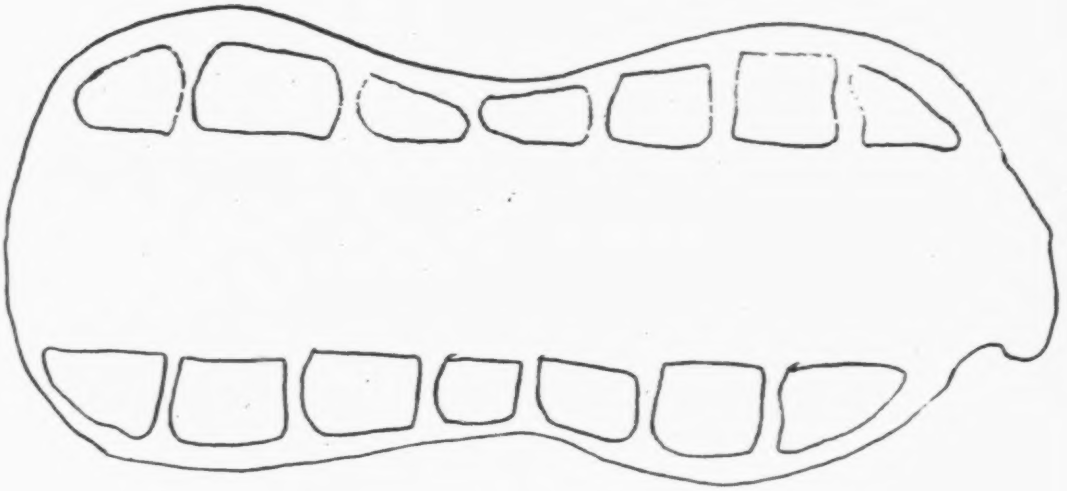
MATERIALS:
1. Poster board with picture of elephant on it.
2. Paper bag labeled "peanuts."
3. Construction paper peanuts for students to write problems on.

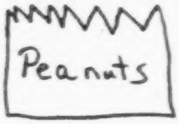
DIRECTIONS: Students write their own word problems on a piece of paper with the solution included. The teacher "edits" the problem before the student writes it on a peanut for the bag. Students are allowed during free or enrichment time to get a peanut and work the problem. The author of the problem checks the work before the worked peanut is turned in to the teacher.

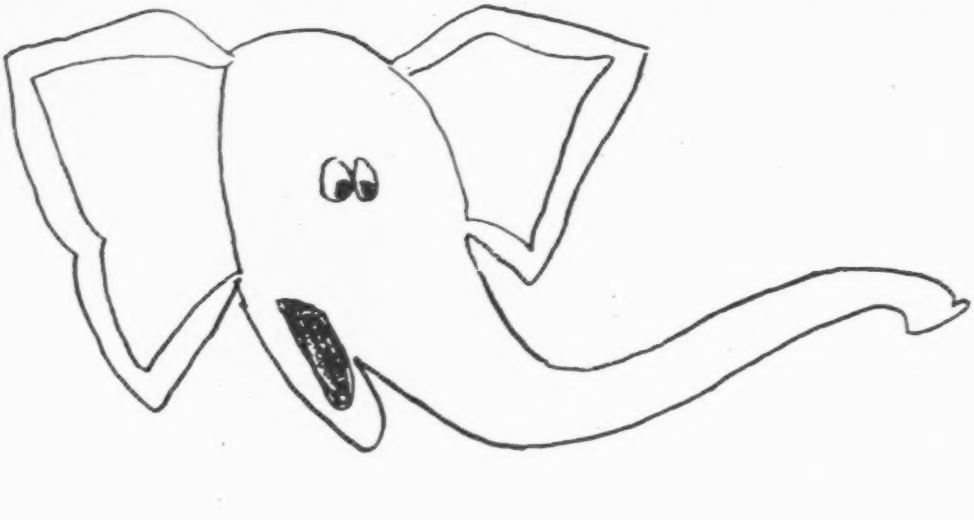
NOTE: Since this is an activity-oriented board, it should be made as sturdy as possible.



FREE TIME UNIT





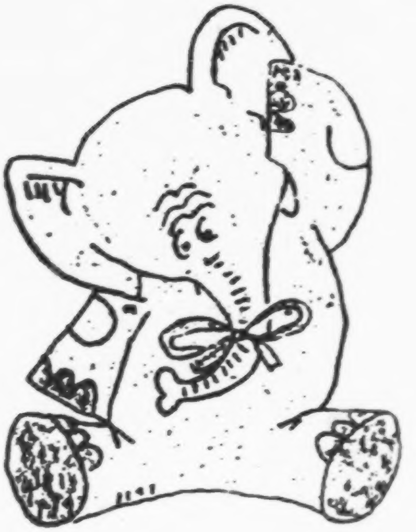


TONGUE TIED TWISTERS

PURPOSE: To give practice in writing word problems.

MATERIALS: Poster board with a picture of a puzzled animal on it and a list of twisters.

DIRECTIONS: Students write their own problems using the words of a twister. The problems may be placed in an envelope and left for other students to solve or the problems may be shared with the class by having students read the problems to the class. You might have each student read a problem written by another student. The class may work on the solution or a group of students might solve the problem.



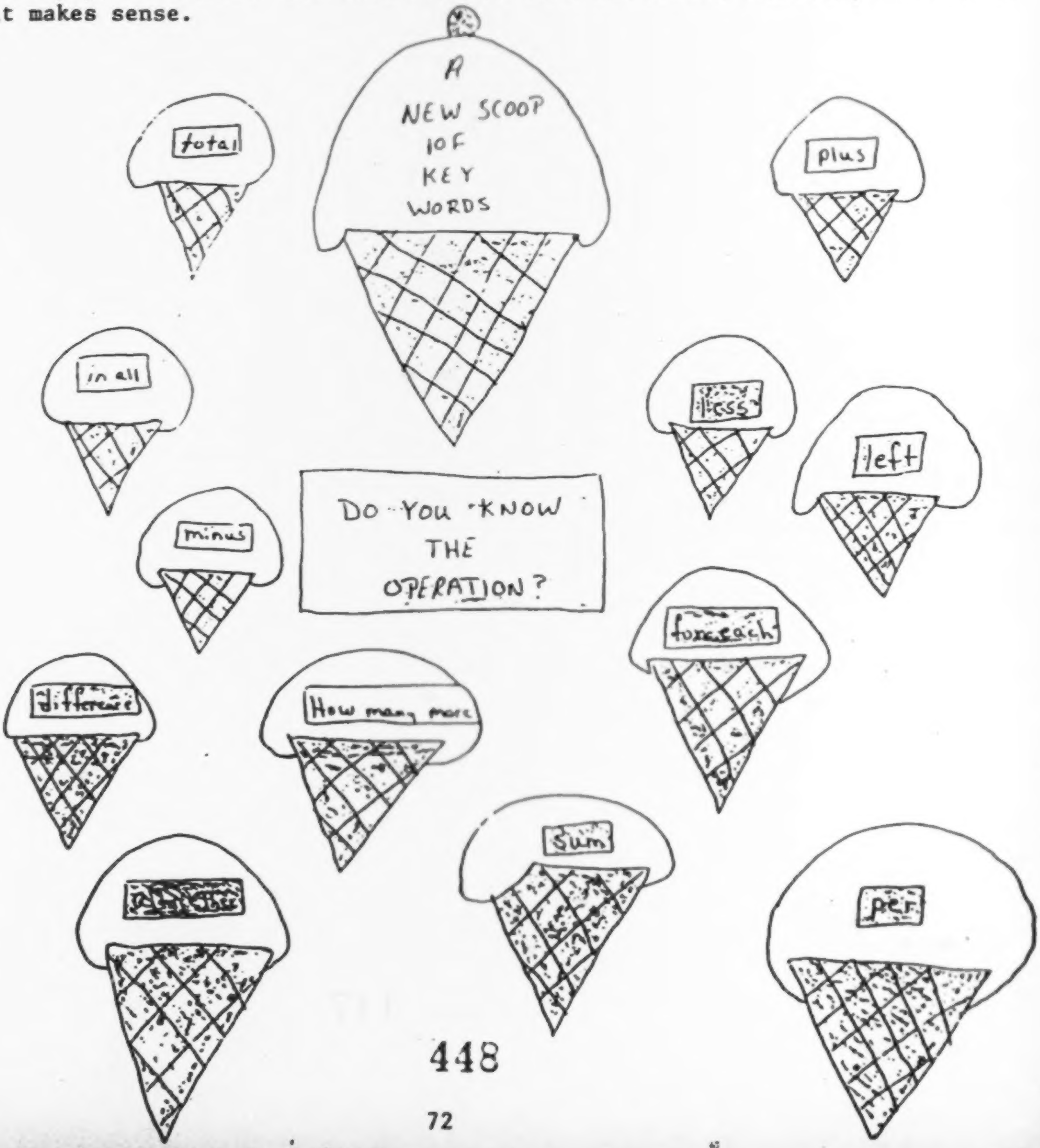
1. SAM SUPER SOLVER SOLVED.........
2. RHONDA RAWLY ROWED..............
3. PERFECT PETER PLAYED............
4. SIMPLE SUE SWALLOWED............
5. ANGRY ANDREW ATE................
6. GORDY GREW GORGEOUS.............
7. BILLY BUCK BATTED...............
8. BEULAH BUTTON BLEW..............

KEY OR CLUE WORDS FOR PROBLEM SOLVING

PURPOSE: To introduce key words of problem solving in the areas of addition, subtraction, multiplication, and division.*

MATERIALS:
1. brown construction paper cut out into the shape of cones
2. cotton for scoops of ice cream
3. colored strips of paper with key words written on them (see sample below)
4. colored strips of paper with key words and their operation written on them to go into the "DO YOU KNOW THE OPERATION?" envelope
5. large envelope

*Remind students that sometimes these words do not mean to add, subtract, multiply, or divide. Students need to read the answer carefully and see if it makes sense.


A NEW SCOOP OF KEY WORDS
total
plus
in all
less
left
minus
DO YOU KNOW THE OPERATION?
for each
difference
How many more
sum
per

LUCKY IDEAS

PURPOSE: To remind students of possible strategies to use in problem solving.

MATERIALS: Green construction paper cutouts of shamrocks and four-leaf clovers.

DIRECTIONS: Write a problem solving strategy on each shamrock or four-lef clover that you place of the bulletin board.

VARIATION: Have an envelope of problems with each problem written on a shamrock or four-leaf clover. The more difficult problems are written on the four-leaf clovers. A student selects a problem and chooses one of the strategies to use in solving it. The student writes the problem number and the strategy on a strip of paper and places it in the envelope for the class to use in a problem solving discussion. The student should also write their name on the strip of paper so he will get credit for selecting the strategy.

LUCKY IDEAS